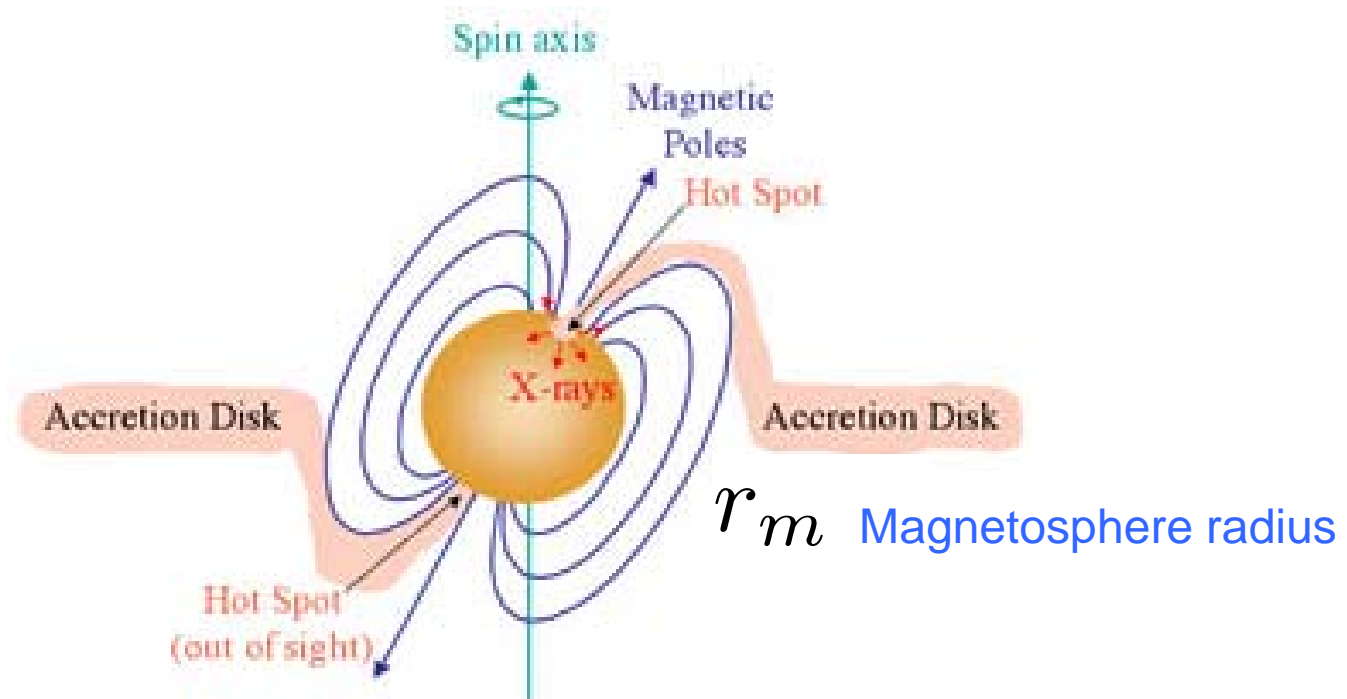


# Magnetic Star - Disk Interaction

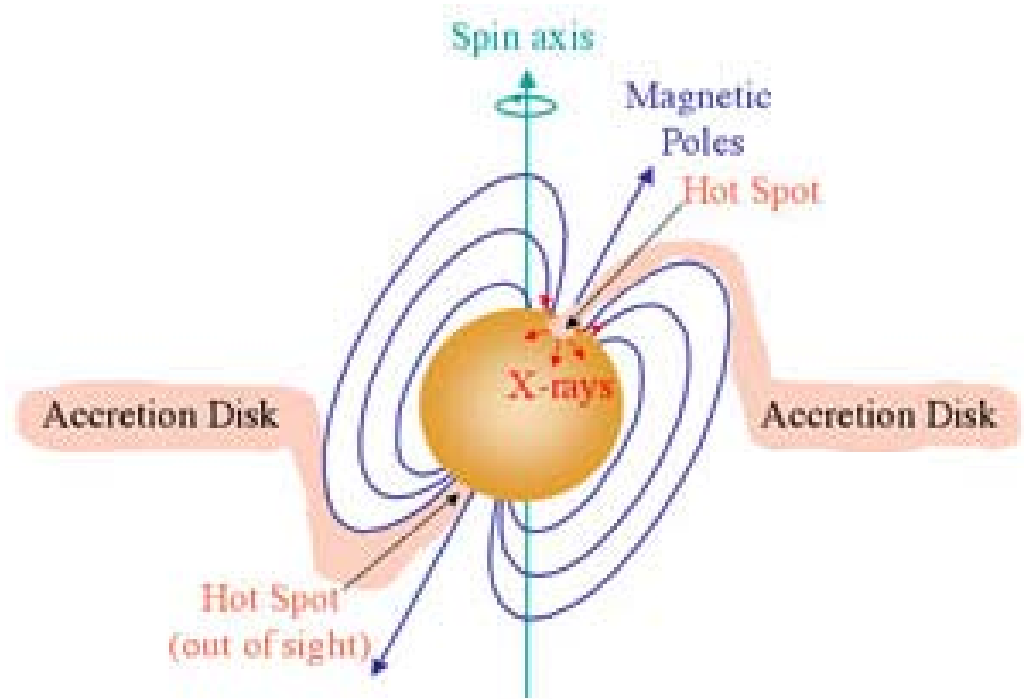
**Dong Lai**  
Cornell University

Tsinghua IAS, 5/18/2014

# Magnetic Star - Disk Interaction: Basic Picture



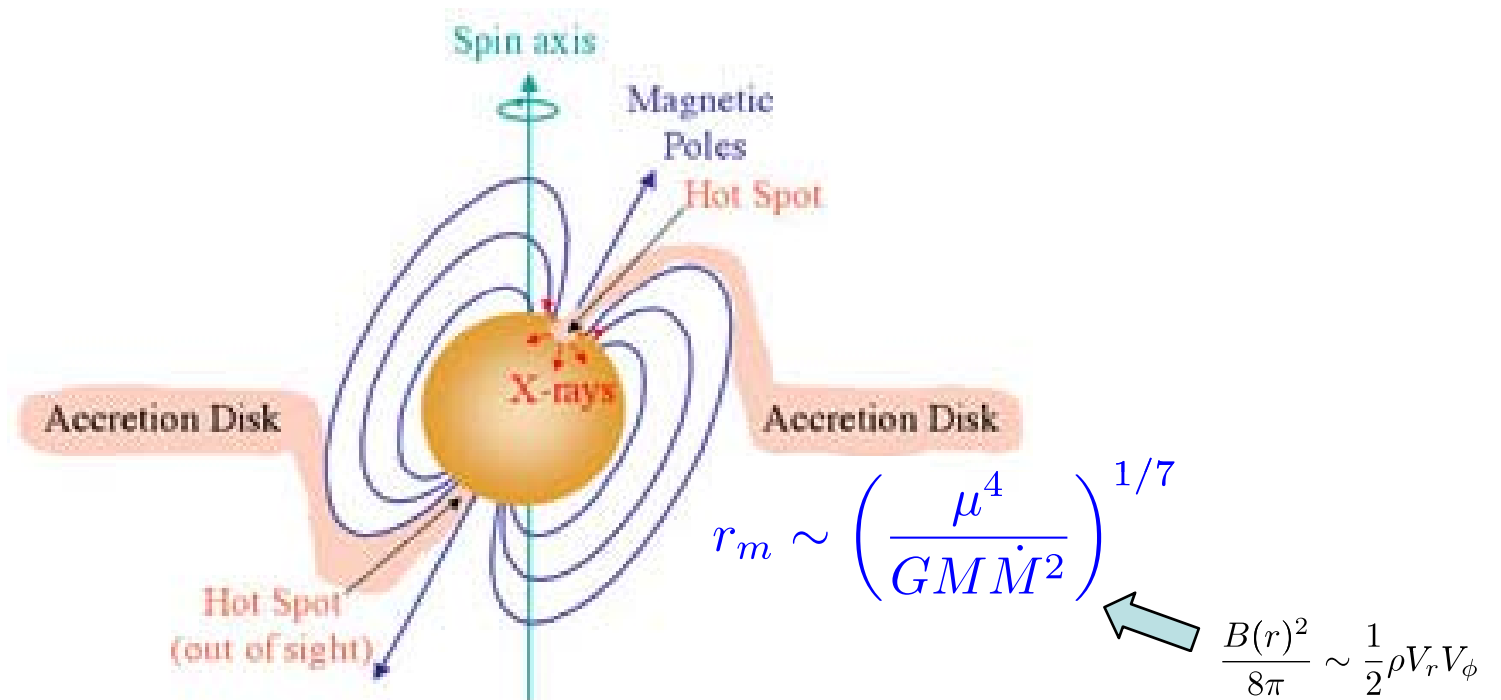
# Magnetic Star - Disk Interaction: Basic Picture



Magnetosphere boundary radius:  
 For spherical accretion:

$$\frac{B^2}{8\pi} \sim \frac{1}{2}\rho V^2 \quad \text{with} \quad B \sim \frac{\mu}{r^3}, \quad V \sim \sqrt{\frac{GM}{r}}, \quad \rho = \frac{\dot{M}}{4\pi r^2 V} \quad \rightarrow \quad r_m \sim \left( \frac{\mu^4}{GM\dot{M}^2} \right)^{1/7}$$

# Magnetic Star - Disk Interaction: Basic Picture



Accreting x-ray pulsars:

$$B_\star \sim 10^{12} \text{G}, \quad r_m \sim 10^2 R_\star$$

Accreting ms pulsars:

$$B_\star \sim 10^8 \text{G}, \quad r_m \sim (\text{a few}) R_\star$$

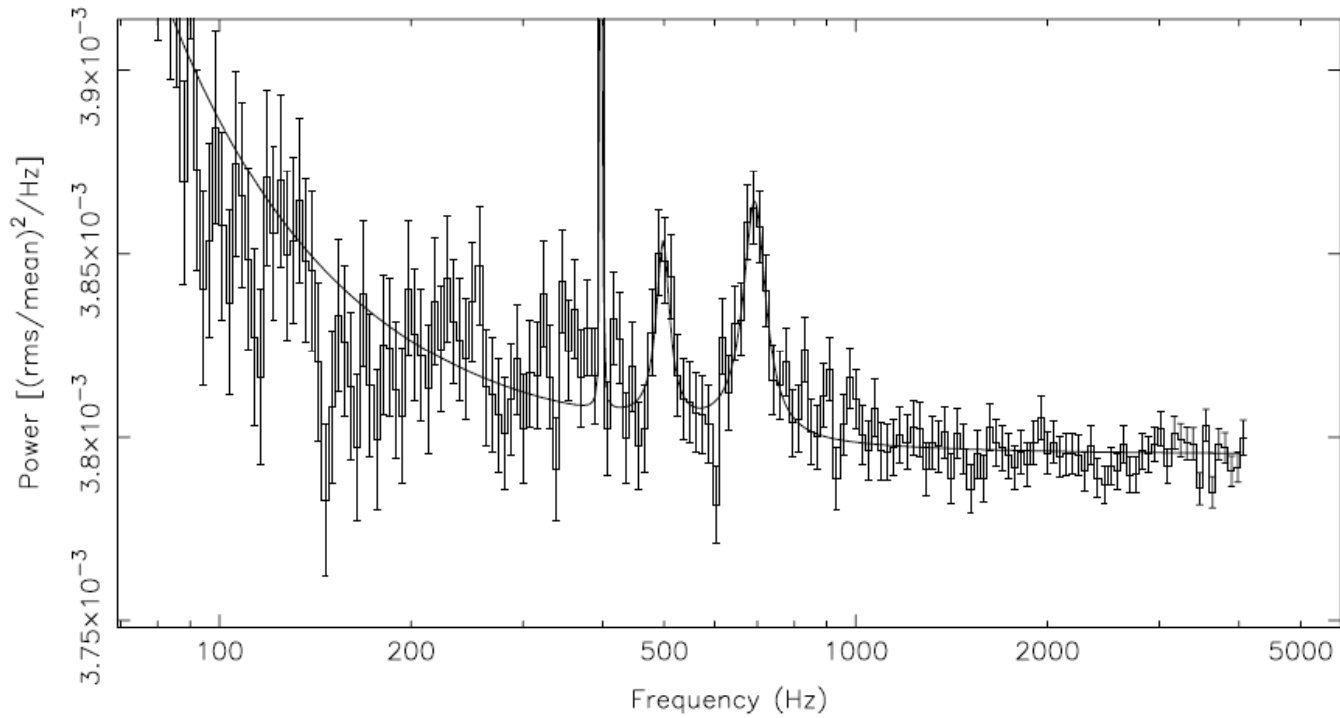
Accreting WDs (Intermediate polars):

$$B_\star \sim 10^7 \text{G}, \quad r_m \sim 10 R_\star$$

Protostars (Classical T Tauri stars):

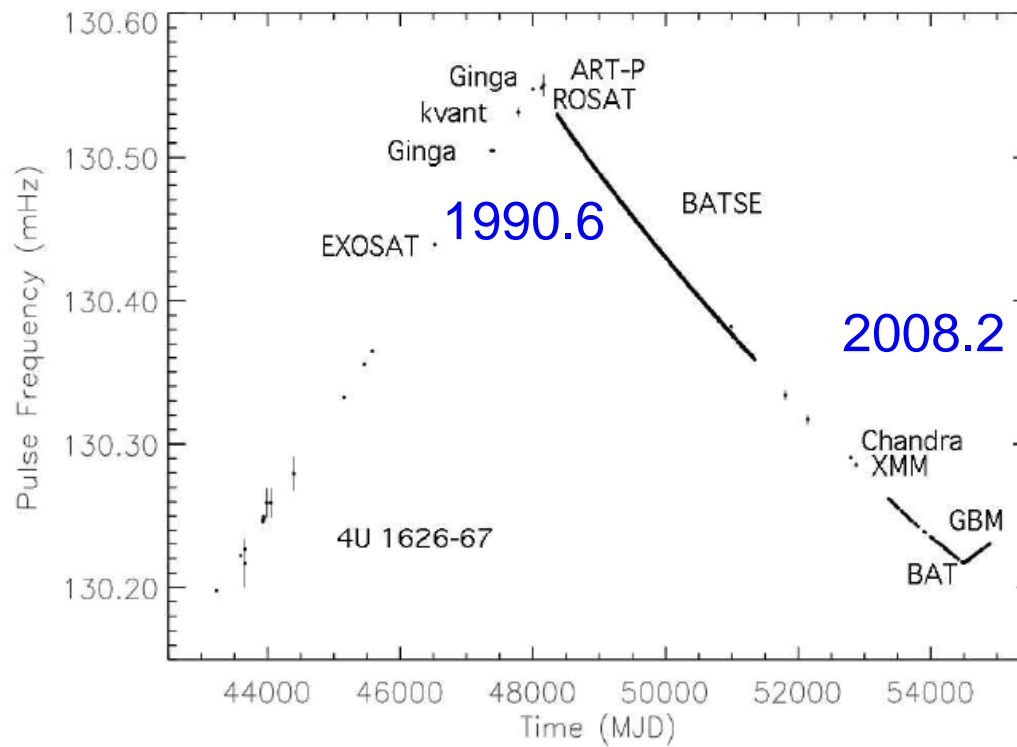
$$B_\star \sim 10^3 \text{G}, \quad r_m \sim (\text{a few}) R_\star$$

# Application: QPOs in Accreting Millisecond Pulsars



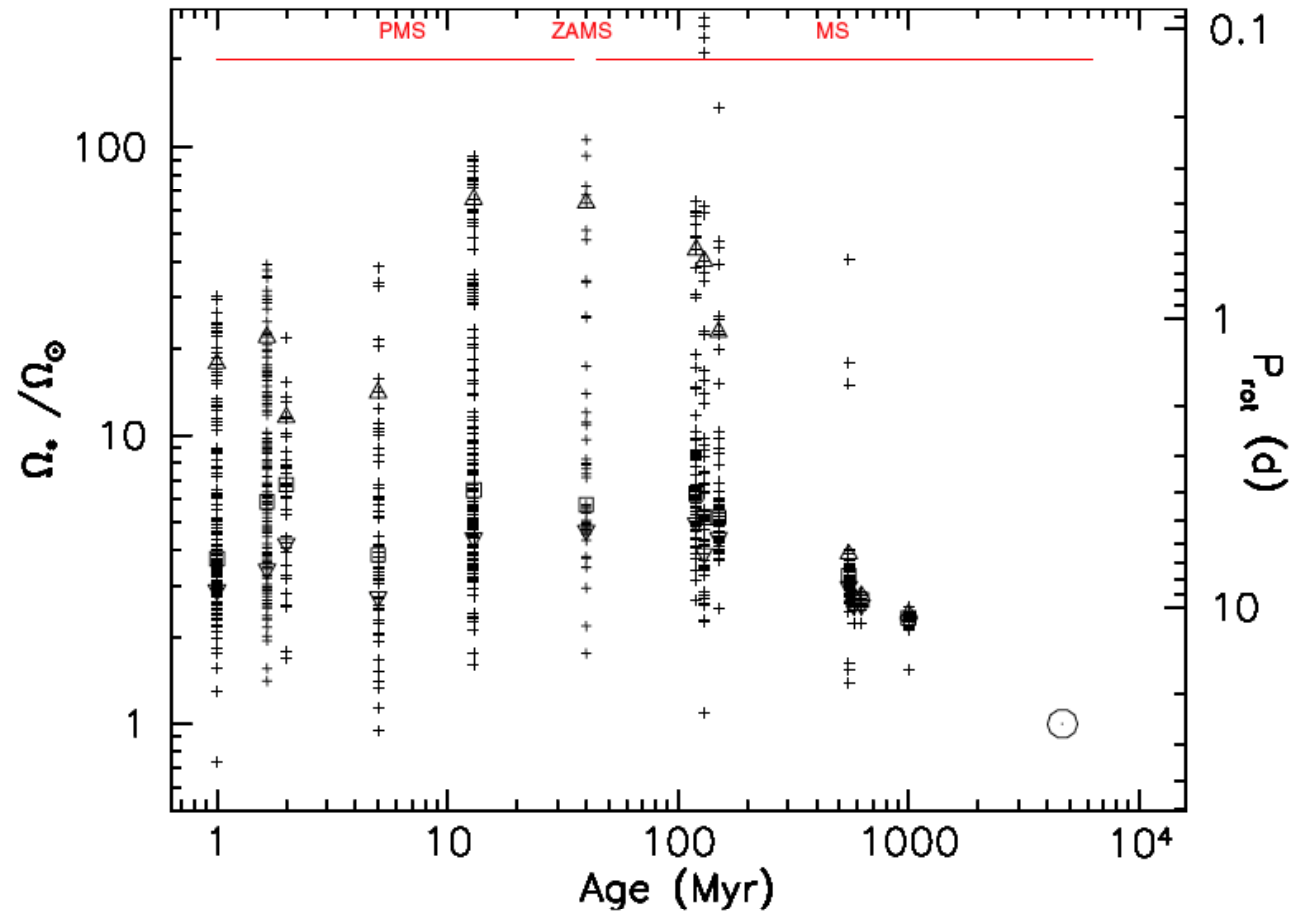
Van der Klis 2005

# Application: Spinup/Spindown of Accreting X-ray pulsars



Camero-Arranz et al. 2010,2012

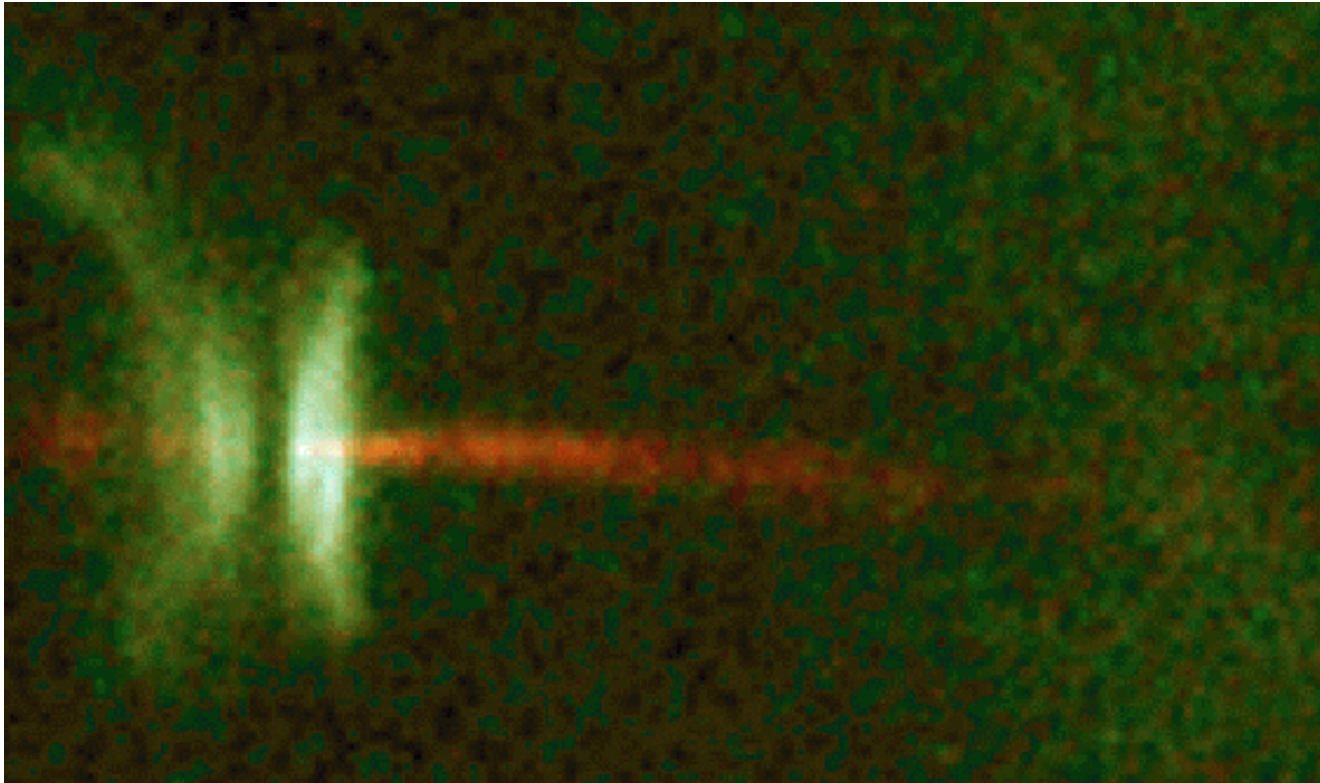
# Application: Rotation of Protostars: why 10% of breakup?



Gallet & Bouvier 2013

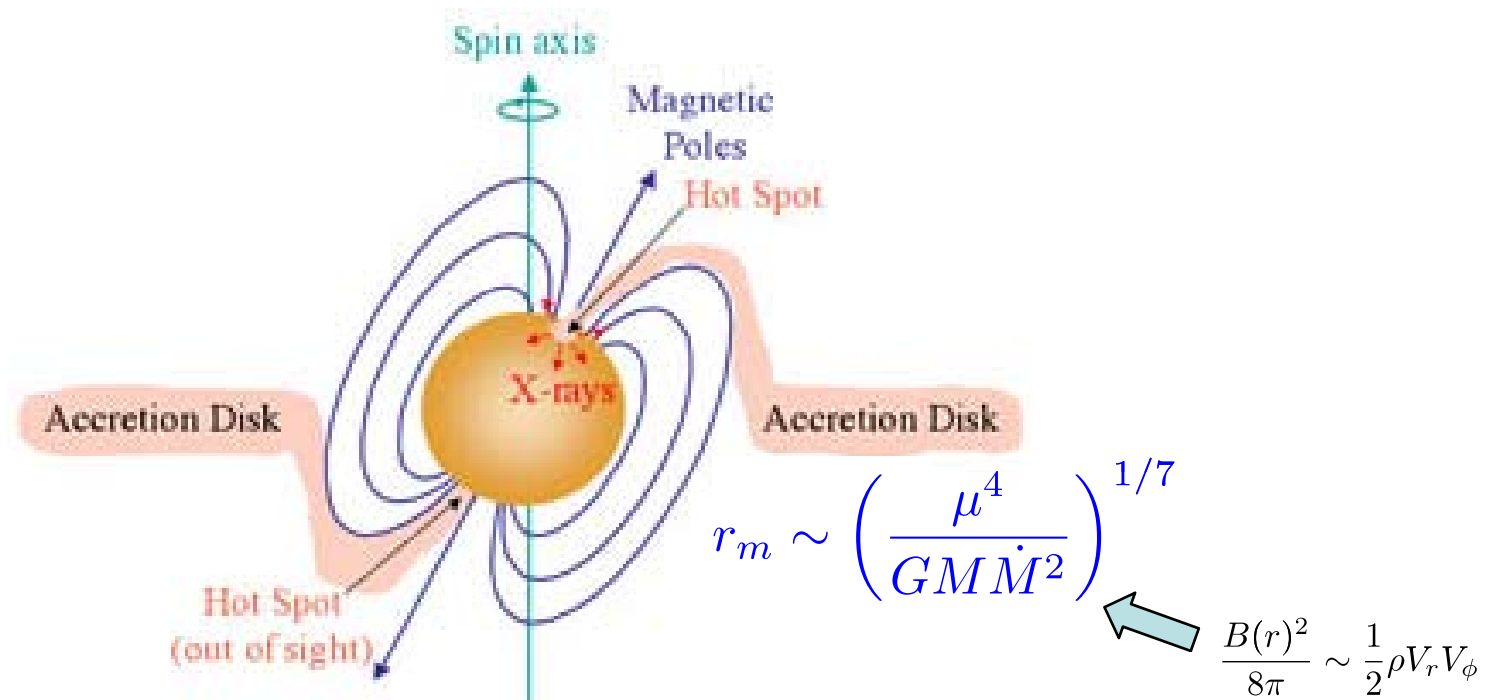
**Application:**

**Outflows/jets from protostars:**





# Magnetic Star - Disk Interaction: Basic Picture



Accreting x-ray pulsars:

$$B_\star \sim 10^{12} \text{G}, \quad r_m \sim 10^2 R_\star$$

Accreting ms pulsars:

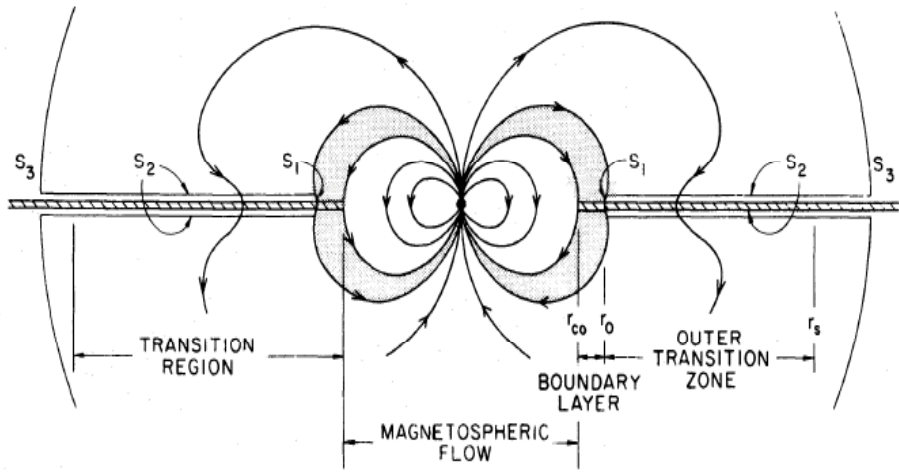
$$B_\star \sim 10^8 \text{G}, \quad r_m \sim (\text{a few}) R_\star$$

Accreting WDs (Intermediate polars):

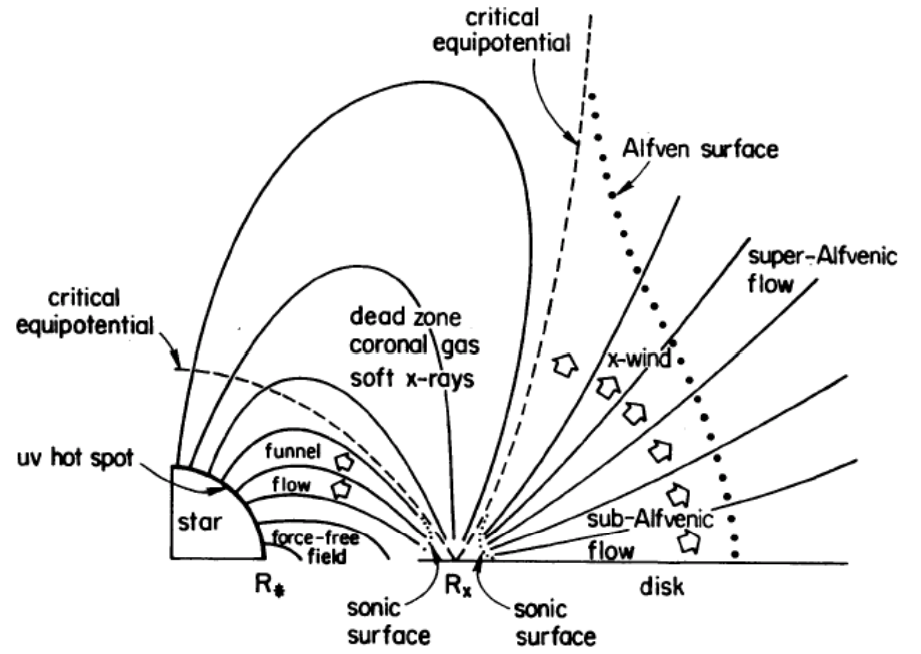
$$B_\star \sim 10^7 \text{G}, \quad r_m \sim 10 R_\star$$

Protostars (Classical T Tauri stars):

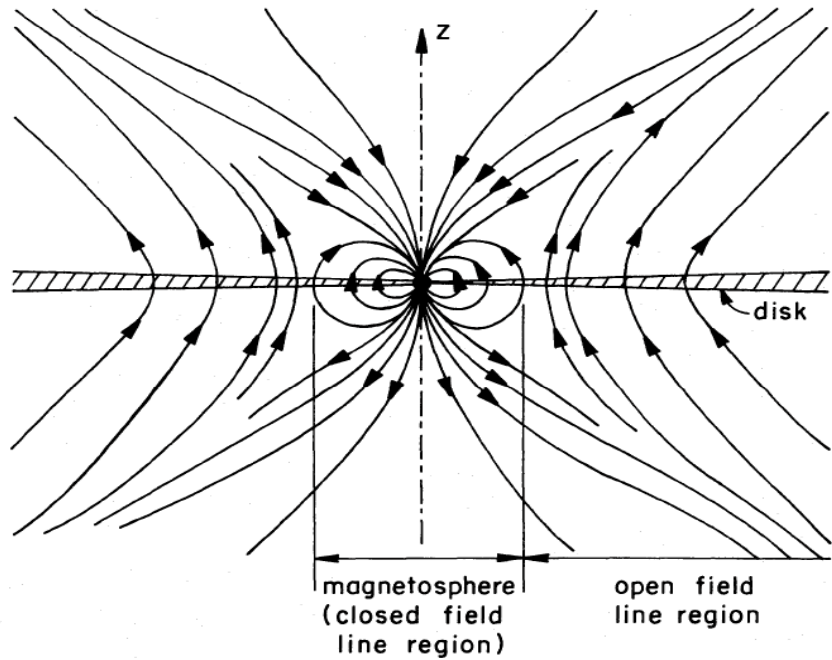
$$B_\star \sim 10^3 \text{G}, \quad r_m \sim (\text{a few}) R_\star$$



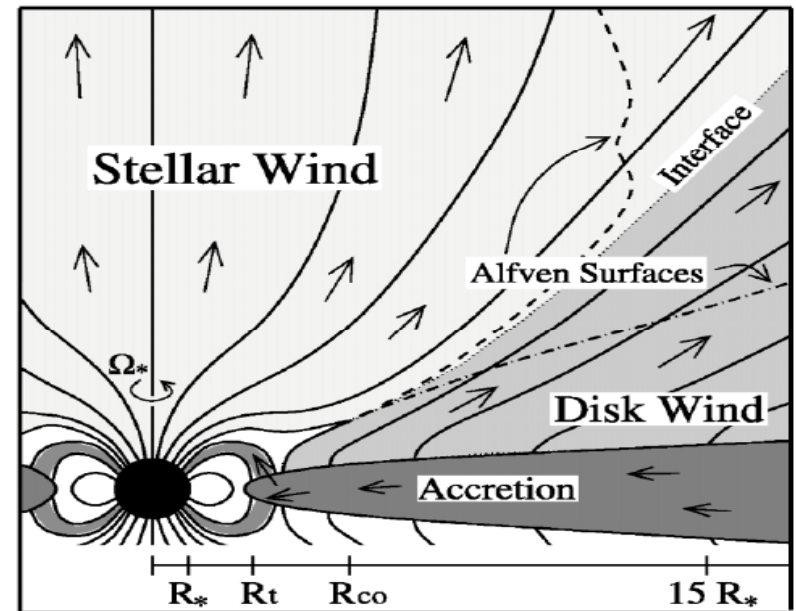
Ghosh & Lamb 1979



Shu et al. 1994



Lovelace et al. 1995



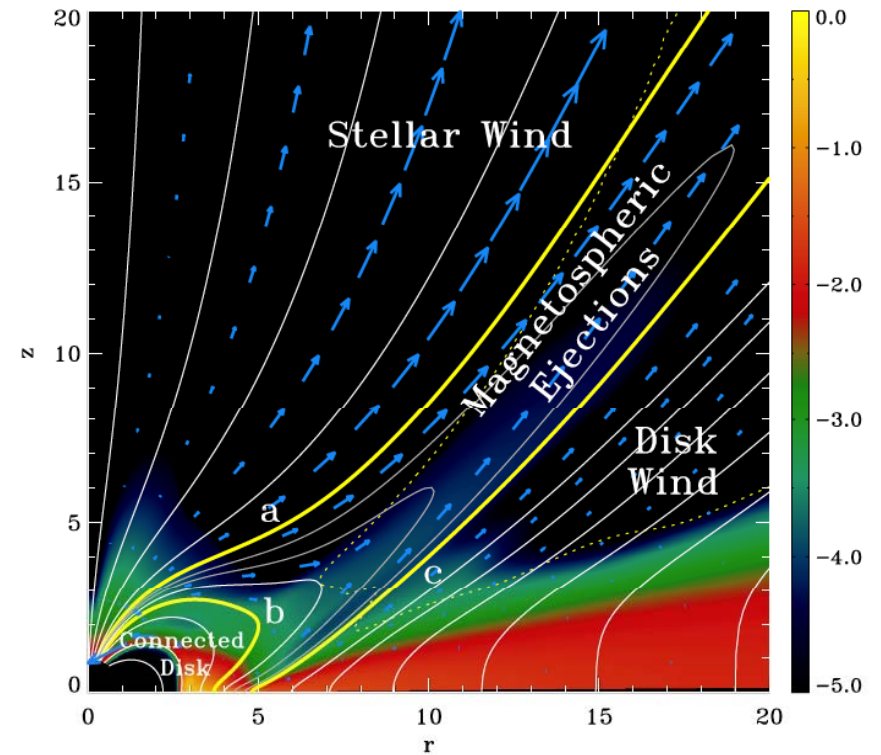
Matt & Pudritz 2005

# Simulations...

Hayashi, Shibata & Matsumoto, Miller & Stone, Goodson, Winglee & Bohm, Fendt & Elastner, Matt et al, Romanova, Lovelace, Kulkarni, Long, Lii et al, Zanni & Ferreira, .....



Romanova et al. 12



Zanni & Ferreira '13

# Outstanding Issues:

(uncertainties, possible applications...)

- Magnetosphere boundary vs disk inner radius
- Magnetic linkage between star and disk  
quasi-cyclic behavior
- Magnetosphere outflows
- Torque on the star: Spinup/spindown
- Misaligned dipole: Effect on disks
- Spin-disk misalignment: Application to exoplanets

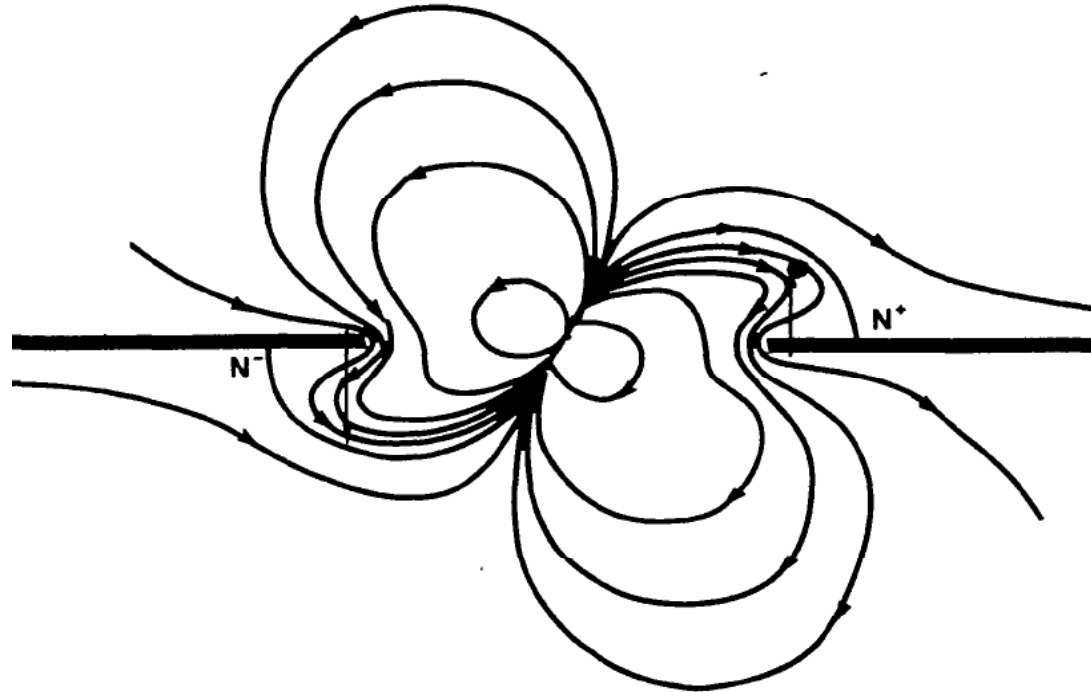
# Dipole Field Invaded by a Conducting Disk

Disk is a good conductor:

Diffusion of B into disk:

$$t_{\text{diff}} \sim \frac{H^2}{\eta} \sim \frac{1}{\alpha\Omega}$$

$$\eta \sim \nu = \alpha H c_s \quad (\text{MRI})$$



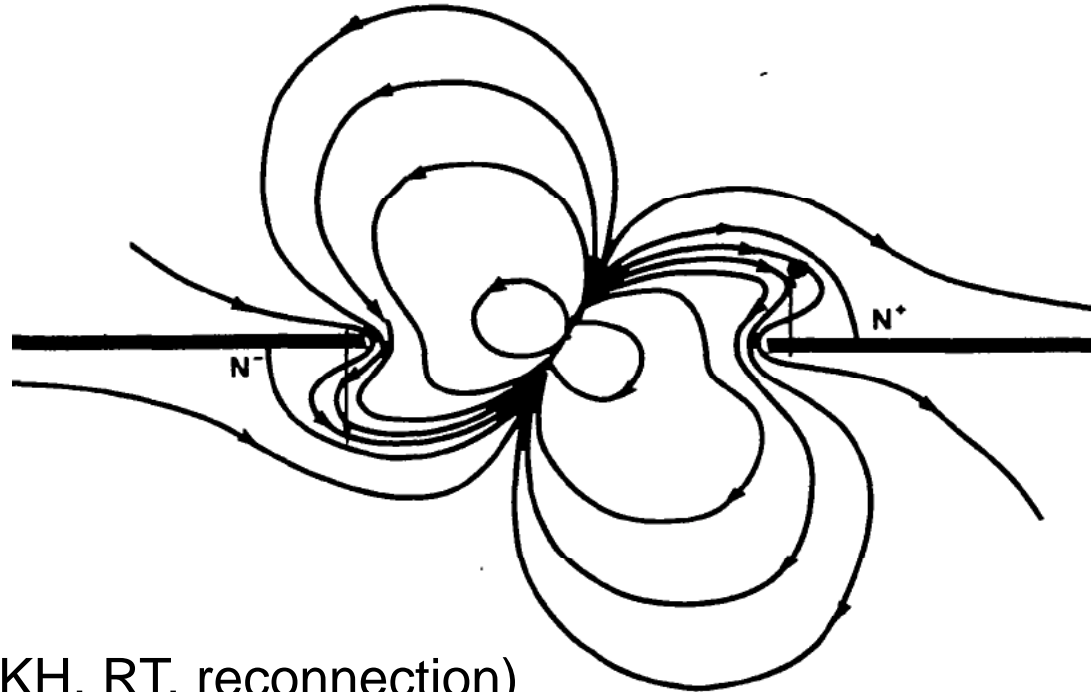
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$$t_{\text{diff}} \sim \frac{H^2}{\eta} \sim \frac{1}{\alpha\Omega}$$

$$\eta \sim \nu = \alpha H c_s \quad (\text{MRI})$$



Instabilities at inner edge (KH, RT, reconnection)

→ **Boundary layer**

# Magnetosphere Boundary Layer

$$r_m \rightarrow r_m + \Delta r_m$$

Transition from  $\Omega_K$  to  $\Omega_*$

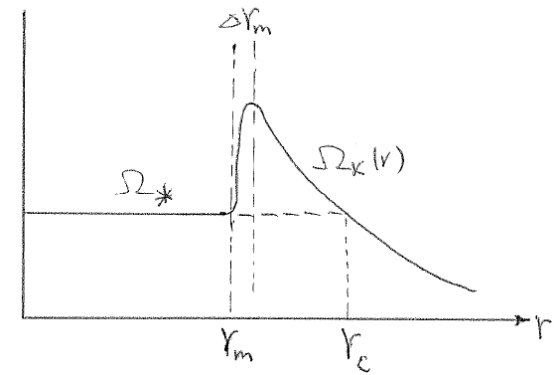
$$B_{\phi+} = -\zeta B_z$$

Magnetic torque on BL (per unit area)

$$r \frac{B_z B_{\phi+}}{2\pi} = \Sigma \frac{d(r^2 \Omega)}{dt} = \Sigma v_r \frac{\partial(r^2 \Omega)}{\partial r}$$

$$\rightarrow r_m \sim \left( \frac{\mu^4}{GM\dot{M}^2} \right)^{1/7}$$

↑  
depends on  $\zeta$ ,  $\Delta r_m$ .

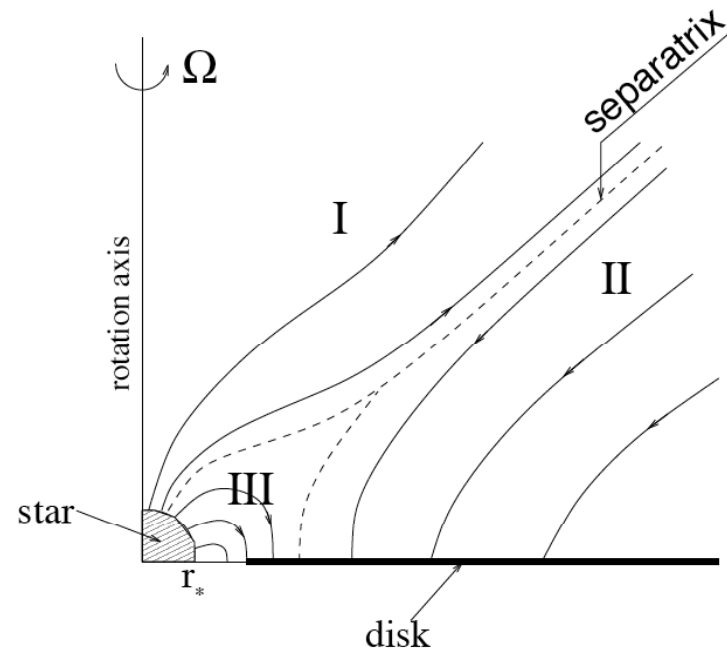
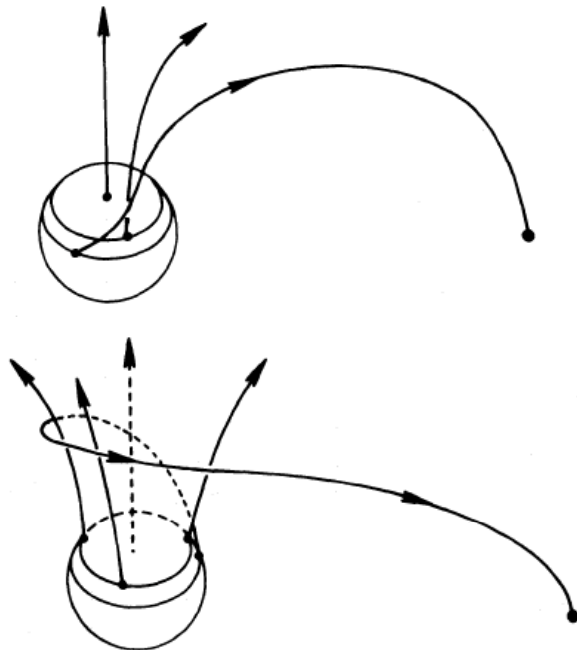


# Star-Disk Linkage

(Width, Time-dependence...)

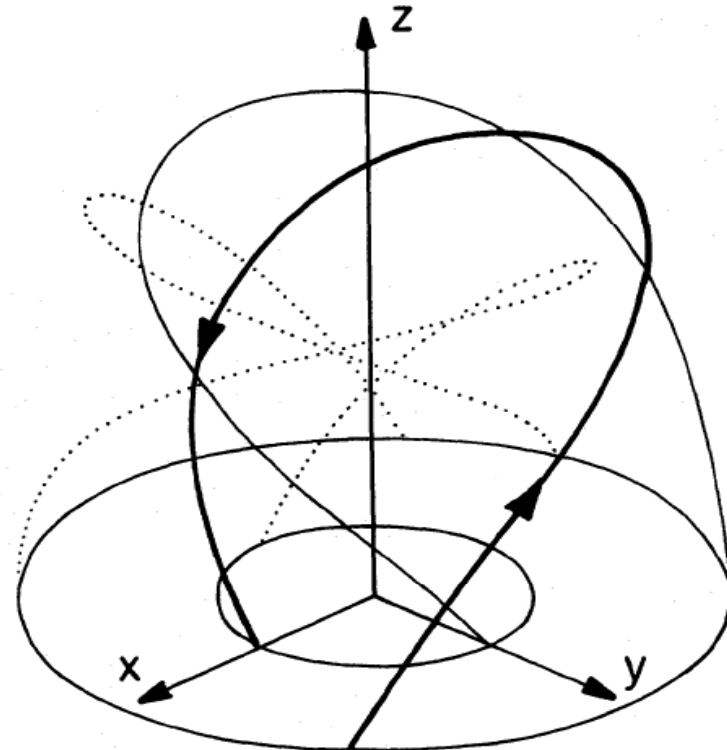
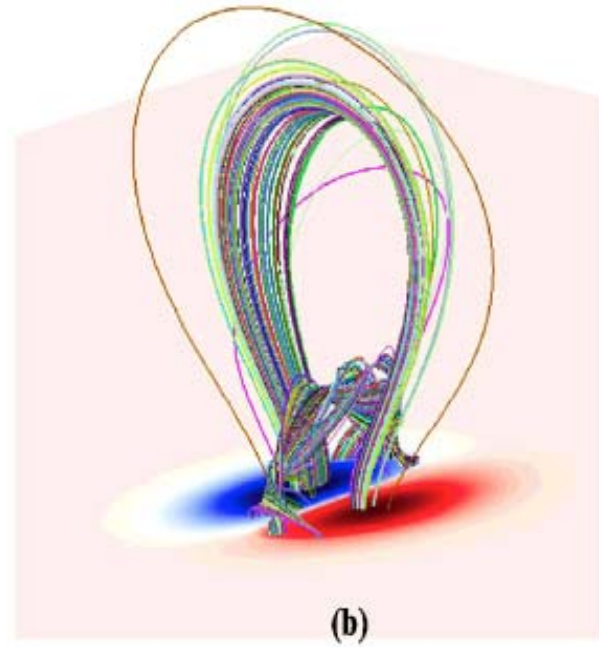
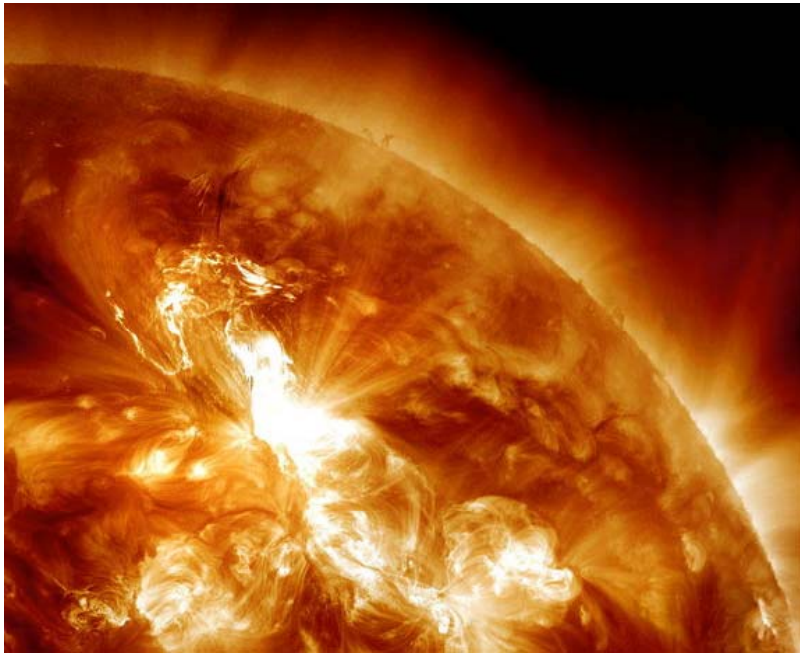
Linked fields are twisted by differential rotation...

→ Field inflates, breaks the linkage



Aly; Lovelace et al.; Uzdensky,...

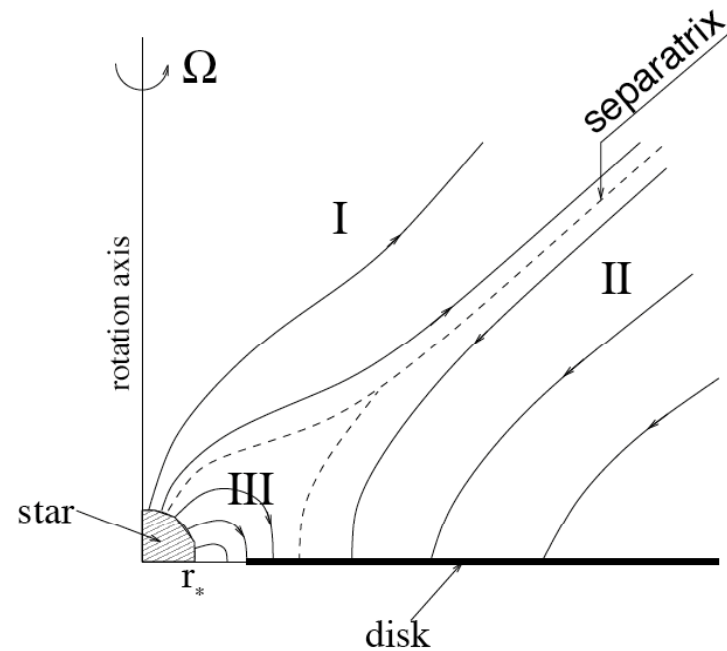
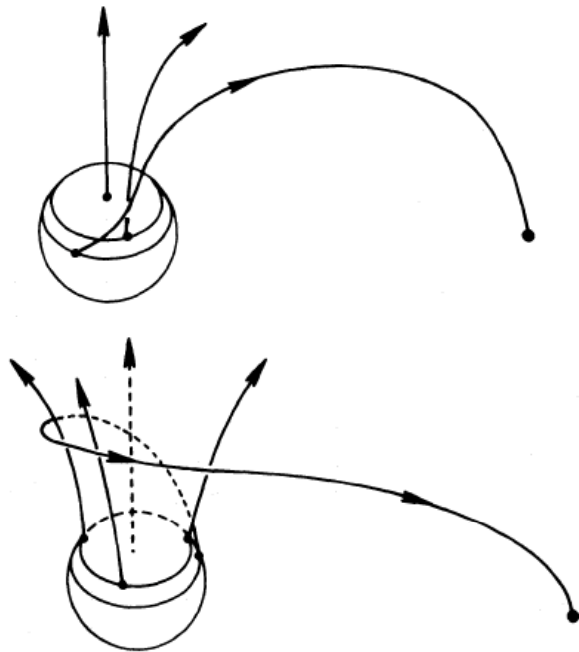




# Star-Disk Linkage

(Width, Time-dependence...)

Linked fields are twisted by differential rotation...  
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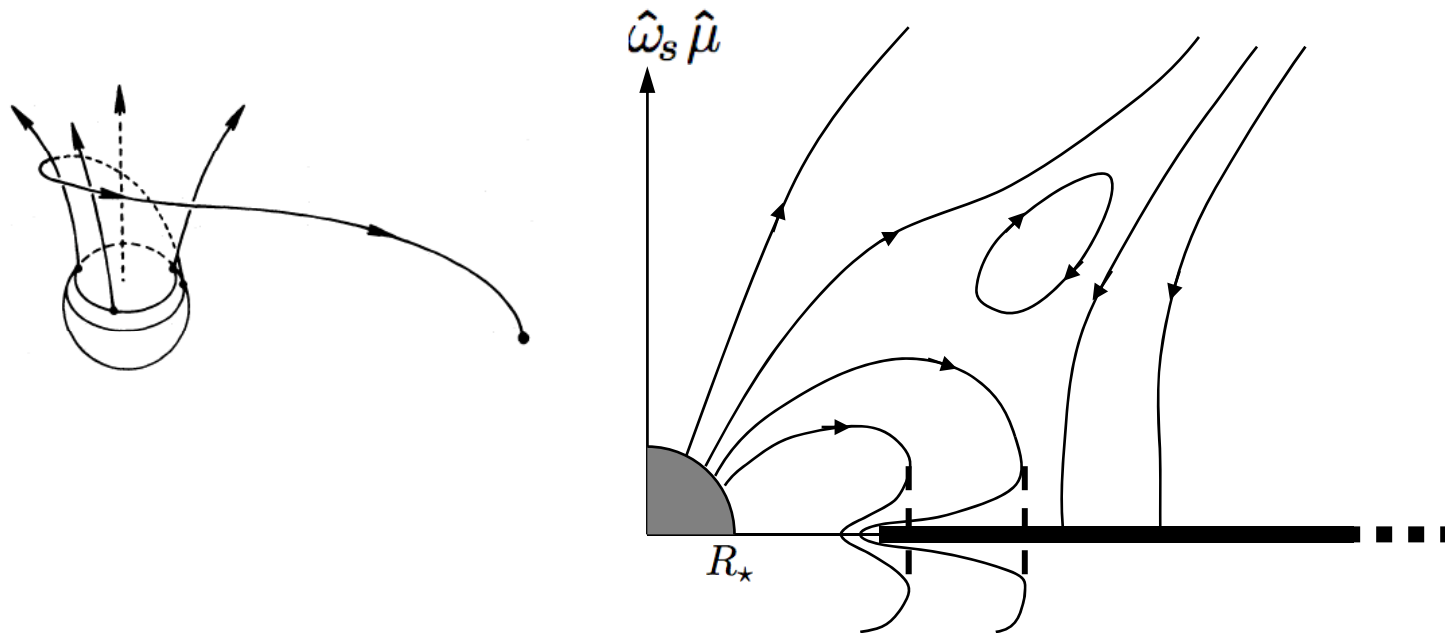


Maximum twist:  $\left| \frac{B_{\phi+}}{B_z} \right|_{\max} \sim 1$

Aly; Lovelace et al.; Uzdensky,...

# Star-Disk Linkage: Quasi-cyclic behavior (Width, Time-dependence...)

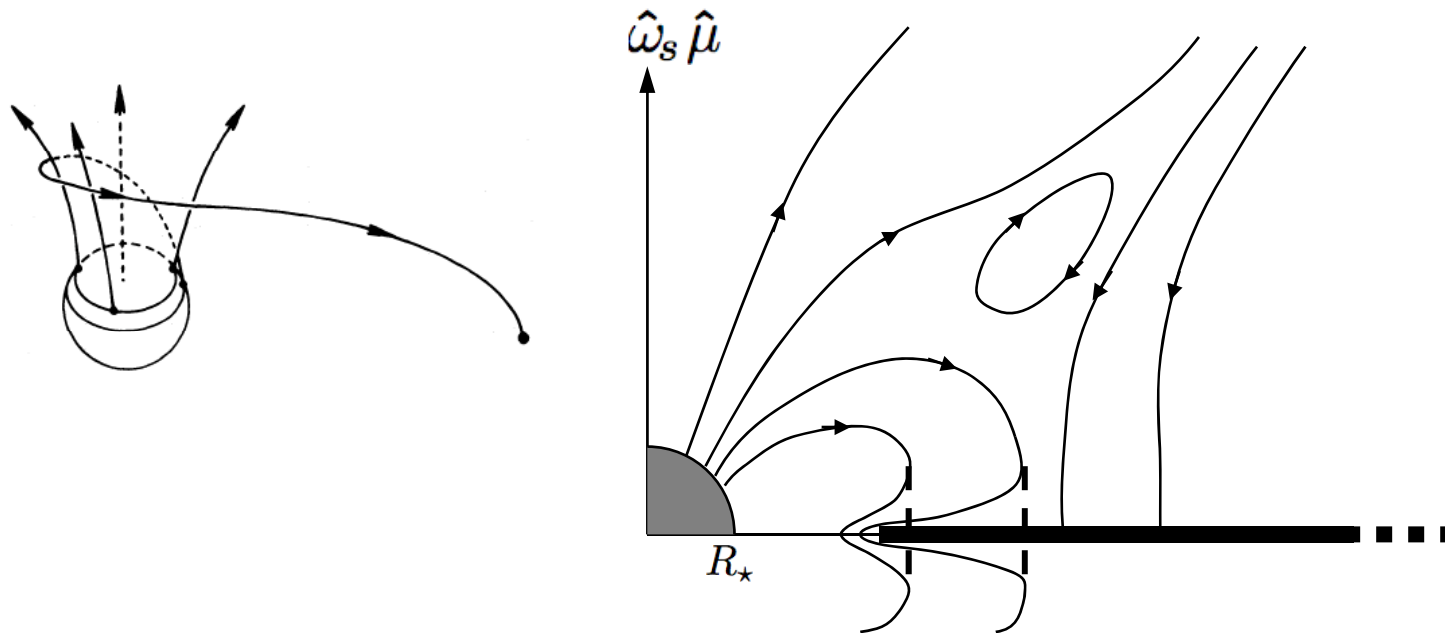
Stellar field penetrates the inner region of disk;  
Field lines linking star and disk are **twisted** --> toroidal field --> field **inflation**  
Reconnection of inflated fields restore linkage



**Inevitable...**

# Star-Disk Linkage: Quasi-cyclic behavior (Width, Time-dependence...)

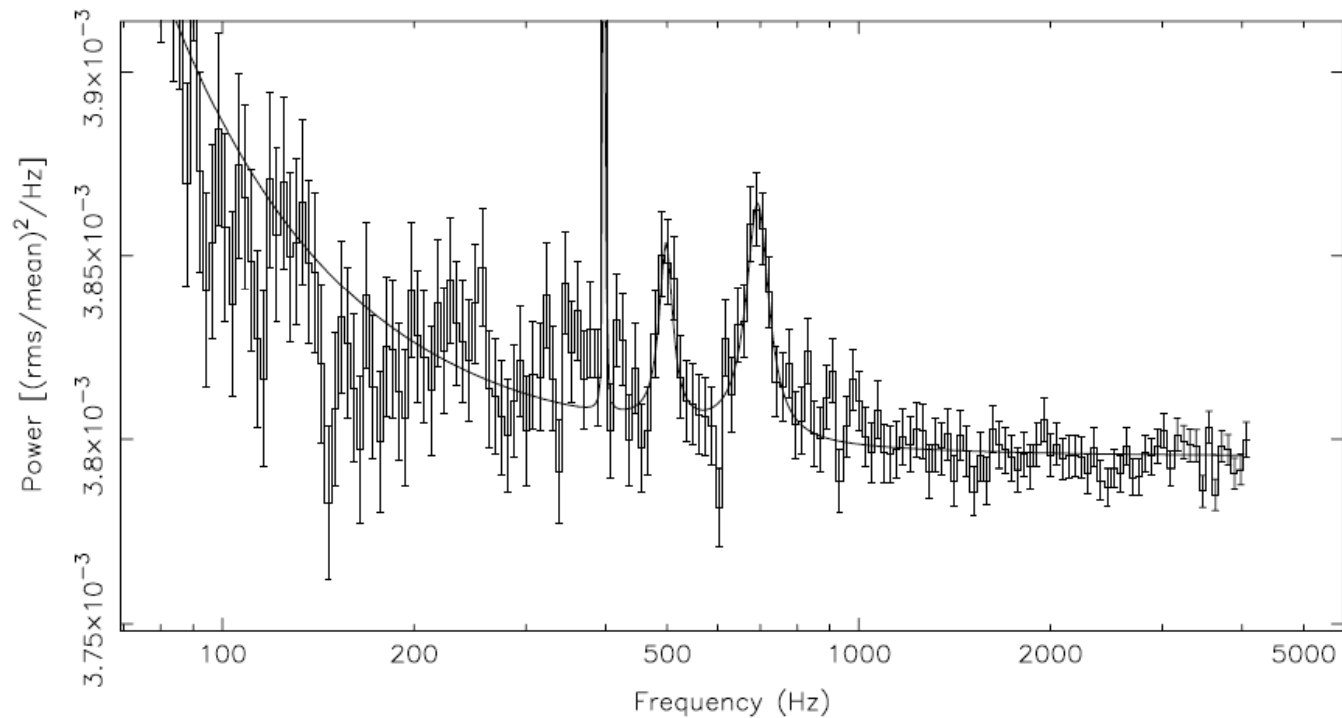
Stellar field penetrates the inner region of disk;  
Field lines linking star and disk are **twisted** --> toroidal field --> field **inflation**  
Reconnection of inflated fields restore linkage



**Application:** Connection with QPOs in LMXBs (and other systems) ?

## Quasi-Periodic Oscillations (QPOs)

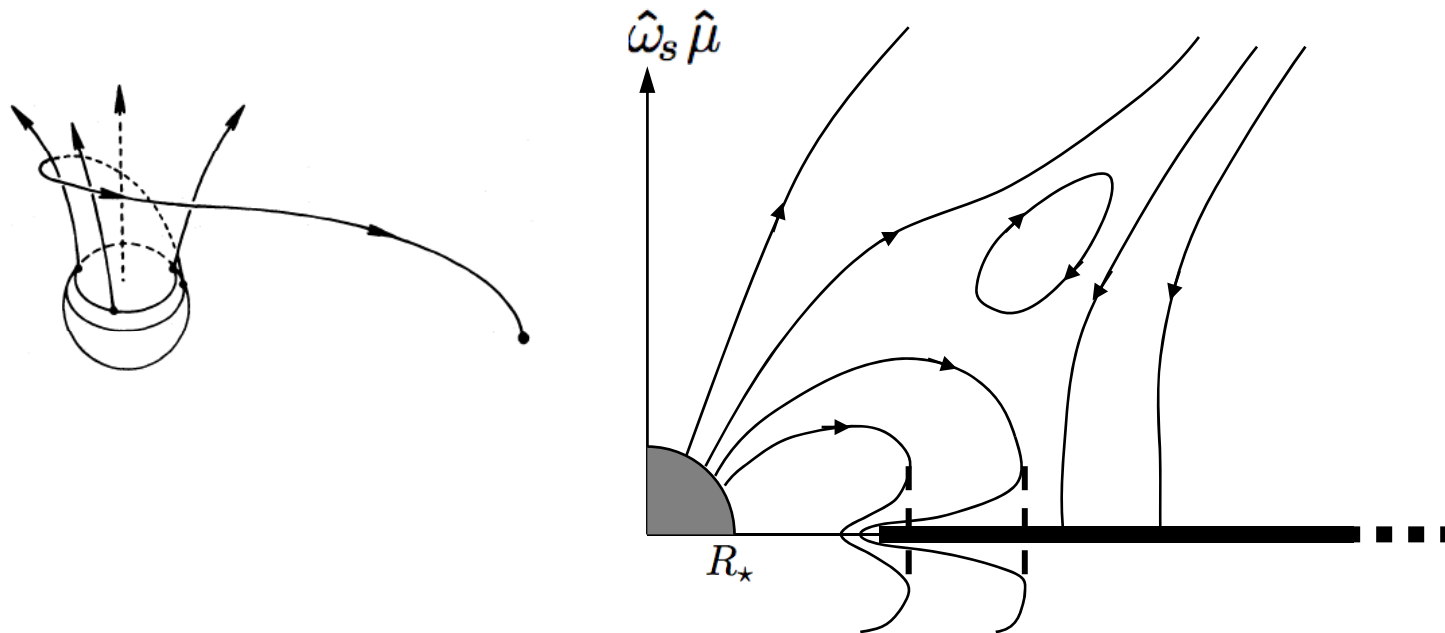
Power density spectrum of x-ray flux variations of accreting millisecond pulsars



Van der Klis 2005

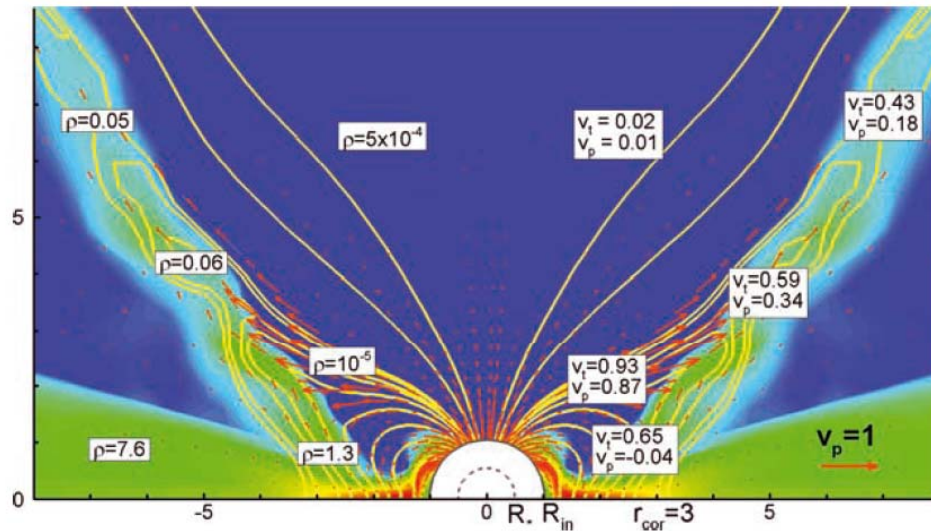
# Star-Disk Linkage: Quasi-cyclic behavior (Width, Time-dependence...)

Stellar field penetrates the inner region of disk;  
Field lines linking star and disk are **twisted** --> toroidal field --> field **inflation**  
Reconnection of inflated fields restore linkage

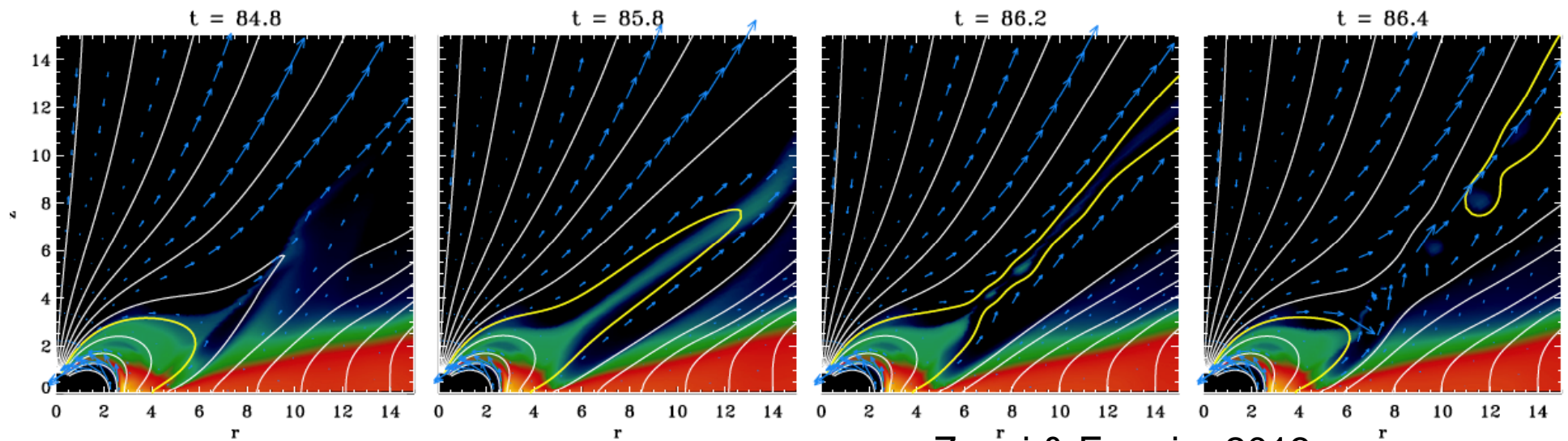


**Application:** Episodic outflow ... Connection with observations?

# Ejection from Magnetospheric Boundary



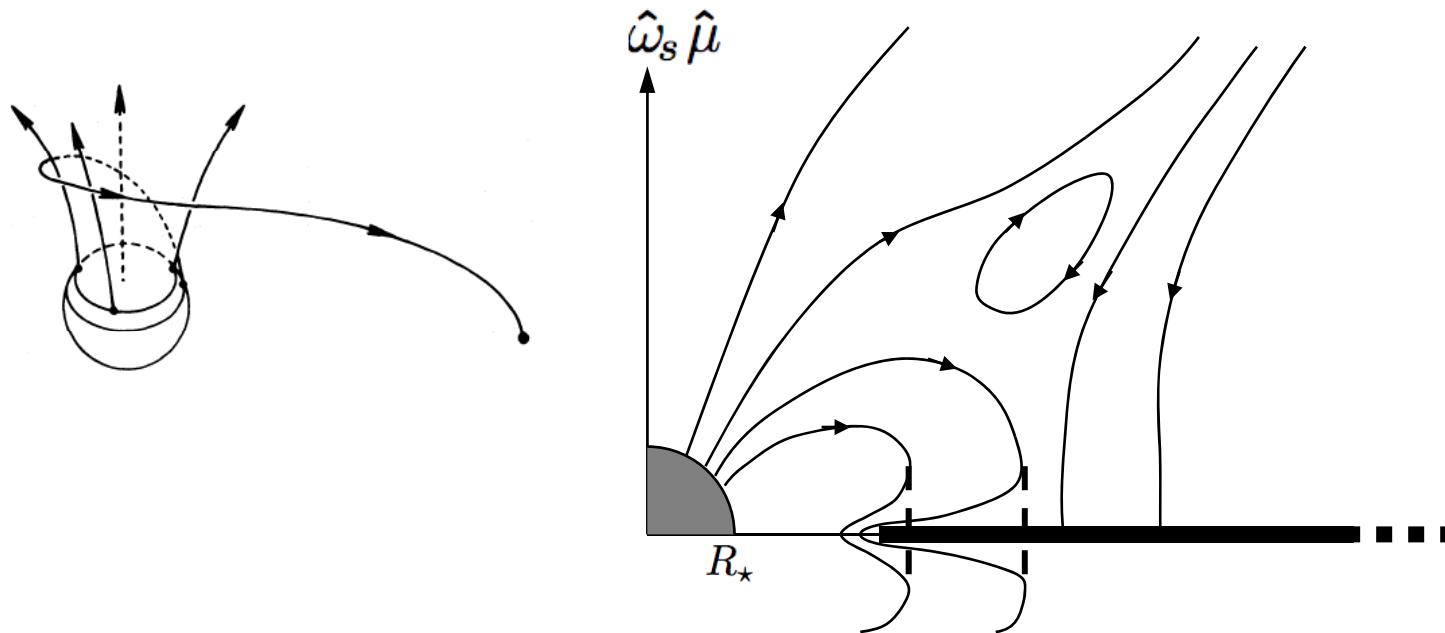
Romanova et al. 2009



Zanni & Ferreira 2013

# Star-Disk Linkage: Quasi-cyclic behavior (Width, Time-dependence...)

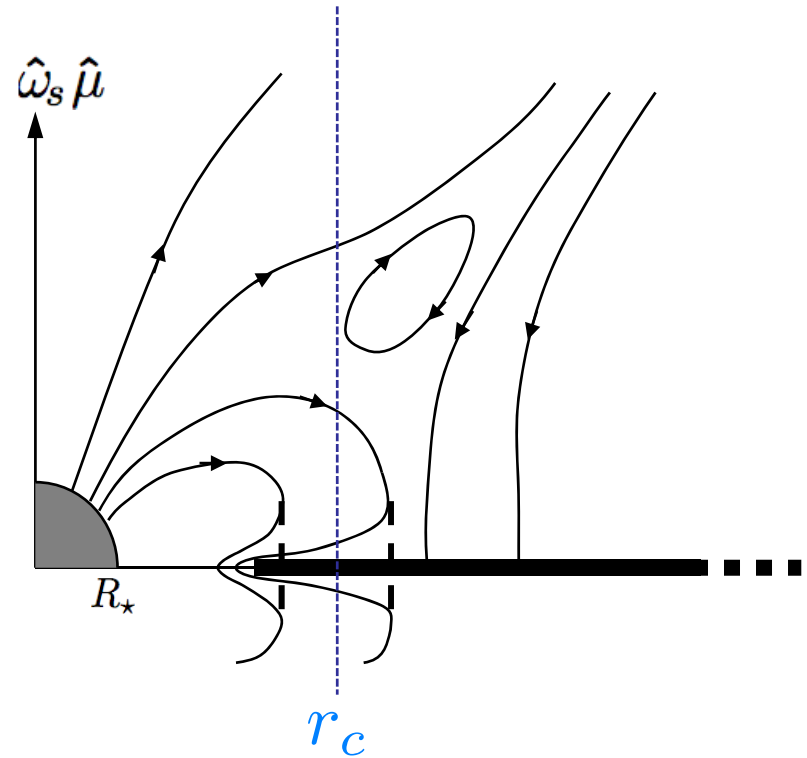
Stellar field penetrates the inner region of disk;  
Field lines linking star and disk are **twisted** --> toroidal field --> field **inflation**  
Reconnection of inflated fields restore linkage



**QUESTION:** On average, what is the width of the linked region?  $\Delta r$



# Torque on Star

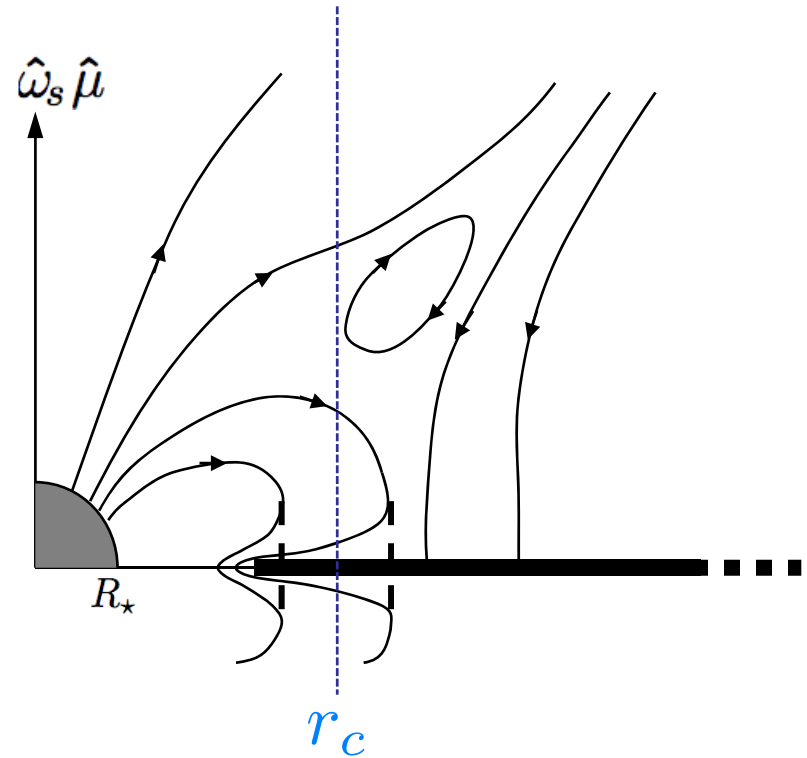


$$\frac{dJ_\star}{dt} \simeq \dot{M}_{\text{acc}} r_m^2 \Omega(r_m) + T_m$$

$$T_m \simeq - \left( r^2 B_z B_{\phi+} \right)_{r_m} \Delta r = \zeta \left( r^2 B_z^2 \right)_{r_m} \Delta r \quad \text{for } B_{\phi+} = -\zeta B_z$$

**Note:**  $|\zeta| \sim 1$ :  $\zeta > 0$  when  $r < r_c$  and  $\zeta < 0$  when  $r > r_c$

# Torque on Star



$$\frac{dJ_\star}{dt} \simeq \dot{M}_{\text{acc}} r_m^2 \Omega(r_m) + T_m$$

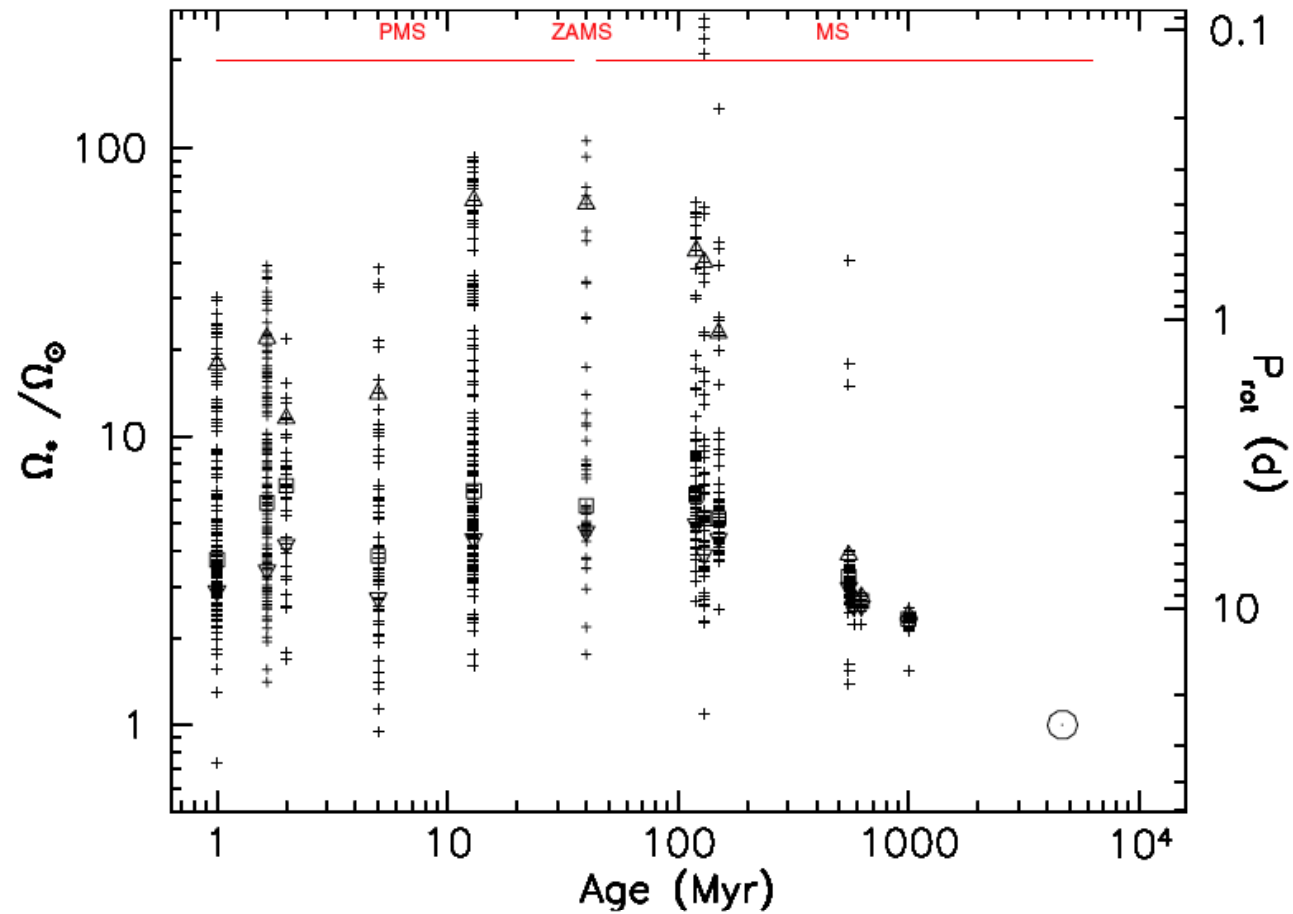
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## Issues:

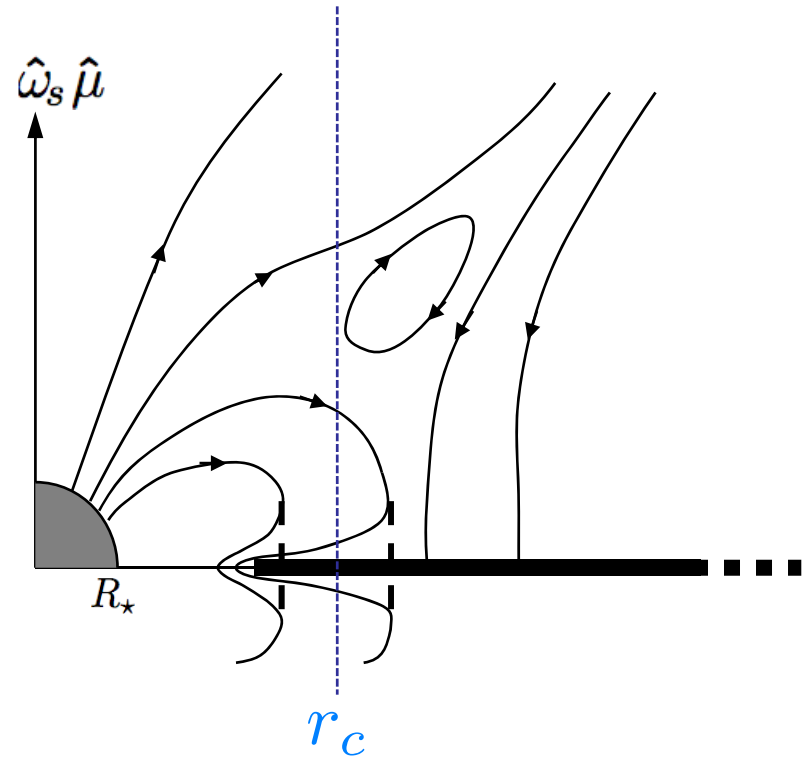
Equilibrium spin: Protostars, millisecond pulsars, long-period pulsars

# Application: Rotation of Protostars: why 10% of breakup?



Gallet & Bouvier 2013

# Torque on Star



$$\frac{dJ_\star}{dt} \simeq \dot{M}_{\text{acc}} r_m^2 \Omega(r_m) + T_m$$

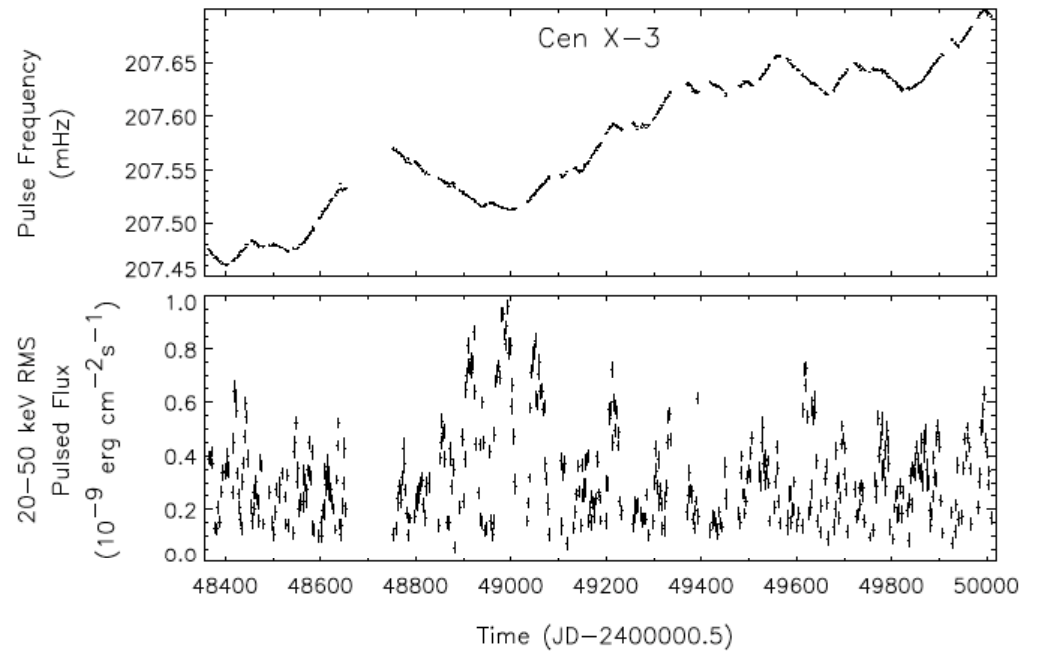
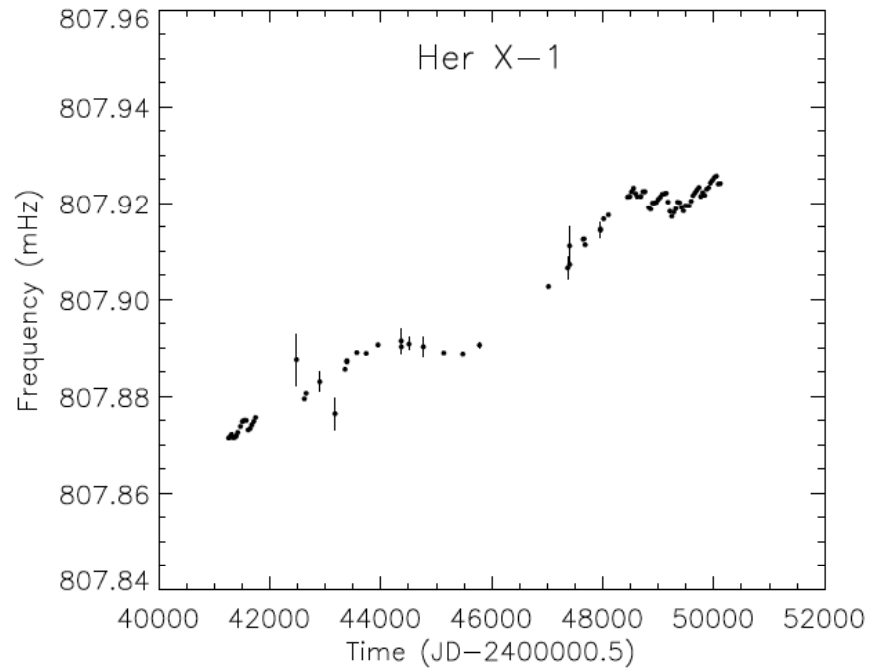
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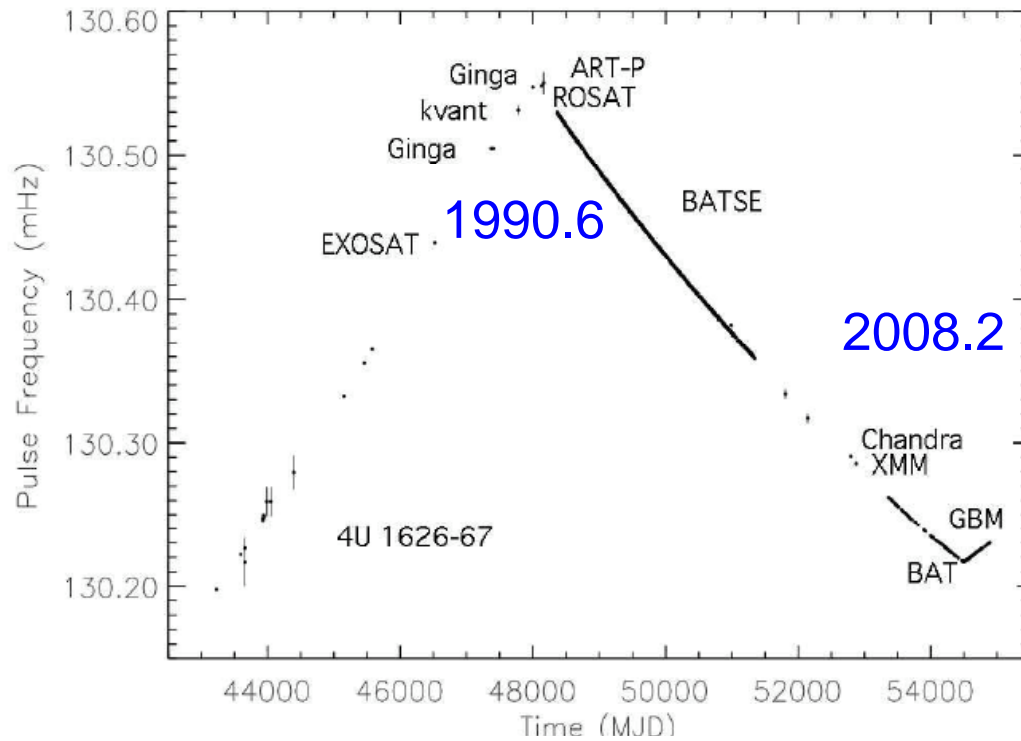
## Issues:

Can we understand spinup/spindown of X-ray pulsars?

# Spinup/Spindown of Accreting X-ray pulsars



Bildsten et al. 1997

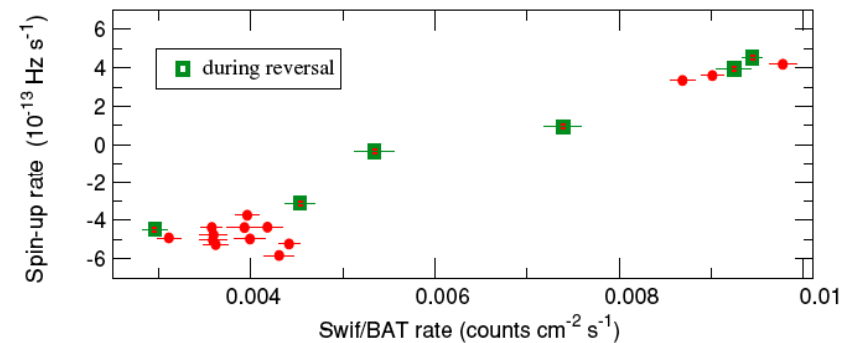
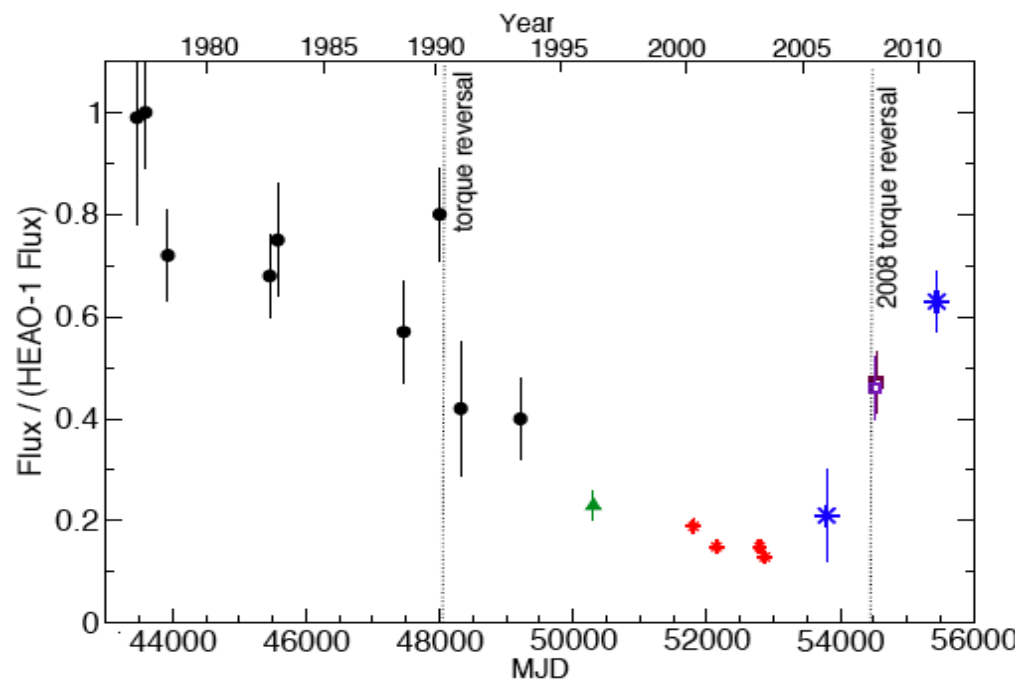


4U1626-67

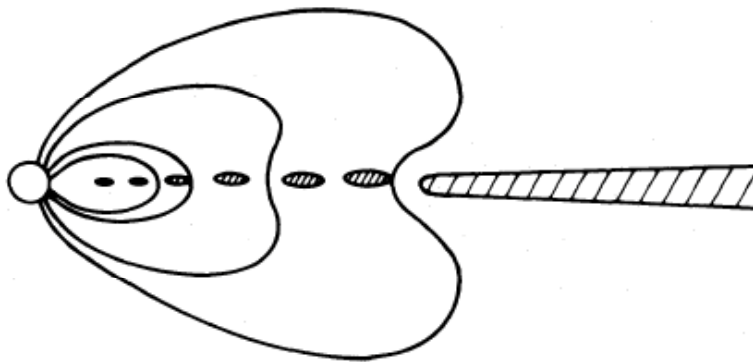
7.66s

Transition lasted 150 days

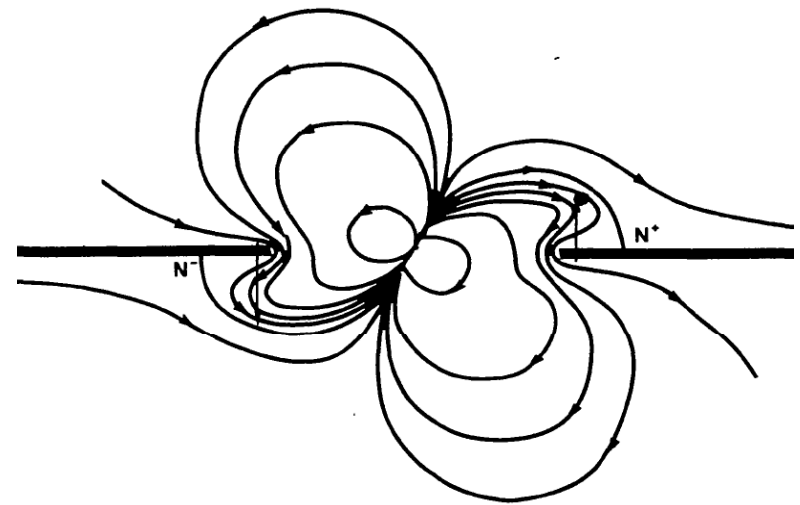
Camero-Arranz et al. 2010,2012



# Misaligned Dipole



Accretion through instabilities



Funnel flow to polar caps

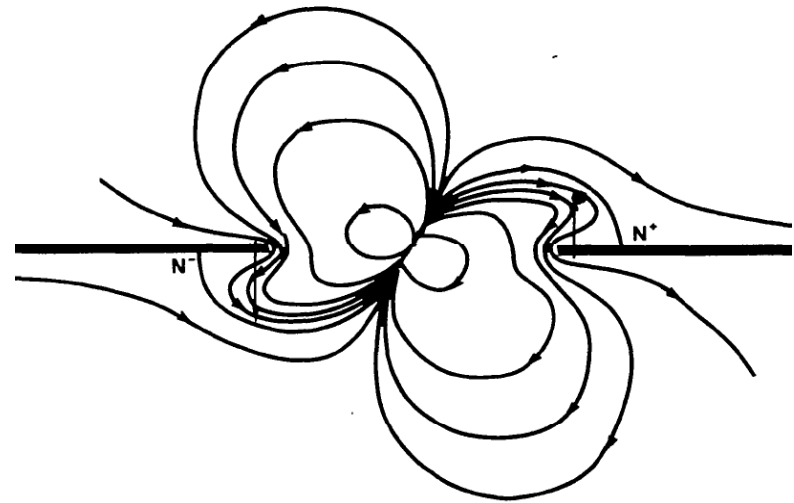
# Misaligned Dipole: Exciting Waves in Disks

Vertical force on disk:

$$F_z(r, \varphi, t) = F_\omega(r) \exp(im\varphi - i\omega t)$$

$$m = 1, \quad \omega = \Omega_*, 2\Omega_*$$

→ Excitation of Bending waves in the disk





# Misaligned Dipole: Exciting Waves in Disks

Vertical force on disk:

$$F_z(r, \varphi, t) = F_\omega(r) \exp(im\varphi - i\omega t)$$

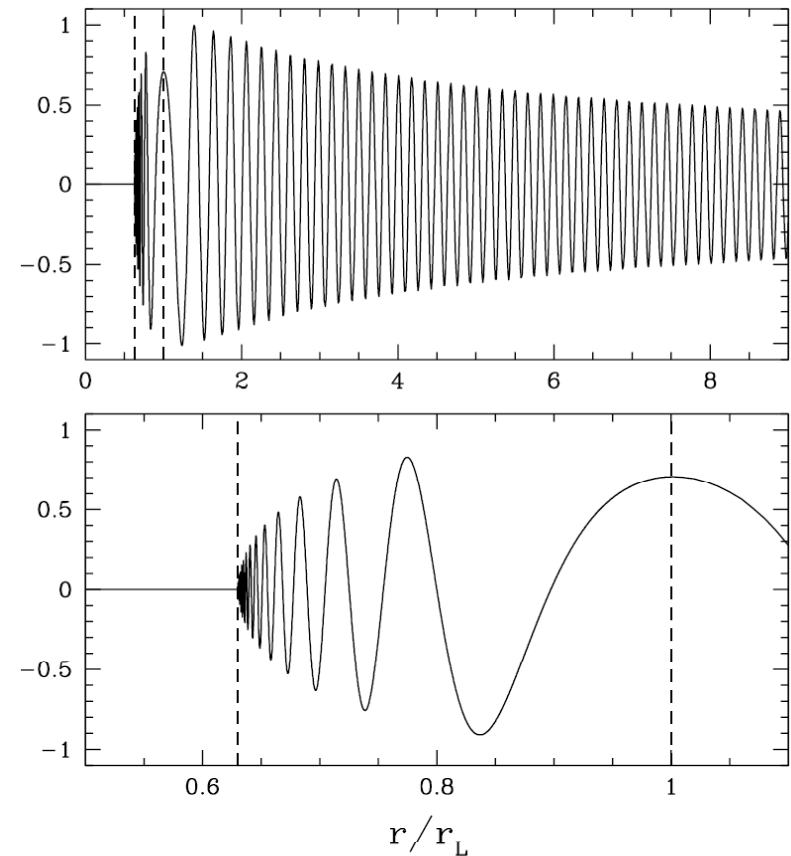
$$m = 1, \quad \omega = \Omega_\star, 2\Omega_\star$$

→ Excitation of Bending waves in the disk

Perturbations most “visible” at  
Lindblad/Vertical Resonance

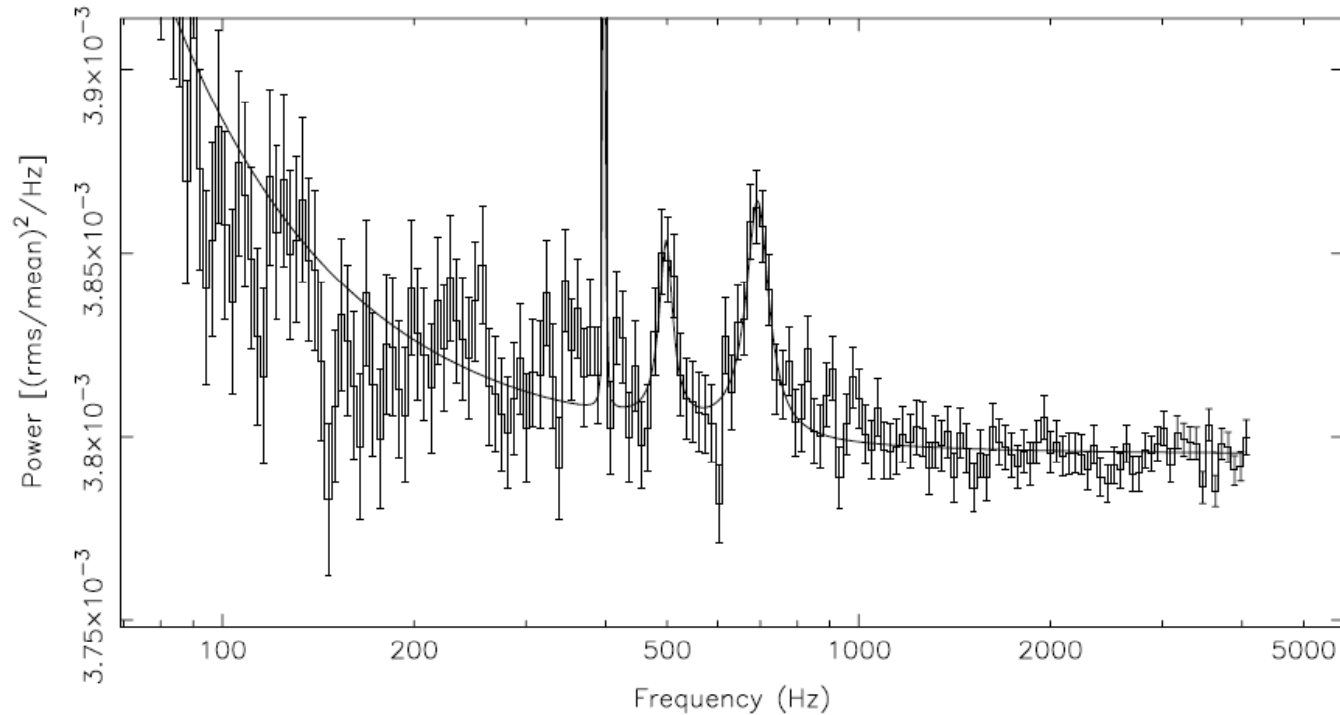
$$\omega - \Omega = \Omega_\perp \simeq \Omega$$

$$\rightarrow \Omega(r_L) = \frac{\omega}{2} = \frac{\Omega_\star}{2}, \Omega_\star$$



**Question: QPOs....**

# kHz QPOs in Accreting Millisecond Pulsars



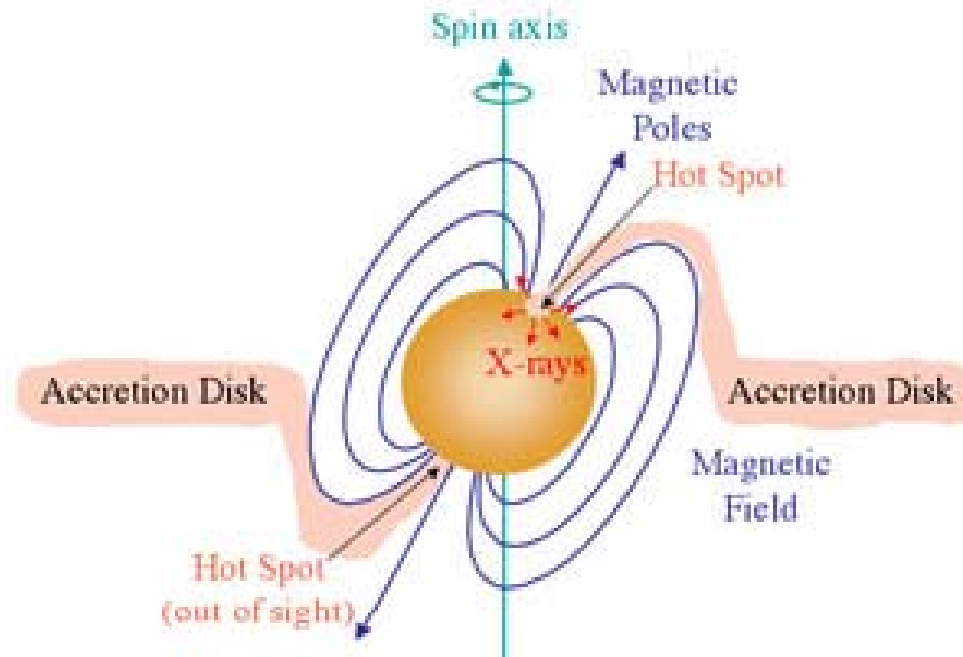
Van der Klis 2005

SAX J1808.4-3658  $\nu_s = 401$  Hz,  $\nu_h - \nu_l \simeq \nu_s/2$  ( $\pm$ a few Hz)

XTE J1807.4-294:  $\nu_s = 191$  Hz,  $\nu_h - \nu_l \simeq \nu_s$

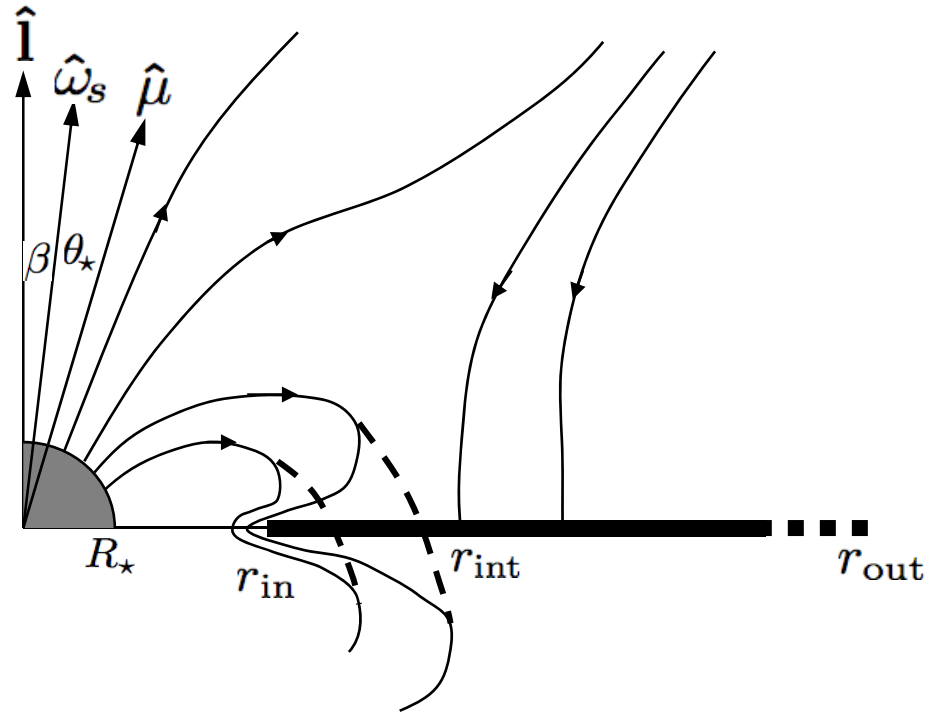
Beating of high-freq. QPO with perturbed fluid at L/R ?

# Stellar Spin – Disk Misalignment

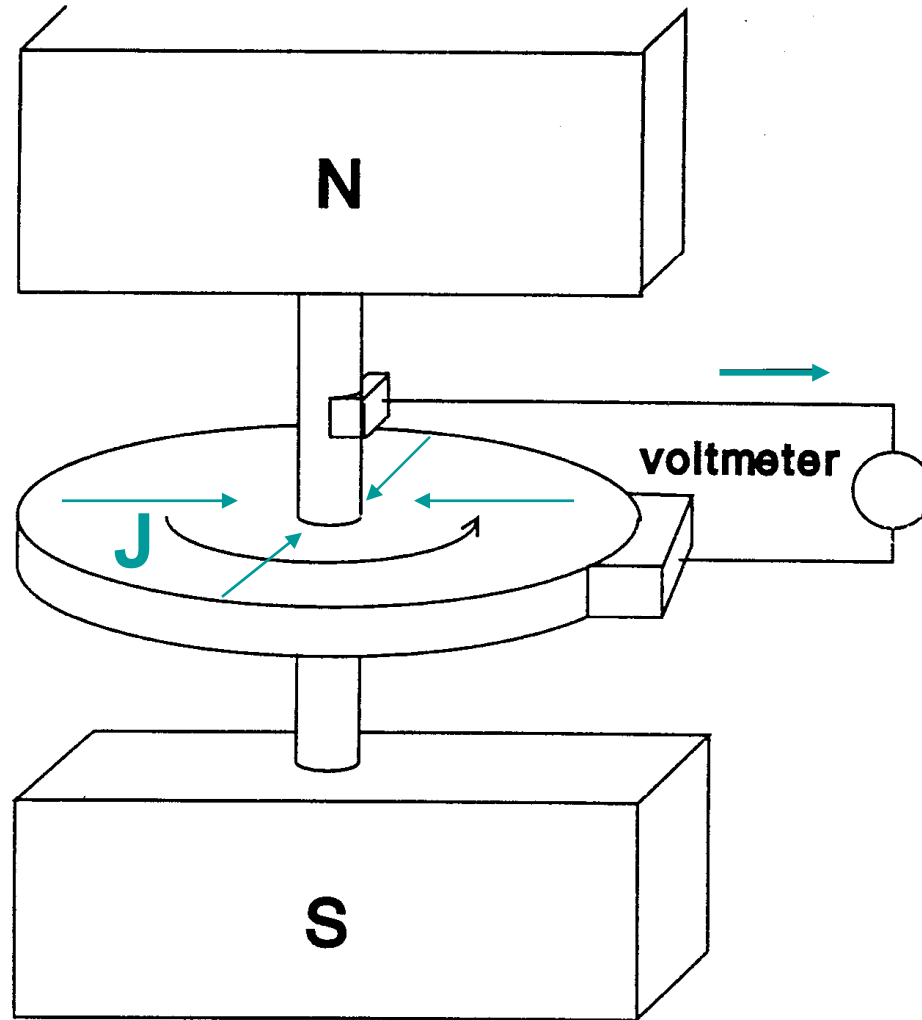


Standard story: **S//L**

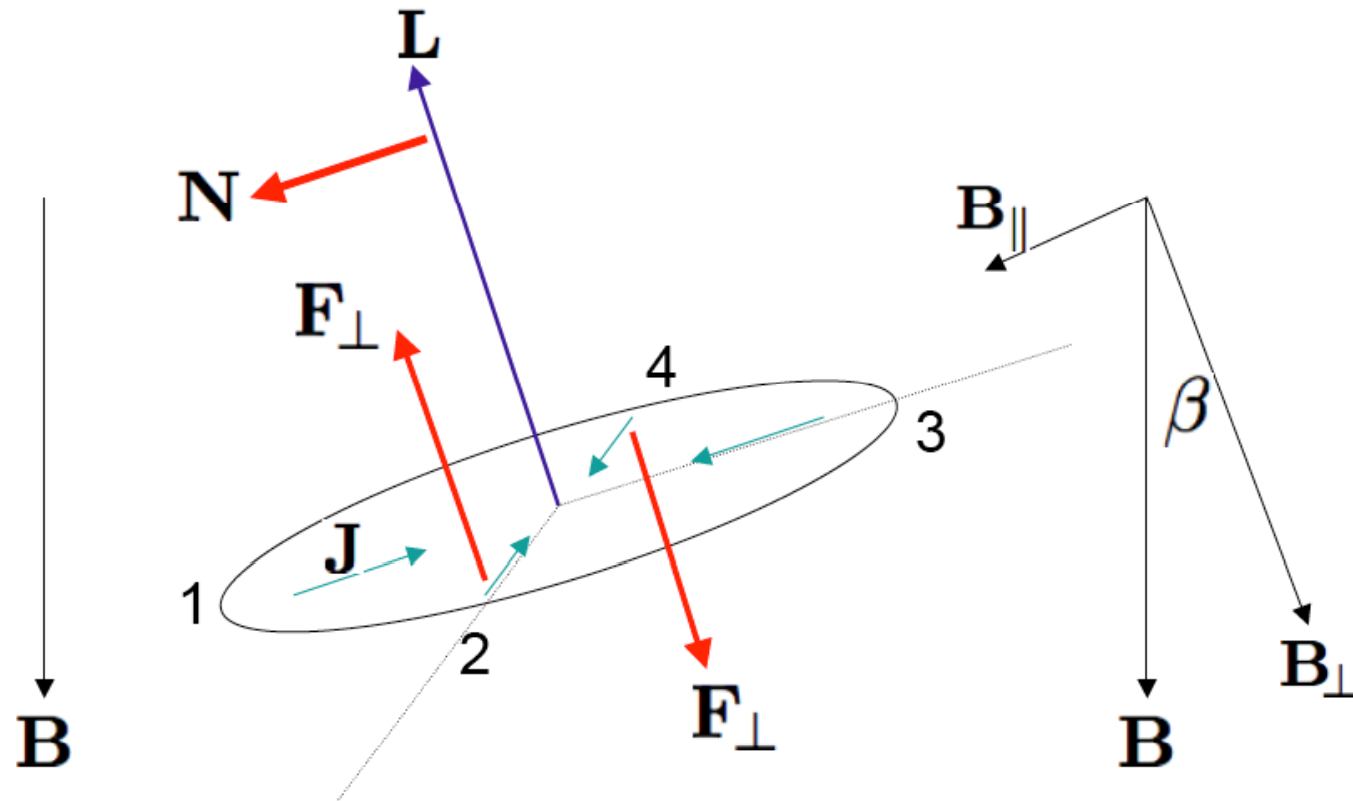
There is a **magnetic torque** which tends to make the inner disk **warp** on timescale  $\gg$  dynamical time (rotation/orbital period)



# A Laboratory Experiment



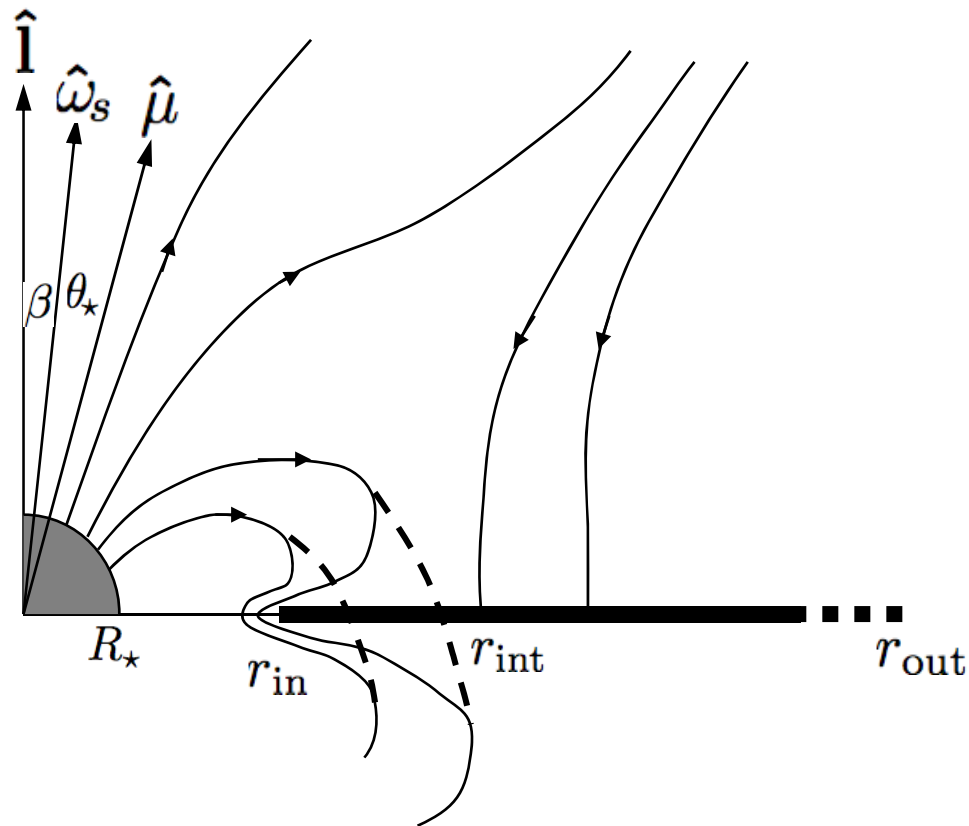
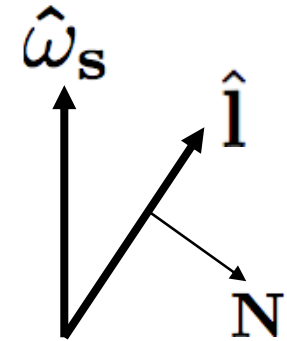
# A Laboratory Experiment



## Magnetically Threaded disk:

Torque on disk (per unit area):  $\mathbf{N} \propto -\hat{\mathbf{l}} \times (\hat{\boldsymbol{\mu}} \times \hat{\mathbf{l}})$   
Averaging over stellar rotation:  $\mathbf{N} \propto -\hat{\mathbf{l}} \times (\hat{\boldsymbol{\omega}}_s \times \hat{\mathbf{l}})$

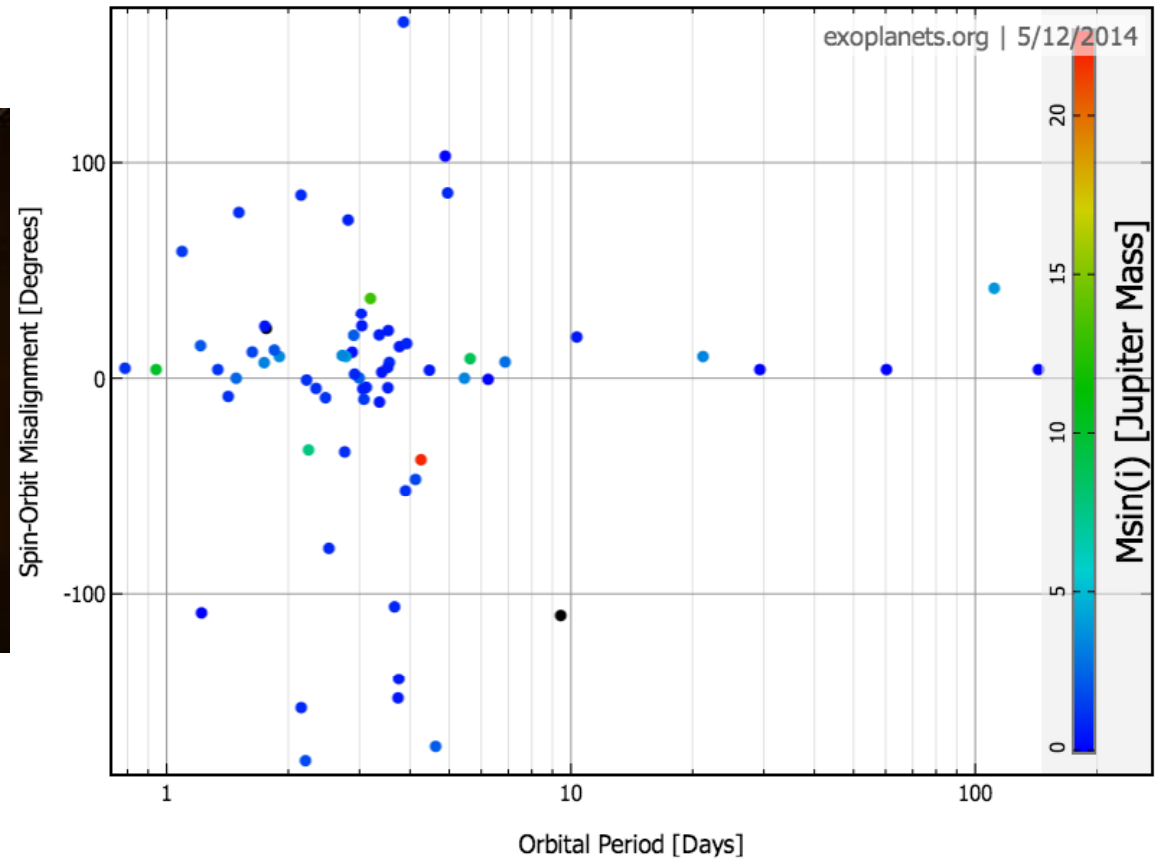
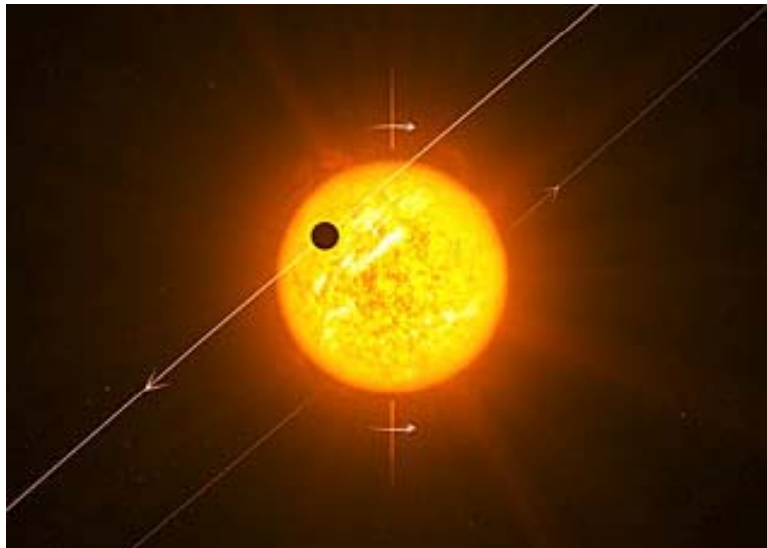
Warping torque



# Possible Connection to (Exo)Planetary Systems



# Many “hot Jupiters” have Misaligned $S^*-L_p$



# **S\*-L<sub>p</sub> misalignment in Exoplanetary Systems**

## **→ The Importance of few-body interactions**

### 1. Kozai + Tide migration by a distant companion star/planet

(e.g., Wu & Murray 03; Fabrycky & Tremaine 07; Naoz et al.12, Katz et al.12)

### 2. Planet-planet Interactions

#### -- Strong scatterings

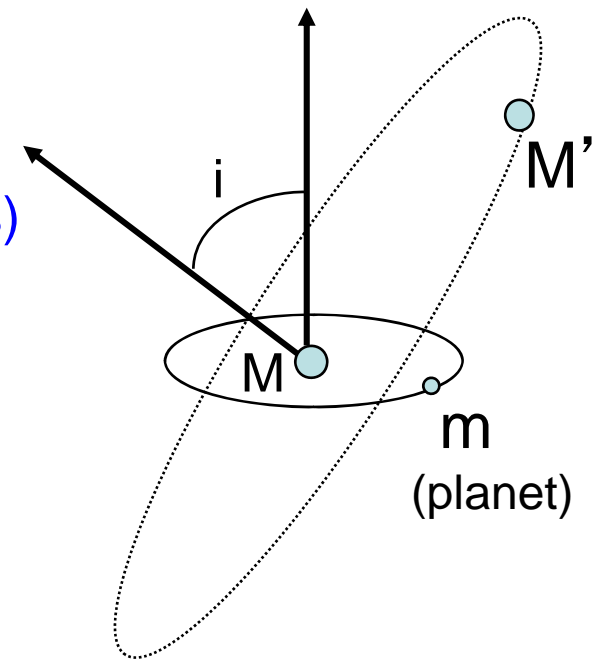
(e.g., Rasio & Ford 96; Chatterjee et al. 08; Juric & Tremaine 08)

#### -- Secular interactions (“Internal Kozai”, chaos)

(e.g Nagasawa et al. 08; Wu & Lithwick 11)

#### -- Chaotic stellar spin evolution during Kozai

(Storch, Anderson & DL 2014)



# Is “High-e Migration” the whole story for producing hot Jupiters and S-L misalignments?

## Likely NO.

- Companion? Initial conditions? (e.g., Knutson et al. 2014)
- Can produce distribution of period, ecc, misalignment? (Naoz+12,Petrovich+14)
- Paucity of high-e proto-hot Jupiters (Socrates et al.2012; Dawson et al.2012)
- Stellar metallicity trend of hot Jupiters → Two mechanisms of migrations (Dawson & Murray-Clay 2013)
- Misaligned multiplanet systems:
  - Kepler-55 (2 planets 10.5 & 21 days →40-55 deg from seismology; Huber et al 2013)
  - Kepler-9 (3-planets; Walkowicz & Basri 2013) ?
  - Other Candidates: Hirano et al. 2014

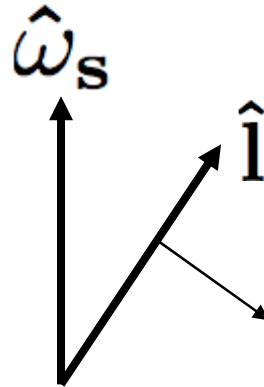
# **Alternative: Primordial Misalignments**

between Stellar Spin and Protoplanetary Disk

-- **Magnetic Star – Disk Interaction** (Lai, Foucart & Lin 2011)

-- **Perturbation of Binary on Disk** (Batygin 2012; Batygin & Adams 2013; **Lai 2014**)

**Recall:** Magnetic torques from the star **want** to make the inner disk warp and precess...



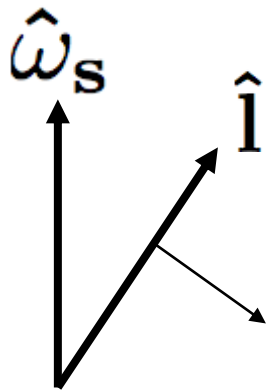
But disk will **want** to resist it by internal stresses (viscosity or bending wave propagation)

$$\frac{\partial}{\partial t} (\Sigma r^2 \Omega \hat{l}) + \frac{1}{r} \frac{\partial}{\partial r} (\Sigma V_R r^3 \Omega \hat{l}) = \frac{1}{r} \frac{\partial}{\partial r} (Q_1 I r^2 \Omega^2 \hat{l}) + \frac{1}{r} \frac{\partial}{\partial r} \left( Q_2 I r^3 \Omega^2 \frac{\partial \hat{l}}{\partial r} + Q_3 I r^3 \Omega^2 \hat{l} \times \frac{\partial \hat{l}}{\partial r} \right) + \mathbf{N}_m$$

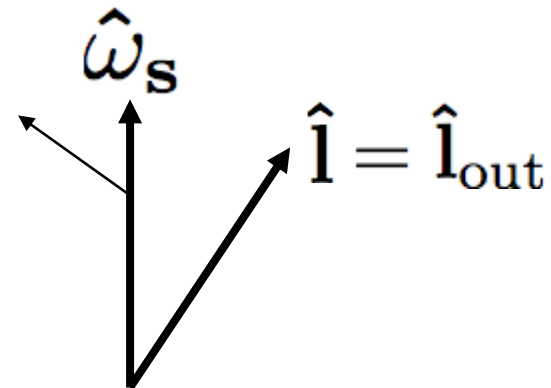
## Back-reaction Torque on Star:

What is happening to the stellar spin direction?

(Is there secular change to the spin direction?)



Warping torque



Back-reaction torque

→ **Stellar spin may be misaligned with disk axis**

Accretion tends to align S & L:

Accretion torque  $\mathcal{N}_{\text{acc}} \simeq \dot{M} \sqrt{GM_{\star} r_{\text{in}}}$

Magnetic misalignment torque:  $\mathcal{N}_{\text{mag}} \sim \mu^2 / r_{\text{in}}^3$

For  $r_{\text{in}} \sim \left( \frac{\mu^4}{GM_{\star} \dot{M}^2} \right)^{1/7}$

$\rightarrow \mathcal{N}_{\text{acc}} \sim \mathcal{N}_{\text{mag}}$

## Evolution of the stellar spin

$$\frac{d}{dt} (J_s \hat{\omega}_s) = \mathcal{N} = \mathcal{N}_{\text{acc}} + \mathcal{N}_m + \mathcal{N}_{\text{sd}}$$

$$\mathcal{N}_{\text{acc}} = \lambda \dot{M} \sqrt{GM r_{\text{in}}} \hat{l}_{\text{in}}, \quad \lambda \sim 1 \text{ (or less)}$$

$\mathcal{N}_m$  = backreaction of magnetic (warping & precessional) torques

$$\mathcal{N}_{\text{sd}} = -|\mathcal{N}_{\text{sd}}| \hat{\omega}_s$$

(Each term is of order  $\mathcal{N}_0 = \dot{M} \sqrt{GM r_{\text{in}}}$ )

$$\implies \frac{d \cos \theta_{\text{sd}}}{dt} = \frac{\mathcal{N}_0}{J_s} \sin^2 \theta_{\text{sd}} \left( \lambda - \tilde{\xi} \cos^2 \theta_{\text{sd}} \right)$$

$$\bar{\xi} = \frac{\zeta \cos^2 \theta_{\star}}{6\eta^{7/2}} (\sim 1)$$

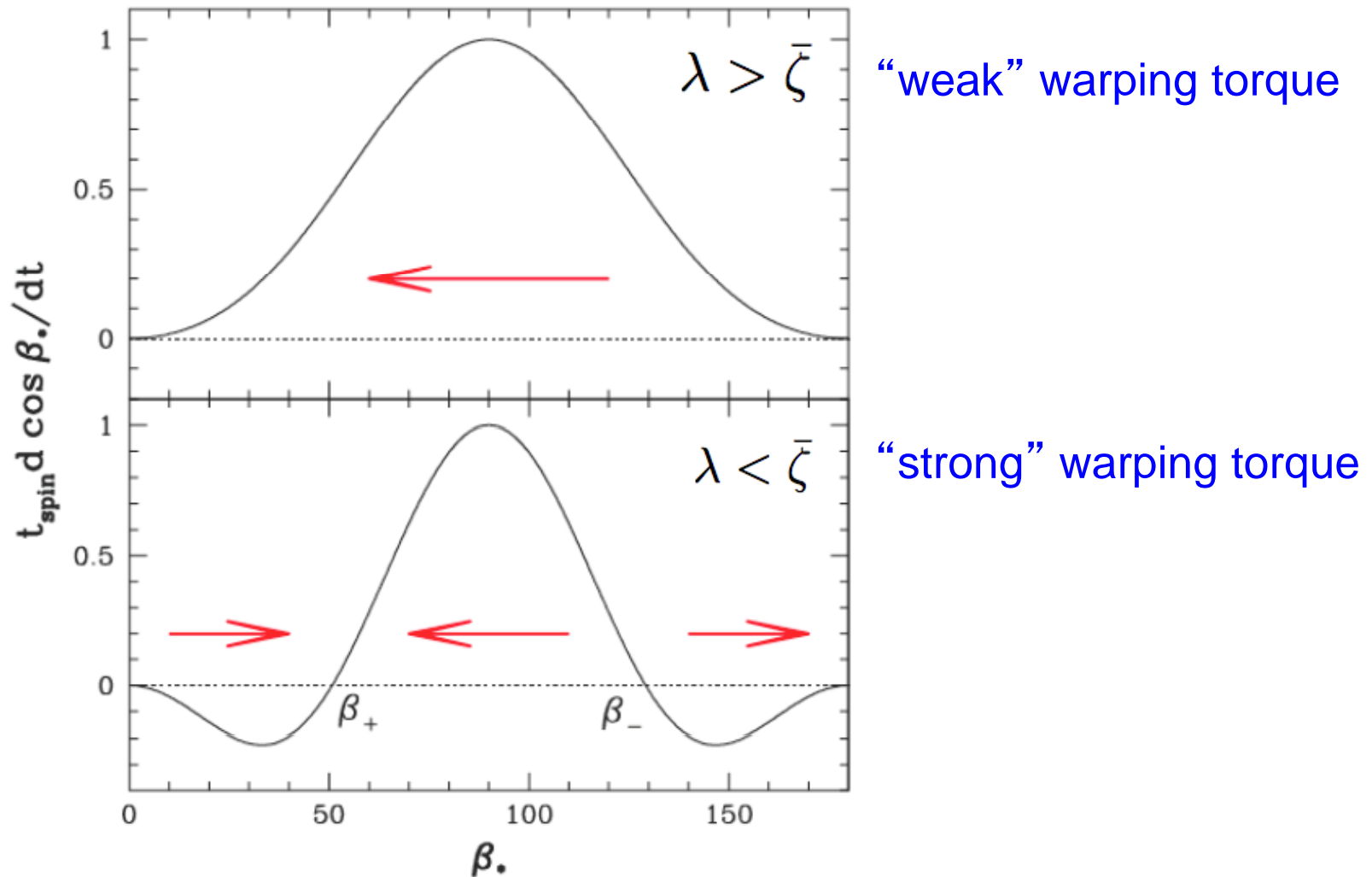
Spin evolution timescale:

$$t_{\text{spin}} = (1.25 \text{ Myr}) \left( \frac{M_{\star}}{1 M_{\odot}} \right) \left( \frac{\dot{M}}{10^{-8} M_{\odot} \text{ yr}^{-1}} \right)^{-1} \left( \frac{r_{\text{in}}}{4 R_{\star}} \right)^{-2} \frac{\omega_s}{\Omega(r_{\text{in}})}$$



## Evolution of the stellar spin

$$\frac{d \cos \theta_{\text{sd}}}{dt} = \frac{\mathcal{N}_0}{J_s} \sin^2 \theta_{\text{sd}} \left( \lambda - \tilde{\xi} \cos^2 \theta_{\text{sd}} \right) \quad \bar{\zeta} = \frac{\zeta \cos^2 \theta_{\star}}{6\eta^{7/2}} (\sim 1)$$



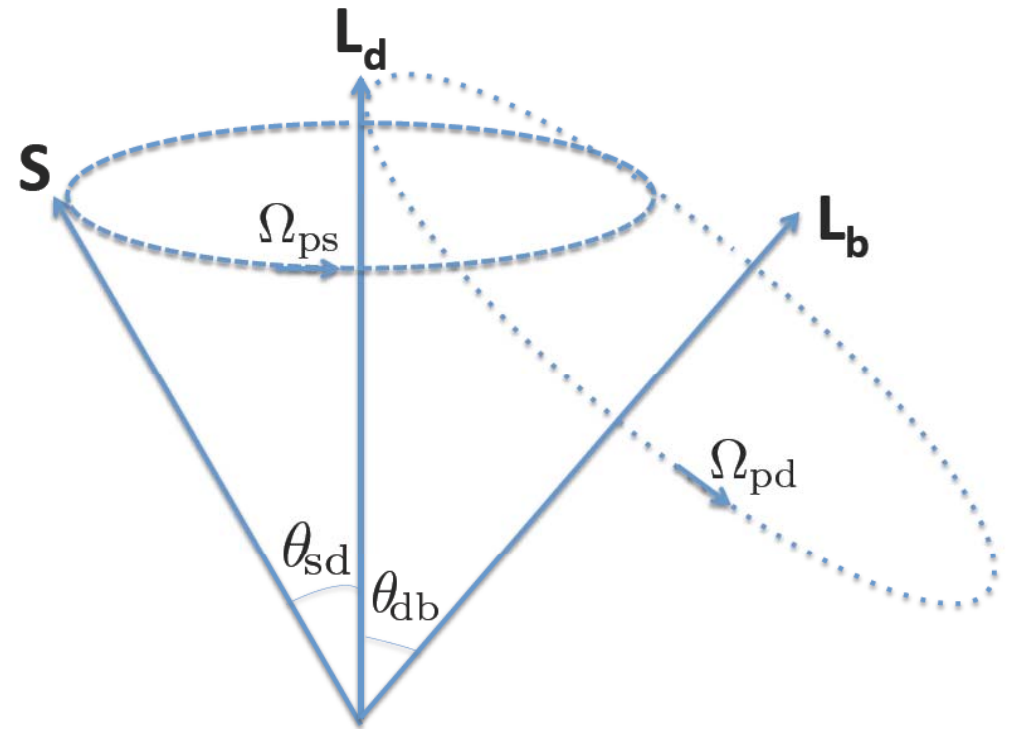
## **For Isolated star-disk systems:**

Magnetic torque tends to produce spin-disk misalignment,  
But competes with accretion

→ May or may not produce small/modest misalignment  
(e.g., Solar system 7 degree?)

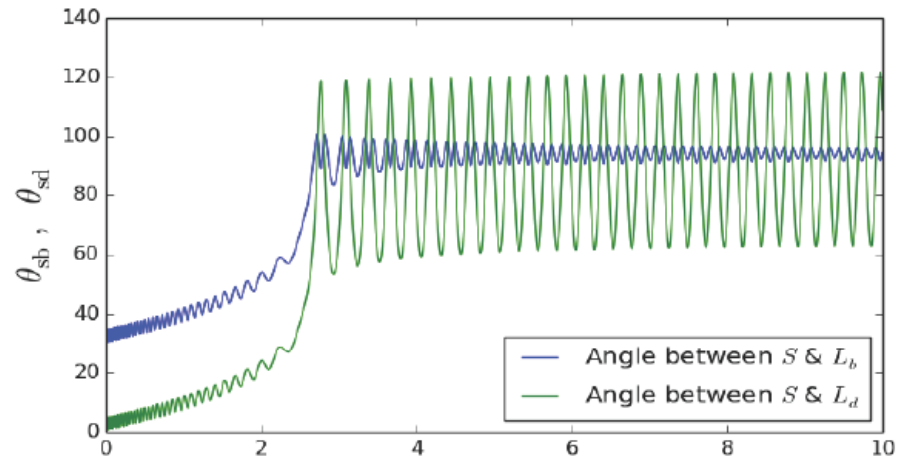
# Star-Disk-Binary Interactions

Gravitational interactions...

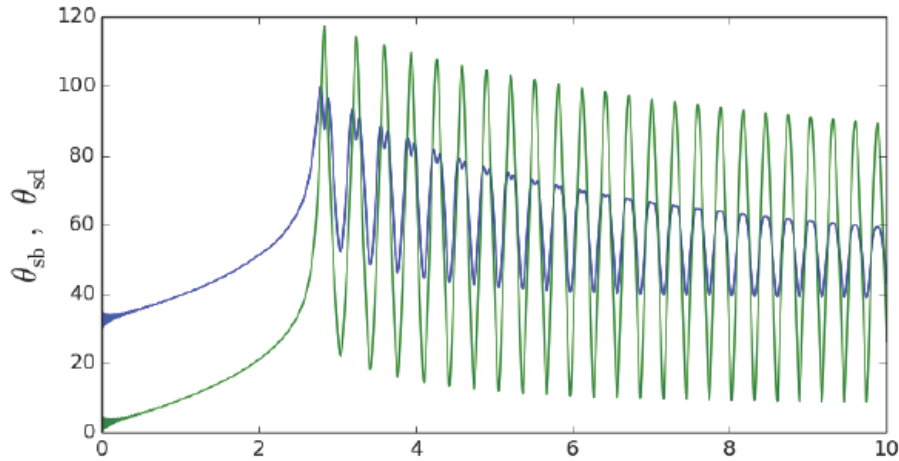


Now include Accretion and Magnetic Torques

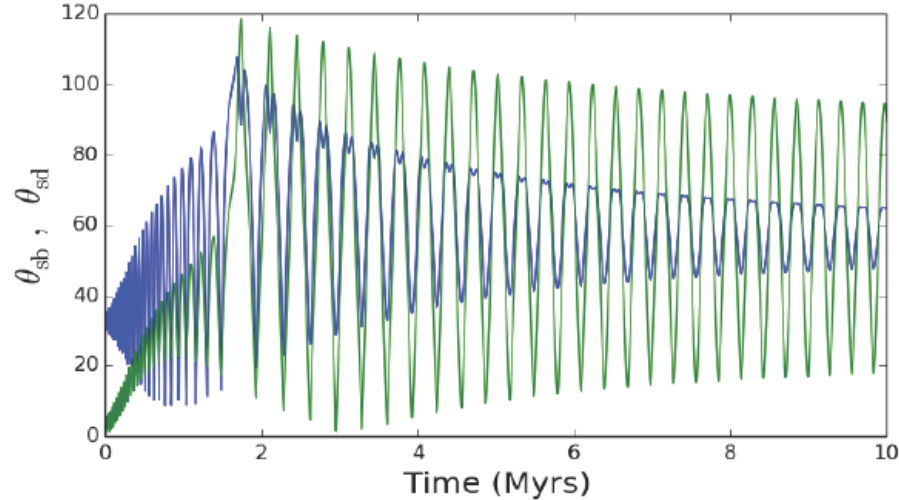
No accretion/magnetic



Accretion/magnetic  
damps SL-angle



Accretion/magnetic  
increases SL-angle



# Summary

## Magnetic star - disk interaction

**Rich MHD and plasma physics:** connection to other field

Interchange instabilities, reconnection, field winding/inflation, outflows, waves, resonances...

**Wide Applications (accreting NSs, WDs, protostars):**

Variabilities (QPOs), jets, spin equilibrium, spinup/down, warp/precession, exoplanets, protoplanetary disks

THANKS