Spring School on Planet Formation, IASTU, Beijing, 05/2014

Early Stages of Planet Formation

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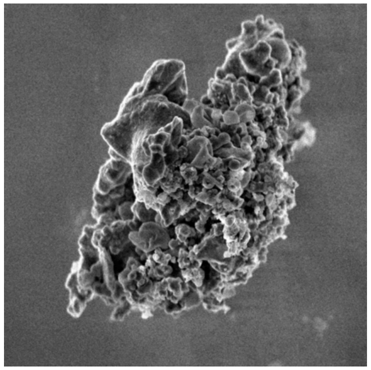
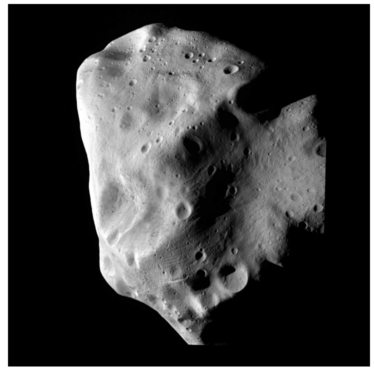
Stages of planet formation

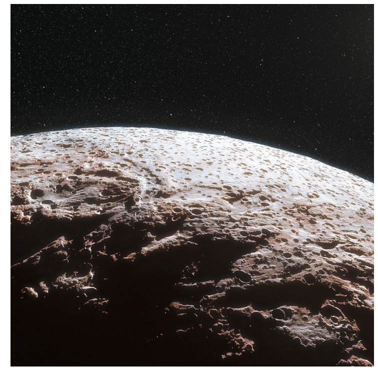


μm

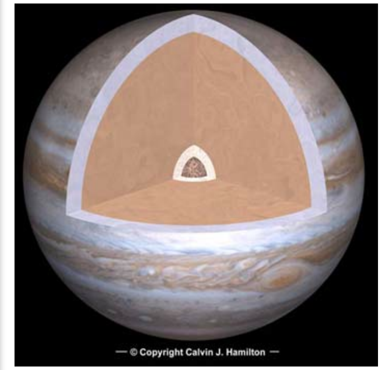
cm



km



10^3km



10^5km

Grain growth

Planetesimal formation

Planetesimal growth to cores

growth/accretion to gas giants

Sticking by surface forces

Miracle?

Gravitational accretion

Topics to be covered

- Aerodynamics of dust/solids
- Grain growth
- Planetesimal formation
- Planetesimal growth
- Summary

Dust particles: aerodynamics

Aerodynamic drag: $$\boldsymbol{F}_{\rm drag} = -M \frac{\Delta \boldsymbol{v}}{t_{\rm stop}}$$

For subsonic motion, $\Delta v \ll c_s \sim v_{\rm th}$:

Epstein regime: particle size a << $\lambda_{\rm mfp}$, drag due to molecular collisions

$$F_{\rm drag} \sim nma^2[(v_{\rm th} + \Delta v)^2 - (v_{th} - \Delta v)^2] \sim \rho_g a^2 v_{\rm th} \Delta v$$

Stokes regime: particle size a >> $\lambda_{\rm mfp}$, fluid drag via molecular viscosity

$$F_{\rm drag} \sim \rho_g \nu \frac{\Delta v}{a} a^2 \sim \rho_g \lambda_{\rm mfp} a v_{\rm th} \Delta v$$

For supersonic motion or motion with high Reynolds number:

$$F_{\rm drag} \sim \rho_g \Delta v^2 a^2$$

Dust particles: aerodynamics

Aerodynamic drag: $$\boldsymbol{F}_{\rm drag} = -M\frac{\Delta \boldsymbol{v}}{t_{\rm stop}}$$ (Weidenschilling 1977)

For subsonic motion, $t_{\rm stop}$ only depends on particle size (a):

Epstein regime: $$t_{\rm stop} = \frac{\rho_s a}{\rho_g c_s} \qquad a < 9\lambda_{\rm mfp}/4$$

Stokes regime: $$t_{\rm stop} = \frac{4a}{9\lambda_{\rm mfp}}\frac{\rho_s a}{\rho_g c_s} \qquad a \geq 9\lambda_{\rm mfp}/4$$

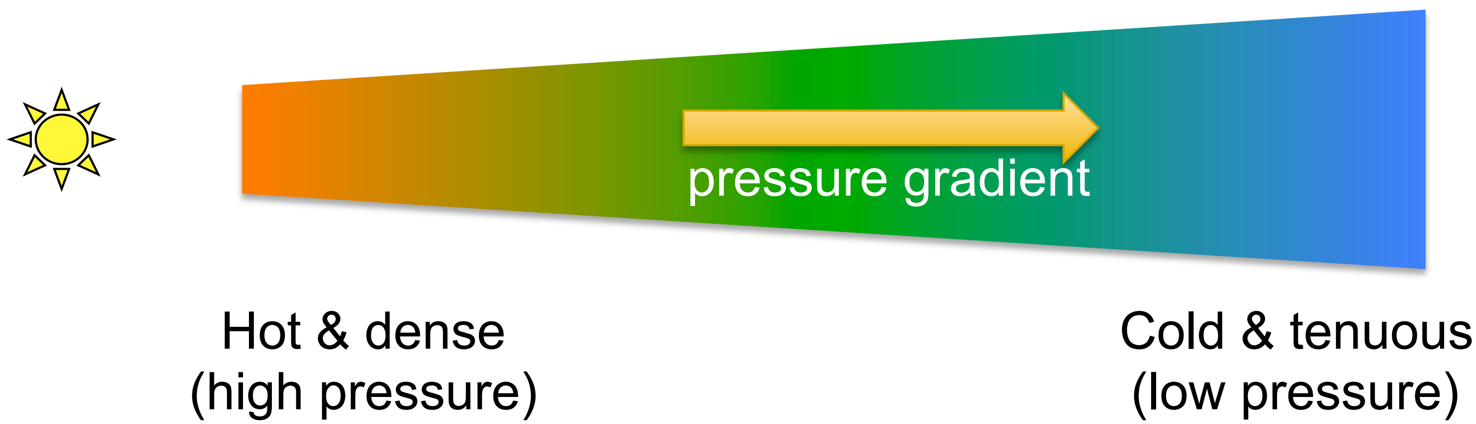
Define dimensionless stopping time, and for MMSN disk at midplane:

$$\boxed{\tau_s \equiv \Omega t_{\rm stop}} = \max\left[4.4\times 10^{-3} a_{\rm cm} r_{\rm AU}^{3/2},\ 1.4\times 10^{-3} a_{\rm cm}^2 r_{\rm AU}^{-5/4}\right]$$

Epstein Stokes

For mm size particles: $\tau_s \sim 10^{-3}$ at 1AU; $\tau_s \sim 0.1$ at 30 AU.

Radial drift of particles



Gas rotates at sub-Keplerian velocity due to pressure support.

$$\Delta u_\phi \equiv -\eta v_K \quad \text{where for MMSN:} \quad \frac{\eta v_K}{c_s} = -\frac{1}{2}\frac{d\ln P}{d\ln R}\frac{c_s}{v_K} \approx 0.05\ r_{\rm AU}^{1/4}$$

Solid particles feel headwind => lose angular momentum and drift inward.

In general, particles drift toward regions with higher pressure.

Radial drift velocity

Consequence: particles spiral inwards, gas slowly drifts outward.

$$u_r = -\eta v_K \frac{2\tau_s}{(1+\epsilon)^2 + \tau_s^2}$$

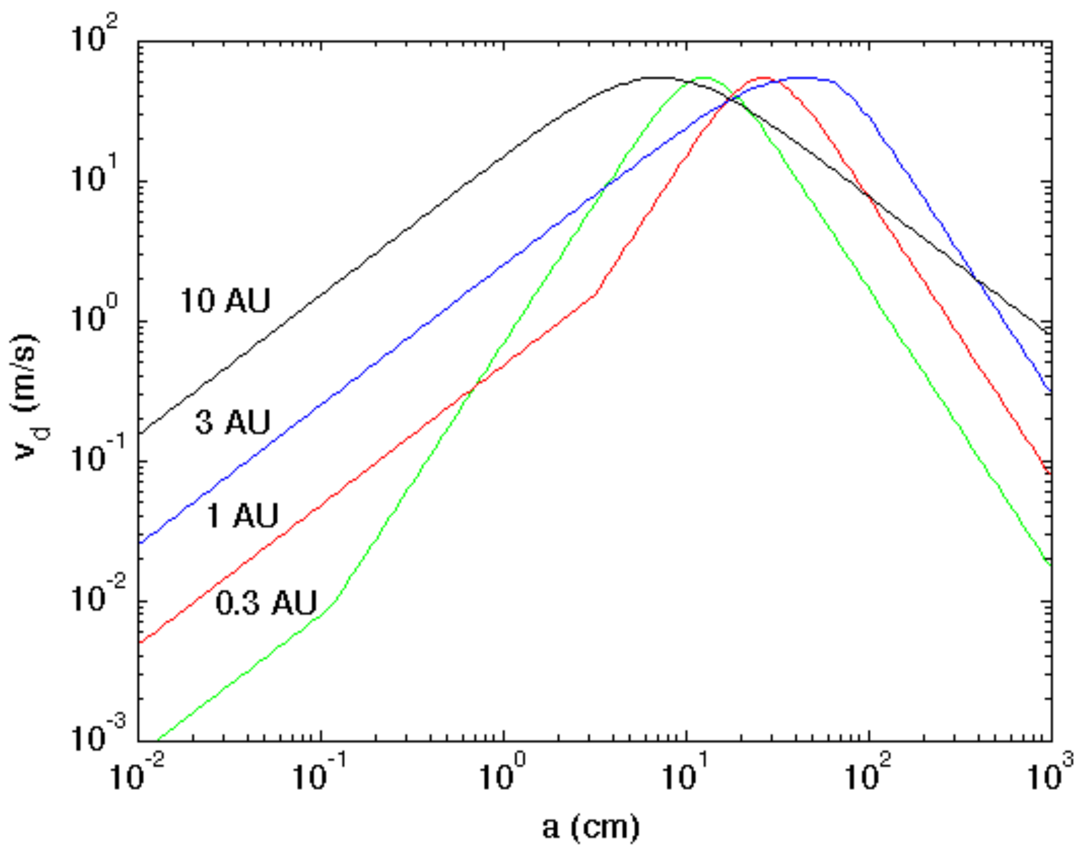
(Nakagawa, Sekiya & Hayashi 1986)

ε: solid to gas density ratio

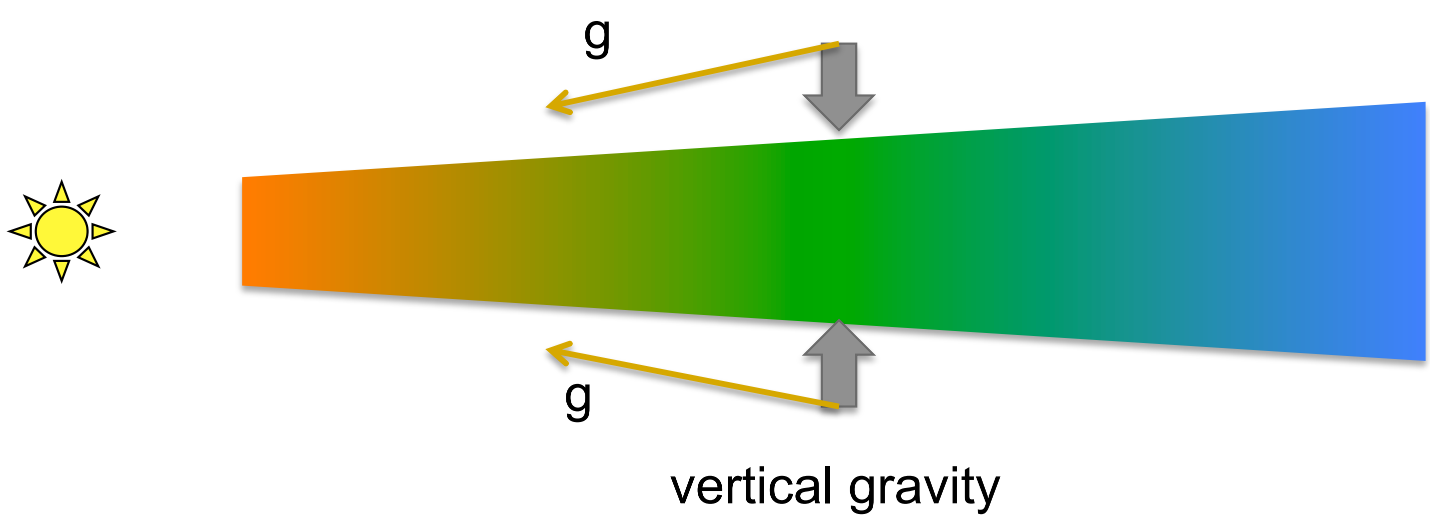
=> dust feedback to gas

Characteristic lifetime for meter sized bodies is only ~100 years at 1 AU!

Severe constraint on the timescale for planetesimal formation!



Vertical settling



Dust feels vertical stellar gravity + gas drag (but no gas pressure).

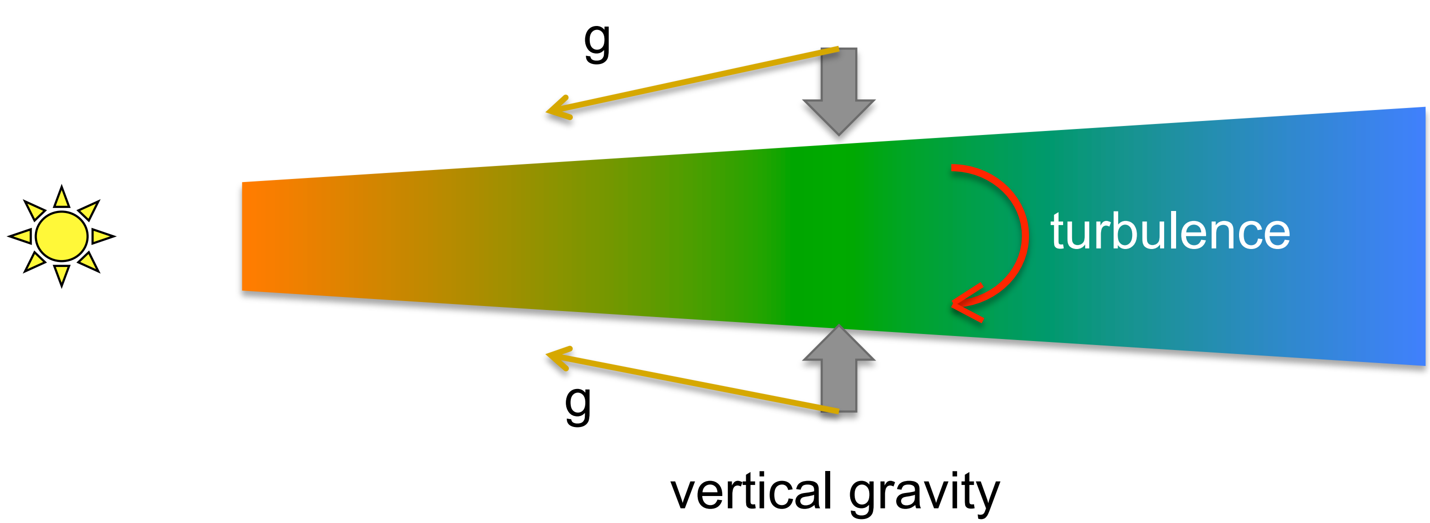
$$\frac{dv_{d,z}}{dt} = -\Omega^2 z - \frac{v_{d,z} - v_{g,z}}{t_{\rm stop}}$$

Large grain => vertical damped oscillation settling timescale ~τ_s

Small grain => terminal velocity: $v_{d,z} = -(\Omega z)\tau_s$ settling timescale ~$1/\tau_s$

Overall settling timescale: $\Omega t_{\rm sett} \sim \tau_s + 1/\tau_s$

Turbulent diffusion



Turbulent diffusion prevents particles from settling indefinitely, which occurs on timescale of

$$t_{\rm diff} \sim H_p^2 / D_p$$ where D_p is the particle diffusion coefficient.

$D_p \sim D_g$ for small particles (τ_s<<1), but $D_p < D_g$ for big particles.

Particle vertical scale height: $$H_p \sim \sqrt{\frac{D_g}{\Omega \tau_s}}$$

Note:
$$D_g \sim \nu \sim \alpha c_s H$$

Grain growth

Coagulation equation:

(Smoluchowski 1916)

$$\frac{\partial n(m)}{\partial t} = \frac{1}{2}\int_0^m A(m', m-m')n(m')n(m-m')dm' - n(m)\int_0^\infty A(m', m)n(m')dm'$$

$n(m)dm$: number density of solids in mass range between m and $m+dm$.

$A(m_1,m_2)$: coagulation kernel

$$A(m_1, m_2) = P(m_1, m_2, \Delta v)\Delta v(m_1, m_2)\sigma(m_1, m_2)$$

Further generalizations/complications to account for:

- Fragmentation/bouncing
- Different kinds of grains (composition/porosity, etc.)
- Distribution of collision velocity
- Radial and vertical transport/diffusion of grains
- Coupled with global disk evolution.

Relative velocities

(e.g., Birnstiel+ 2010)

Differential radial drift:
$$\Delta u_{RD} = \left| \eta v_K \left(\frac{2\tau_{s,1}}{1+\tau_{s,1}^2} - \frac{2\tau_{s,2}}{1+\tau_{s,2}^2} \right) \right|$$

Differential azimuthal motion:
$$\Delta u_{\phi} = \left| \eta v_K \left(\frac{1}{1+\tau_{s,1}^2} - \frac{1}{1+\tau_{s,2}^2} \right) \right|$$

Effective for collisions between big and small grains.

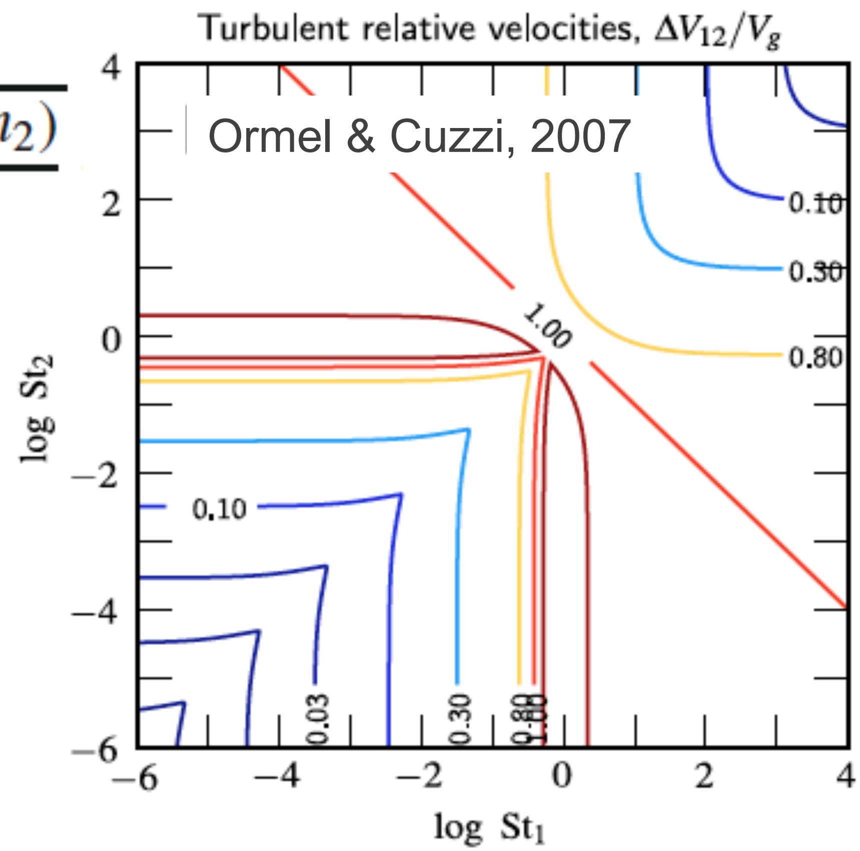
Brownian motion: $\Delta u_{\rm BM} = \sqrt{\frac{8 k_{\rm B}\, T (m_1 + m_2)}{\pi\, m_1\, m_2}}$

Effective for small grains.

Turbulent motion

More complex physics, and depends on the nature of turbulence.

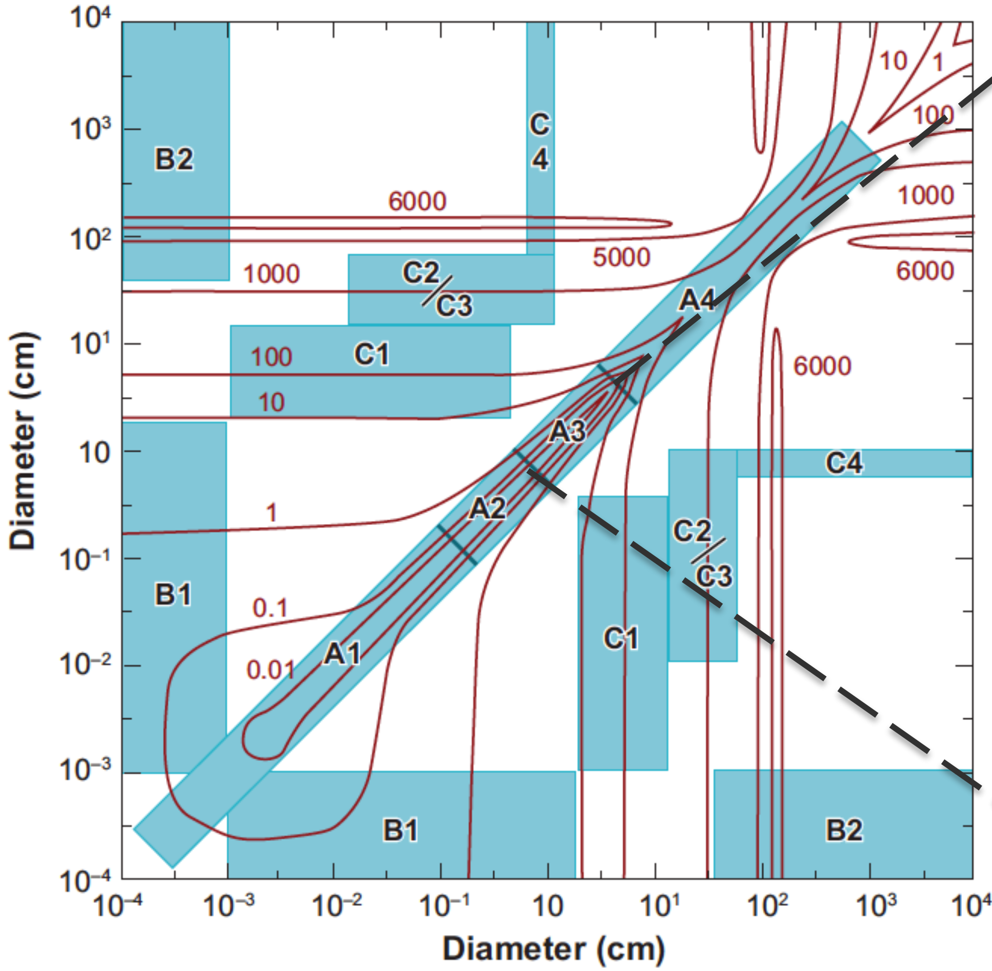
Effective when at least one body is big.



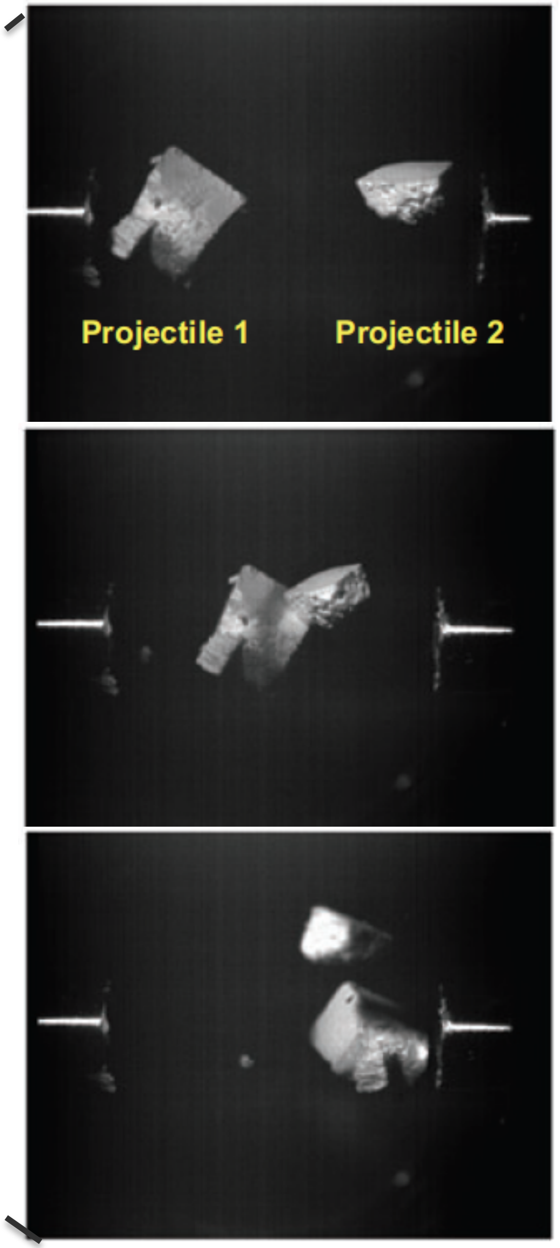
Outcome of collisions: laboratory experiments

Identify the parameter space.

Perform micro-gravity experiments to check outcome.



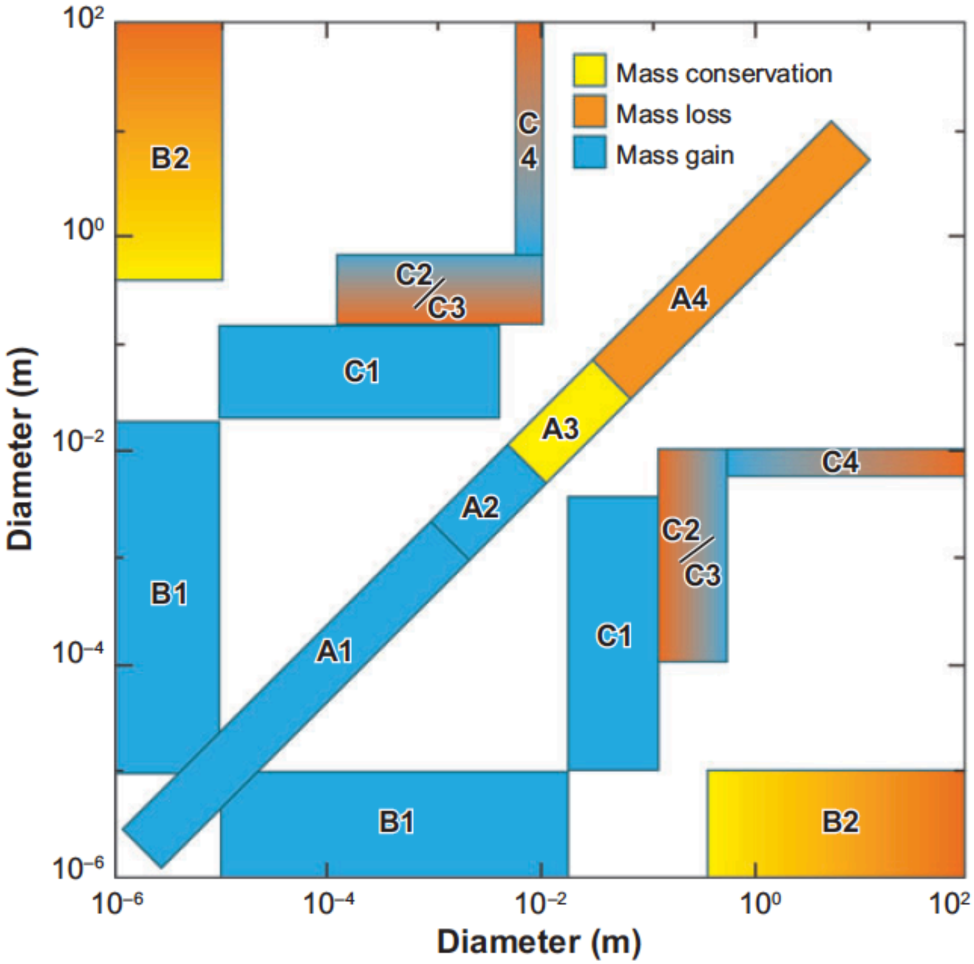
(Blum & Wurm 2008)



Coagulation barrier

Identify the parameter space.

Perform micro-gravity experiments to check outcome.



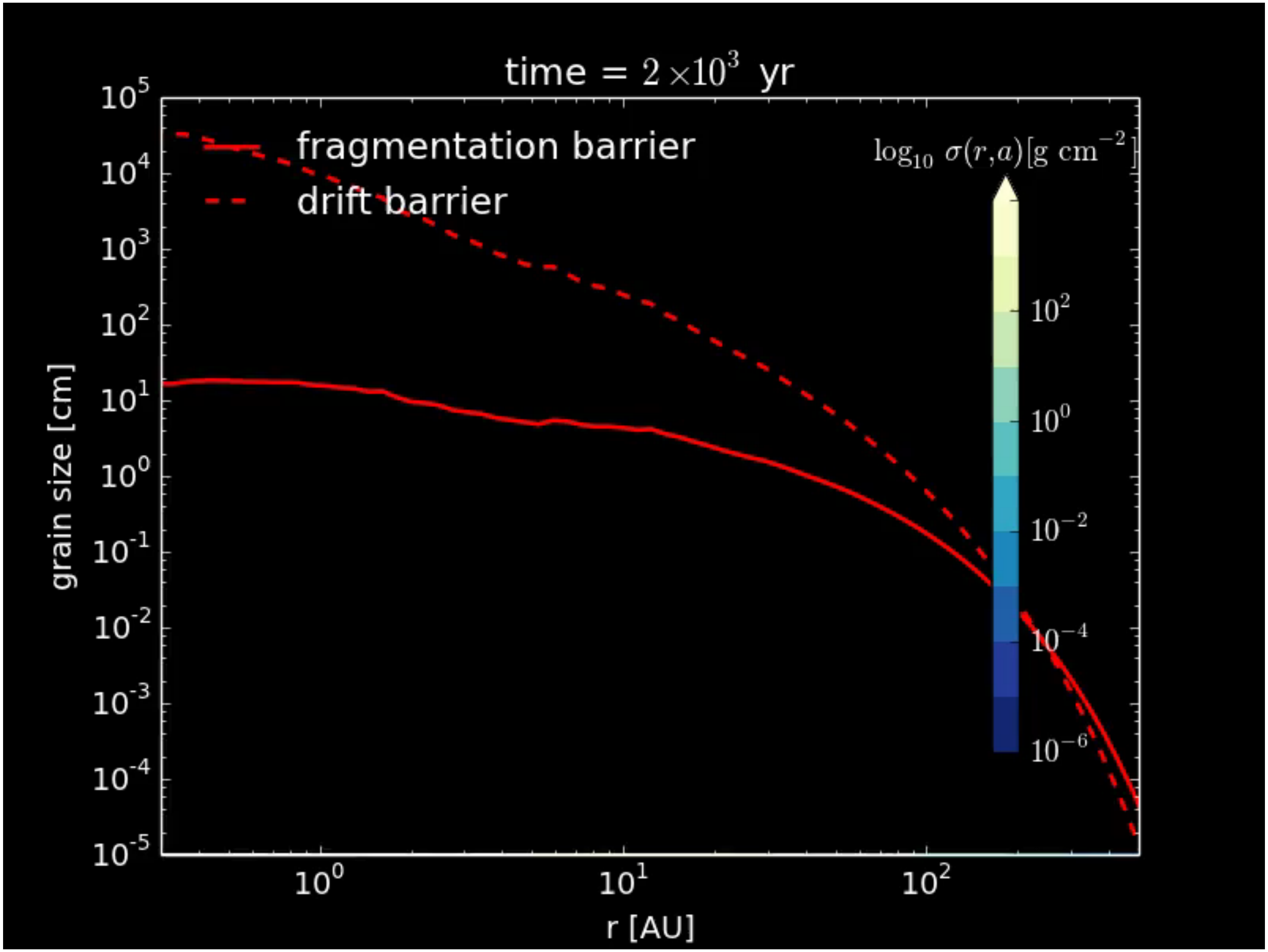
(Blum & Wurm 2008)

Collision velocity >1 m/s typically leads to bouncing or fragmentation.

Difficult to yield particles with size > a few cm due to these barriers.

(Brauer et al. 2008, Birnstiel et al.2010, Guittler et al. 2010, Zsom et al. 2010, 2011)

Coagulation simulations



Credit: Til Birnstiel

Can solids grow bigger?

Can larger objects also grow via coagulation? For an estimate, let us consider a population of boulders of size $s = 1$ m orbiting at 1 AU. The collision time scale is still given by Eq. (4.56), but in this size regime the relative velocity is dominated by radial drift due to aerodynamic forces against the disk gas. If we assume that a boulder of this size is close to the peak of the radial drift curve (shown as Fig. 4.2) then a plausible value for the relative velocity is,

$$\Delta v \sim 10^{-3} v_K \sim 3 \times 10^3 \,\mathrm{cm\,s^{-1}}. \qquad (4.64)$$

One may legitimately worry about whether collisions at $30\,\mathrm{m\,s^{-1}}$ will result in growth as opposed to fragmentation.[9] Finessing that question for the time being,

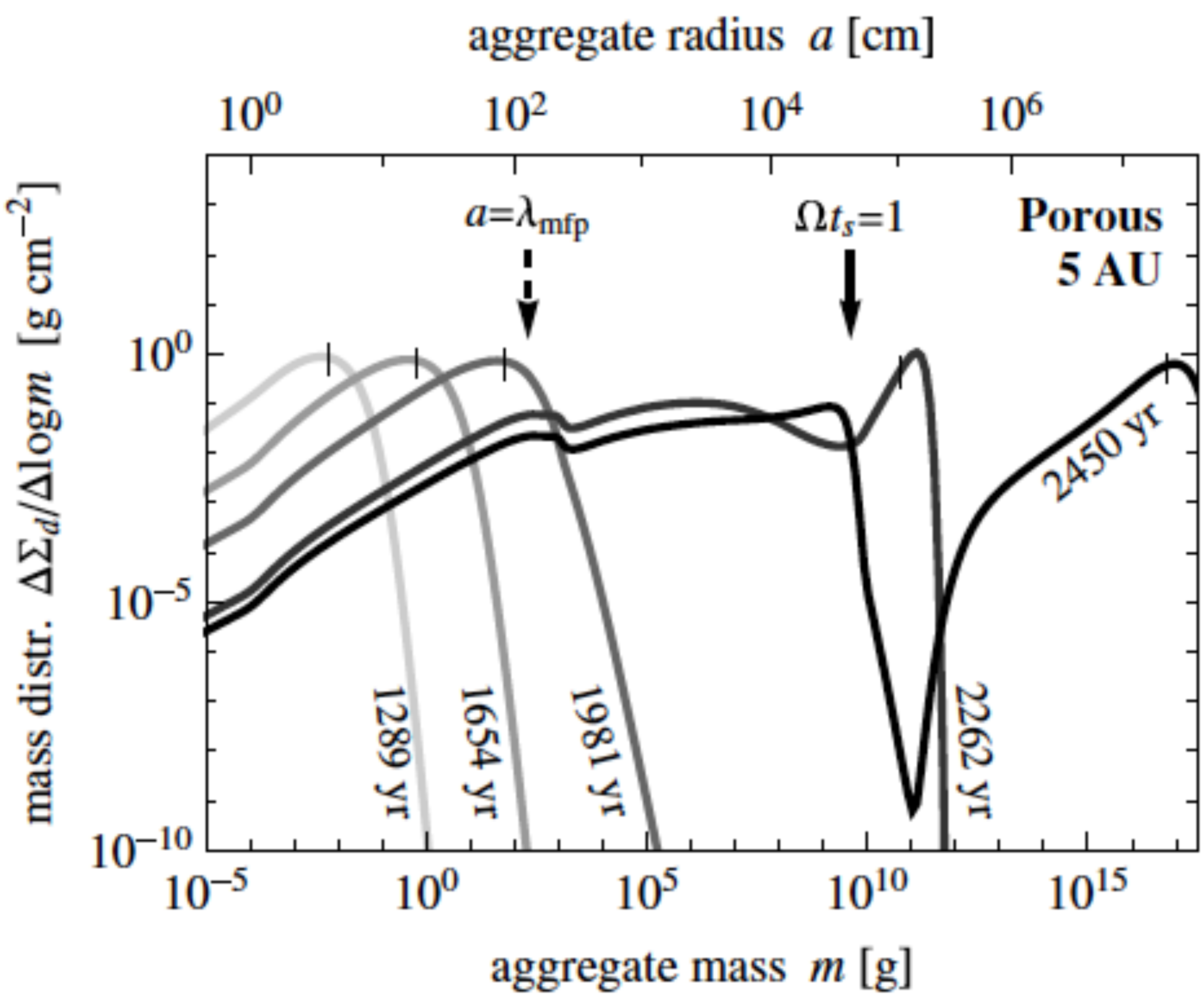
—— P. Armitage's book, p130

[9] The noted astrophysicist Doug Lin has been known to challenge advocates of coagulation theories to retire to the desert and return only once they have gotten such rocks to stick together upon impact!

Can solids grow bigger?

Resolution 1: Porous/fluffy icy grains beyond the snowline

May sustain collision velocity up to 30-70 m/s. (Wada+ 2009)

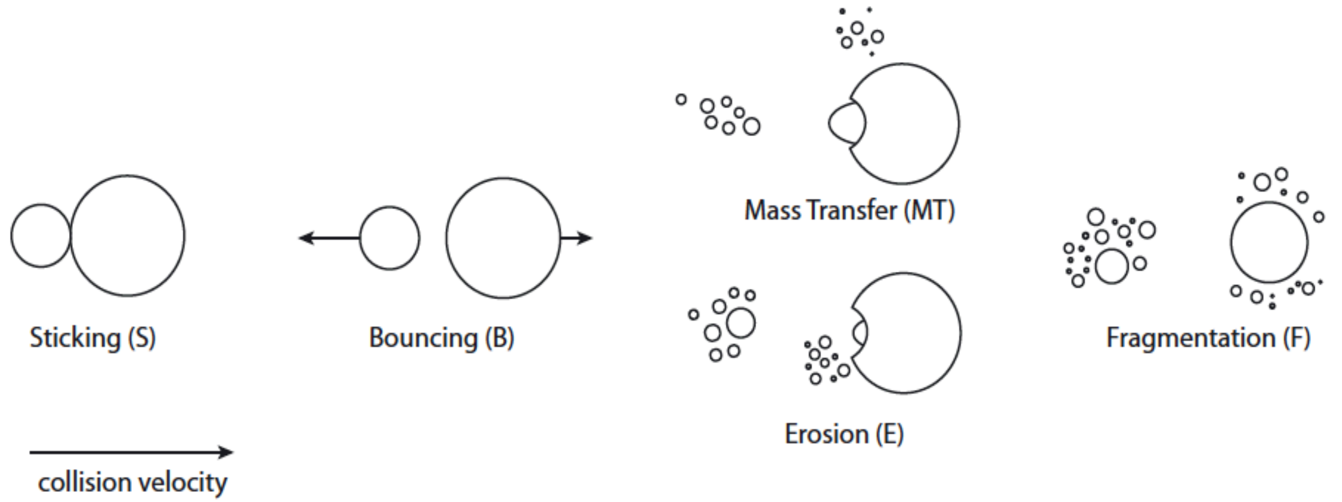


Unlikely to reach the fragmentation barrier, and can overcome the radial drift barrier by transitioning to the Stokes regime.

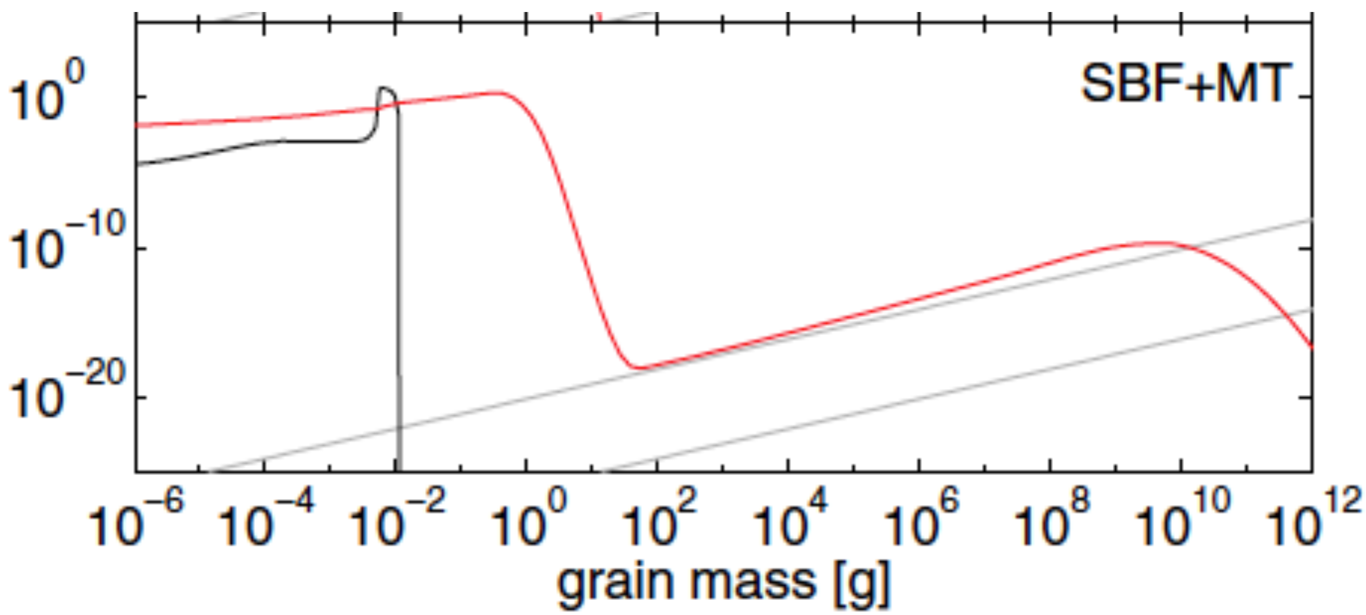
(Okuzumi+ 2012)

Can solids grow bigger?

Resolution 2: New regime of mass transfer + lucky particles

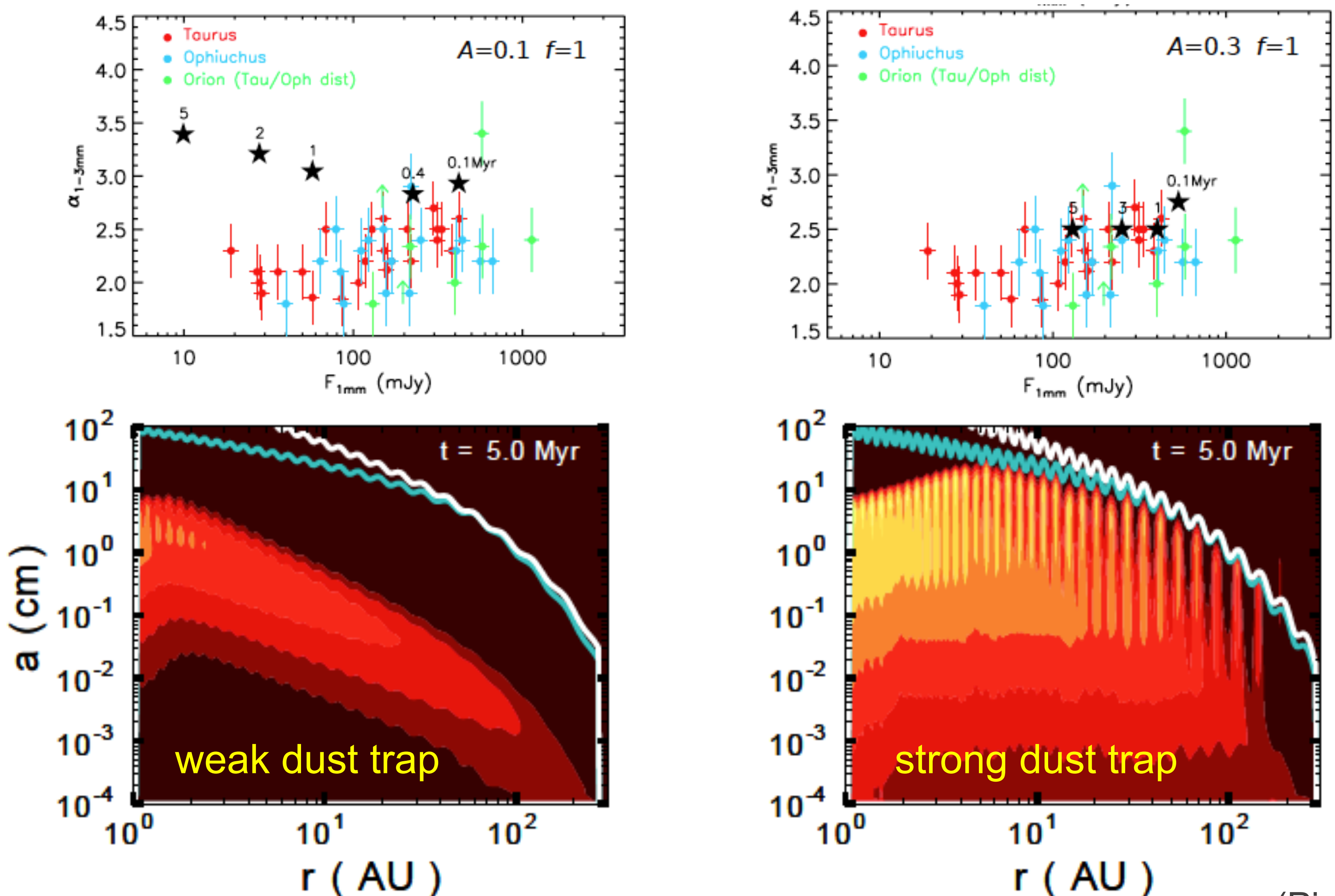


(Guttler+ 2010, Windmark+ 2012)



Radial drift revisited and need for dust traps

Disk observations indicate that disks remain dust-rich for several Myrs.



(Pinila+ 2012)

Dust trap: artificially add radial pressure variations.

Potential sources for the dust traps

- **Zonal flow:** banded flow pattern due to radial pressure variations

  Considered as a general consequence of the MRI turbulence.

  (Johansen+ 2009)

  Likely be enhanced in non-ideal MHD with net vertical magnetic flux, but remains to be convincingly demonstrated.

- **Vortices:** anti-cyclones with pressure maxima at vortex center

  May result from various hydrodynamical instabilities, particularly, the baroclinic vortex amplification. (see the previous lecture)

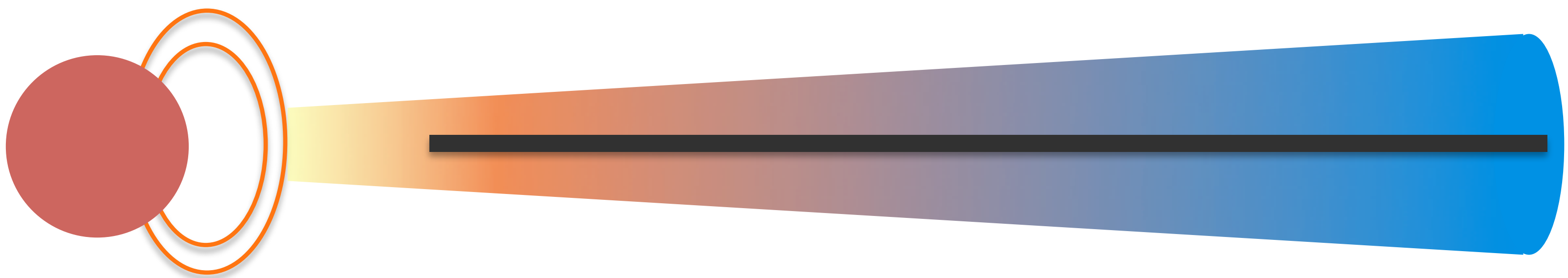
Exercise:

Given the MMSN disk model, for mm sized grains at 100 AU at disk midplane, what is the radial drift timescale?

Planetesimal formation

- Gravitational instability Goldreich & Ward (1973)

Without external turbulence, particles settle to an infinitely thin layer, which is unstable to GI, and collapse to planetesimals.



Instability requires:

$$Q \equiv \frac{c\kappa}{\pi G \Sigma} < 1$$

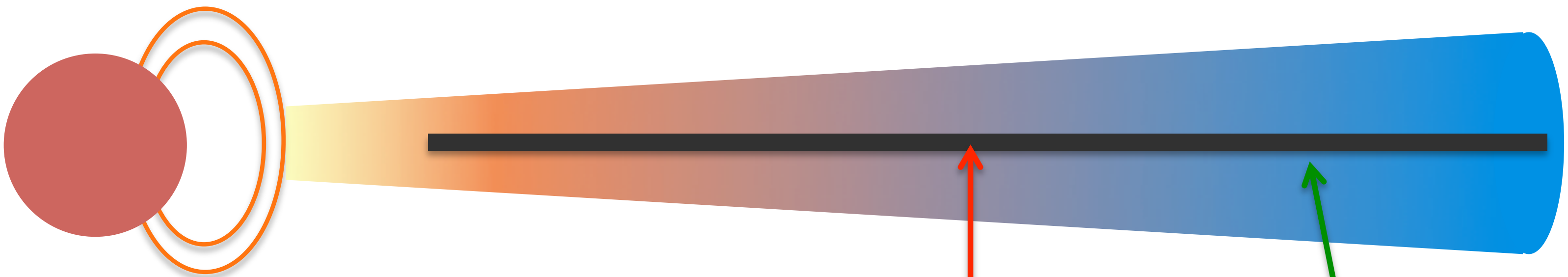
c: velocity dispersion of dust.
κ: epicyclic frequency = Ω
Σ: surface den of the dust layer

Requires excessively low level of turbulence to meet Q<1.

Issue with GI: Kelvin-Helmholtz instability

Before the onset of GI, the dust layer is unstable to the KHI:

(Weidenschilling 1980)



Gas beyond dust layer: sub-Keplerian rotation

Gas in the dust layer: close to Keplerian rotation due to dust feedback

Vertical shear in azimuthal velocity

GI requires enrichment of solids by a factor of at least 4 (Z>0.06) (Lee+ 2010a,b)

Streaming instability (SI)

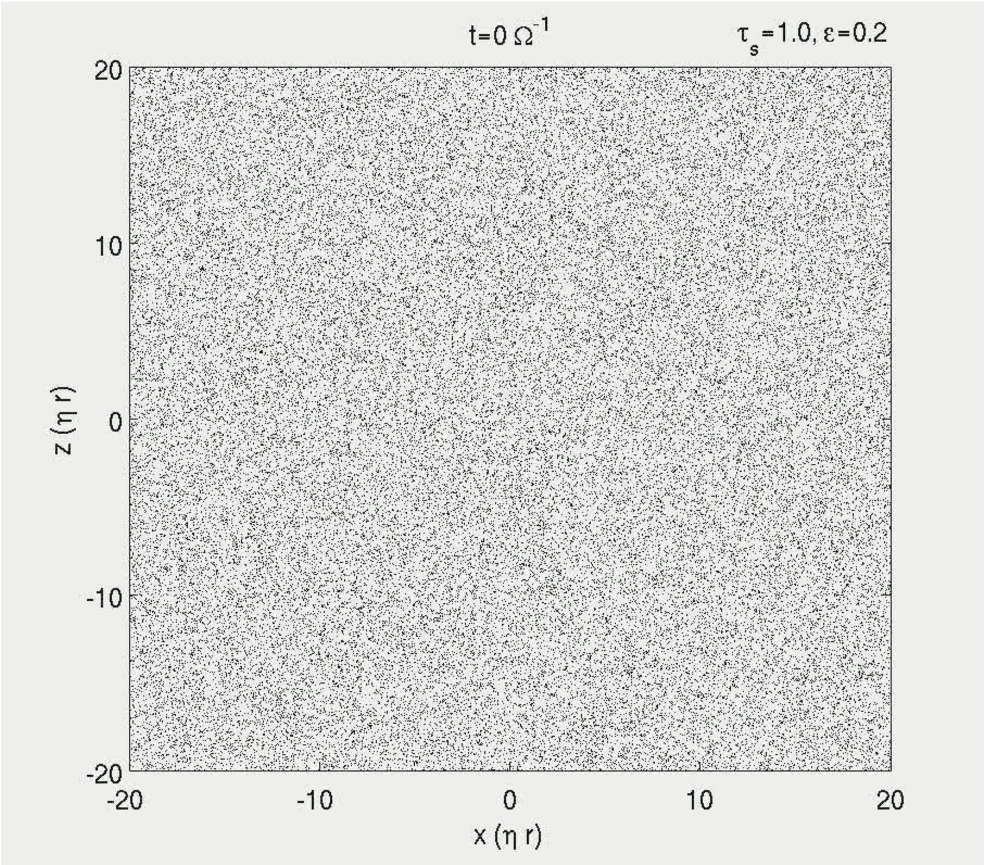
Momentum feedback from particles to the gas leads to a linear instability.

(Goodman & Pindor 2001, Youdin & Goodman 2005, Jacquet et al. 2011)

Most efficient for marginally coupled particles

$$\tau_s \sim 0.1 - 1$$

Energy budget:
radial pressure gradient



(Johansen &Youdin 2007, Bai & Stone 2010a)

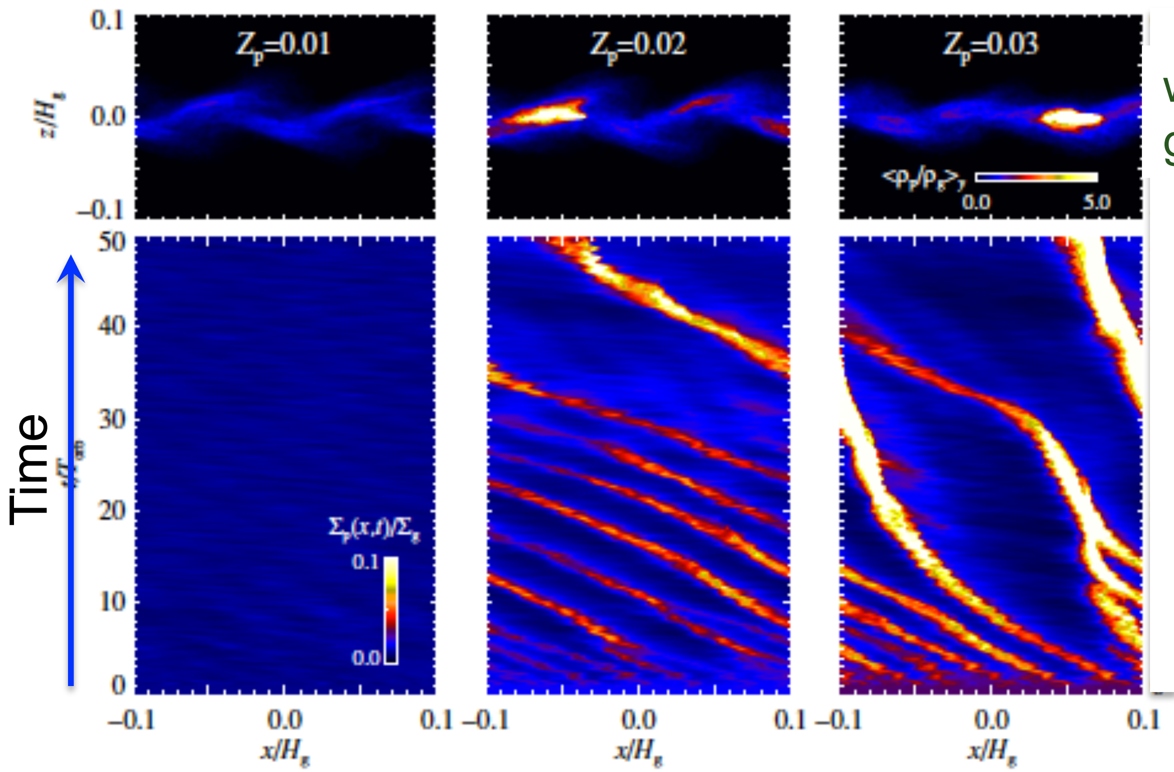
Non-linear evolution can efficiently concentrate particles!

Planetesimal formation from the SI

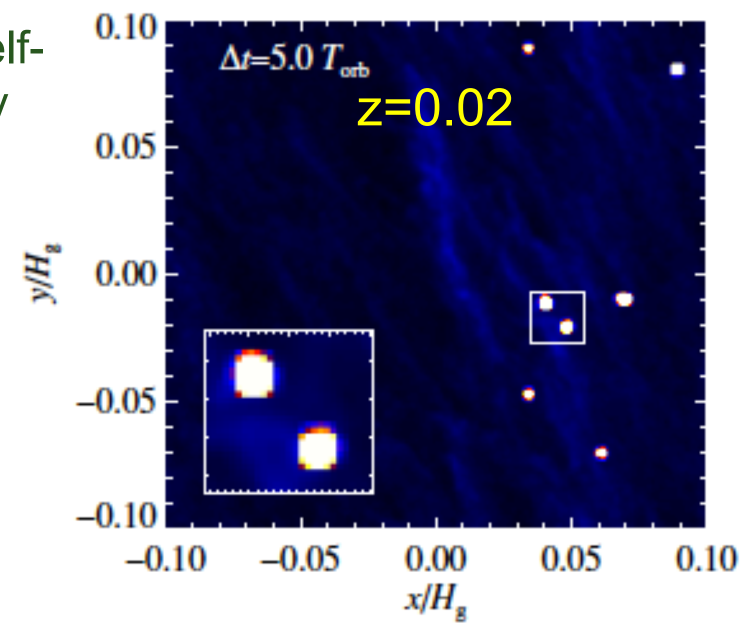
Local shearing box simulations with vertical stratification

Particle stopping time $\tau_s = 0.1 - 0.4$ (Johansen+ 2009,2011,2012)

Height-integrated solid to gas mass ratio (dust abundance): Z=0.01-0.03



w/o self-gravity



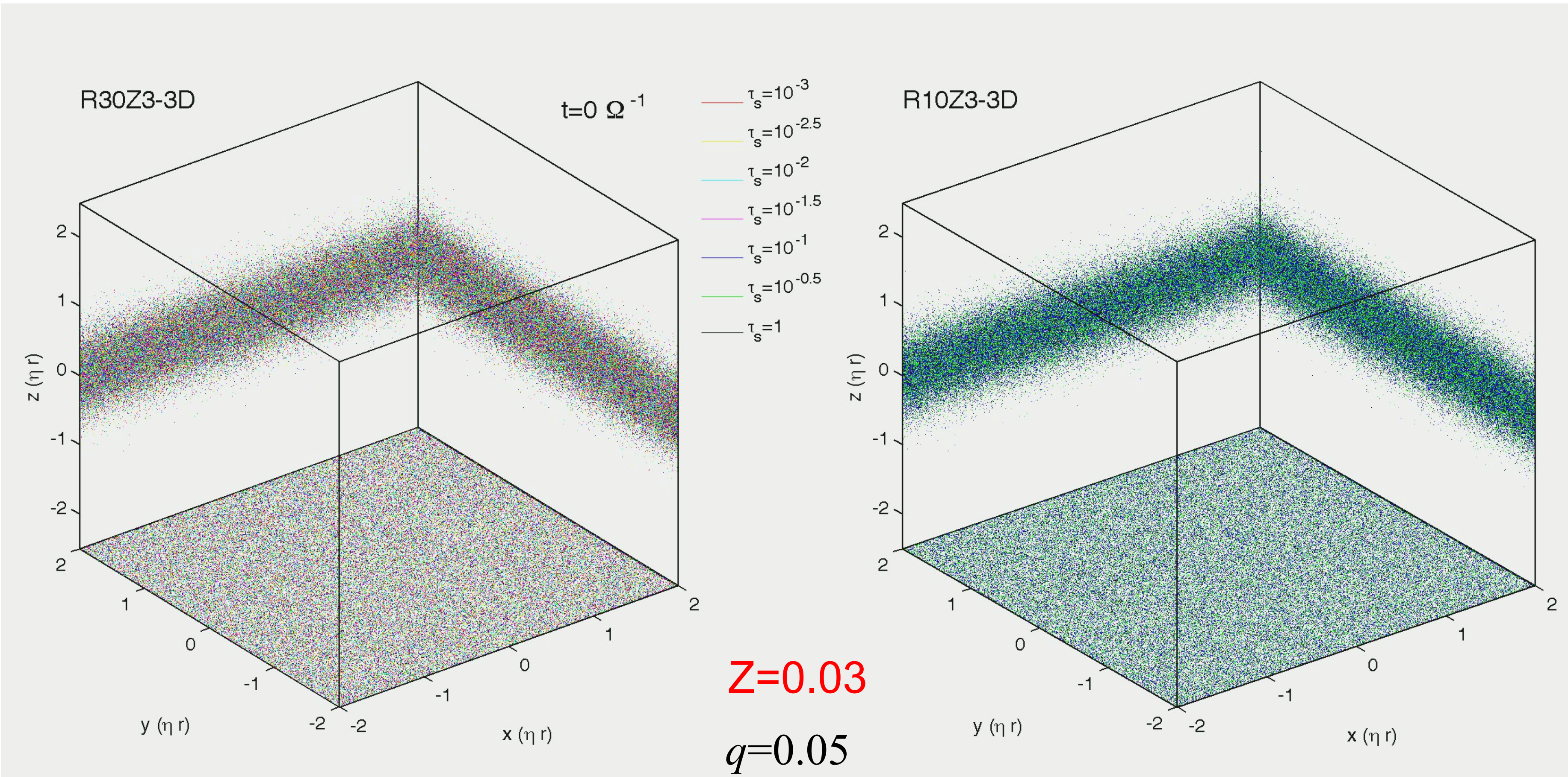
self-gravity turned on after saturation

Strong metallicity dependence!

Planetesimal formed in the computer!

Simulations with broader size distribution

$\tau_s = 10^{-3} - 1$ $\tau_s = 10^{-1} - 1$



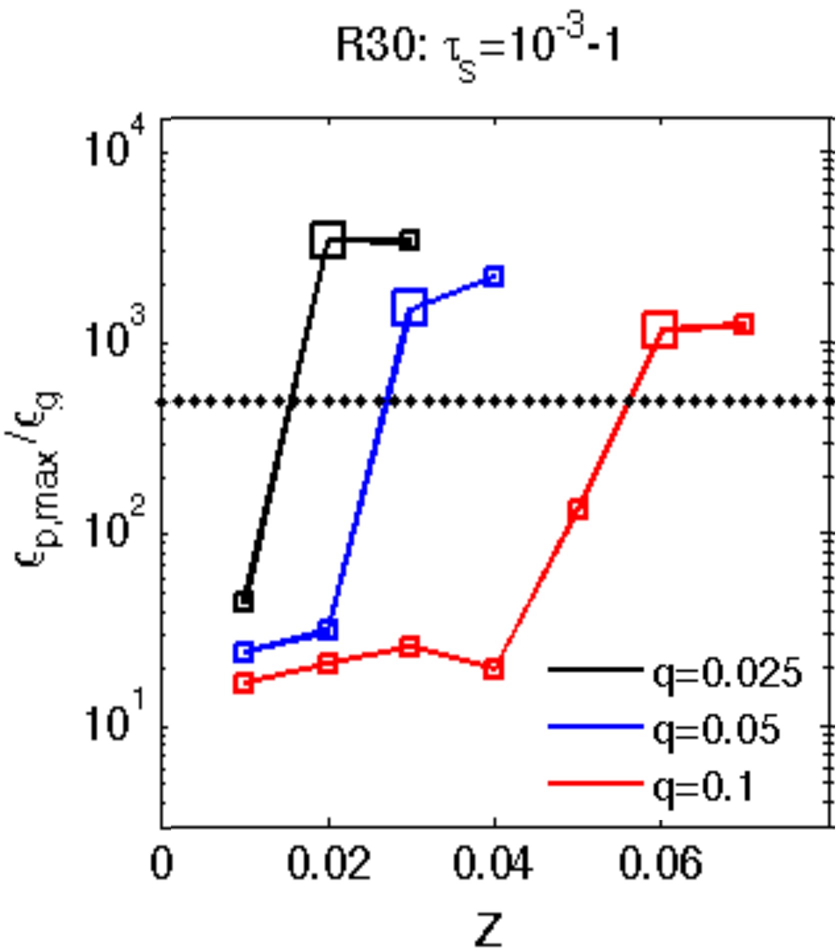
(Bai & Stone, 2010b)

Length unit: qH_g

Marginally coupled particles participate in SI and concentrate into clumps. Smaller particles behave passively.

Particle clumping: parameter dependence

2D simulations



(Bai & Stone, 2010c)

Roche density: $\rho_{\rm Roche} \approx \frac{3M_*}{r^3}$

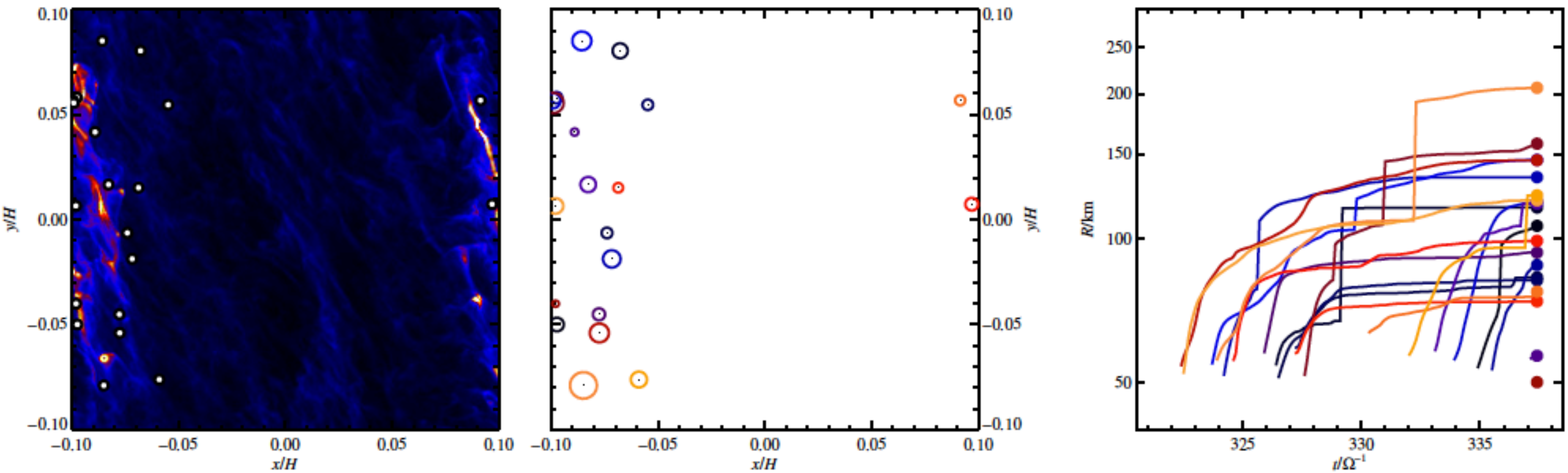
Particle clumping strongly depends on dust abundance

There exists a $Z^{\rm crit}$ above which planetesimal formation occurs.

Particle clumping strongly depends on the pressure gradient

$Z^{\rm crit}$ sensitively depends on pressure gradient, small gradient strongly favors particle clumping.

Initial mass function of planetesimals



Streaming instability simulations + self-gravity typically form planetesimals of the order 100 km in size.

(Johansen+ 2014, PPVI)

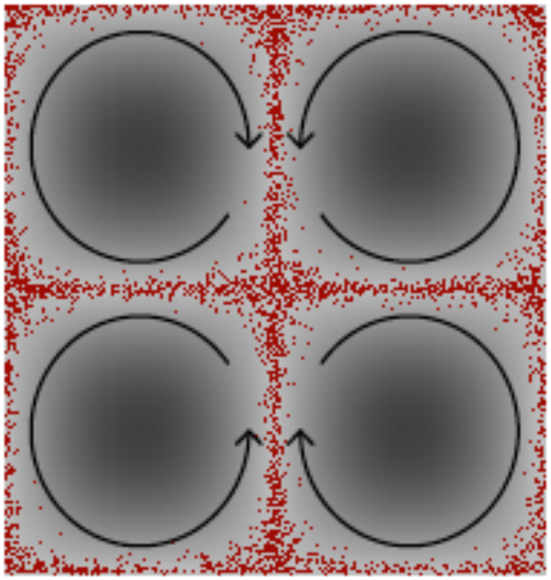
Caveats: more small planetesimals emerge with higher resolution; clumps may fragment into binaries in reality.

Evidence of planetesimals formed big (100-1000km) in asteroid belt (Morbidelli+ 2009)

Evidence of planetesimals formed small (0.4-4km) in the Kuiper belt (Schliching+ 2013)

Planetesimal formation: other channels

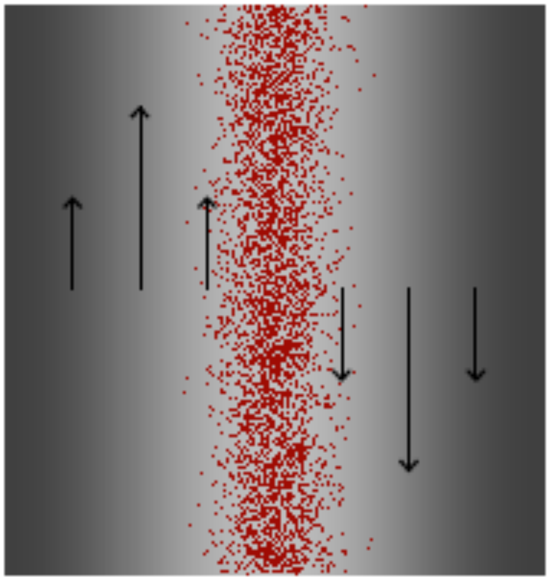
Eddies



$l \sim \eta \sim 1$ km, St $\sim 10^{-5}$–10^{-4}

(Johansen+ 2014, PPVI)

Pressure bumps / vortices



$l \sim 1$–$10\ H$, St ~ 0.1–10

Turbulent concentration:

Particles with stopping time ~ eddy turnover time get clustered between smallest turbulent eddies.

(Cuzzi+ 2001, 2008; Pan+ 2011; Hopkins 2013)

Particles concentration in dust traps:

Traps provided by zonal flows/vortices or at special disk locations.

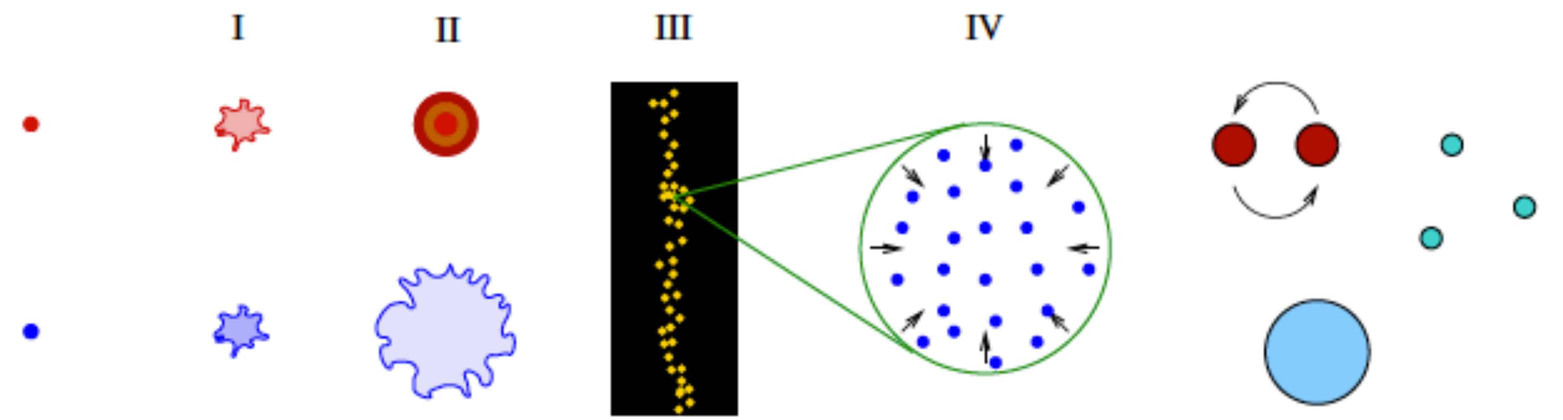
Enhanced collision/growth rate;

Small collisions velocity: avoid fragmentation.

A lot of works ongoing NOW!

Current understanding

Planetesimal formation by coagulation and self-gravity!



I initial growth of fluffy aggregates

II compactification and mass transfer or continued fluffy growth

III concentration in pressure bumps, in vortices, or by streaming instabilities

IV gravitational collapse with collisions to yield planetesimal Initial Mass Function

from Johansen+ 2014, PPVI talk

Very active field of research: involves complex physics of gas dynamics, dust dynamics, and their mutual interaction.

Planetesimals to cores: naive timescales

Consider a disk of planetesimals with size R, surface density Σ (MMSN scaling), velocity dispersion u. Assume collisions always lead to growth.

If collision cross section is geometric:

Collision rate: $n\pi R^2 u$

Planetesimal disk scale height: $h \approx u/\Omega$ ⇨ $n \approx \Sigma/Mh \approx \Sigma\Omega/Mu$

Growth rate:
$$\frac{dR}{dt} = \frac{R}{3M}\frac{dM}{dt} \approx \frac{\Sigma\Omega}{4\rho_p} \approx \frac{30\mathrm{cm}}{\mathrm{yr}}\left(\frac{a}{\mathrm{AU}}\right)^{-3}$$

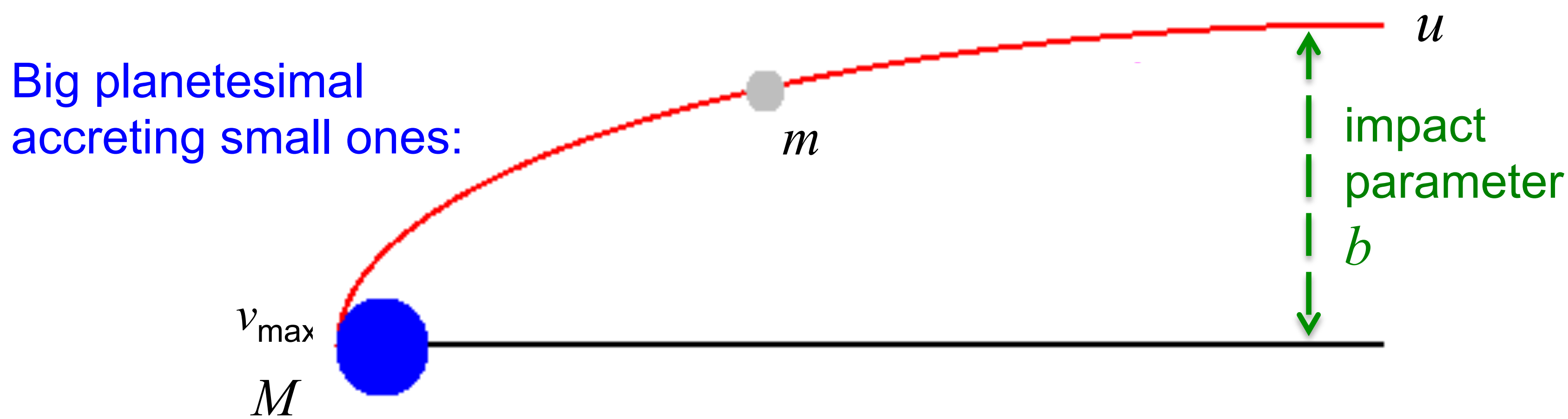
To build Earth:	~ 10^7 yr
Jupiter's core:	~ 10^9 yr
Neptune:	~ 10^{12} yr

Way too long!

Need acceleration!

Gravitational focusing

Consider two-body encounter, ignoring disk rotation for the moment.



For closest approach, angular momentum conservation gives:

$$v_{\rm max} R = ub$$

Energy conservation gives:

$$\frac{u^2}{2} = \frac{v_{\rm max}^2}{2} - \frac{GM}{R}$$

$$b^2 = R^2 + \frac{2GMR}{u^2} = R^2\left(1 + \frac{v_{\rm esc}^2}{u^2}\right)$$ where $$v_{\rm esc}^2 \equiv \frac{2GM}{R}$$

Gravitational focusing

Consider two-body encounter, ignoring disk rotation for the moment.

Enhancement of collisional cross section:

$$b^2 = R^2\left(1+\frac{v_{\rm esc}^2}{u^2}\right) \xrightarrow{\text{cross section}} \sigma = \pi R^2\left(1+\frac{v_{\rm esc}^2}{u^2}\right)$$

where

$$v_{\rm esc}^2 \equiv \frac{2GM}{R}$$

Collision rate: $$f_{\rm coll} = n\sigma u \approx \frac{\Sigma}{mh}\sigma u = \frac{\Sigma\Omega}{m}\sigma$$

Growth rate: $$\frac{1}{R}\frac{dR}{dt} \approx \frac{1}{M}\frac{dM}{dt} \approx \frac{m}{M} f_{\rm coll}$$

$$\Rightarrow \frac{1}{R}\frac{dR}{dt} \approx \frac{\Sigma\Omega}{\rho R}\begin{cases} 1 & \propto R^{-1} \quad (u \gtrsim v_{\rm esc}) \\ (v_{\rm esc}/u)^2 & \propto R \quad (u < v_{\rm esc}) \end{cases}$$

Ordered growth ($u \gtrsim v_{\rm esc}$)

Runaway growth ($u < v_{\rm esc}$)

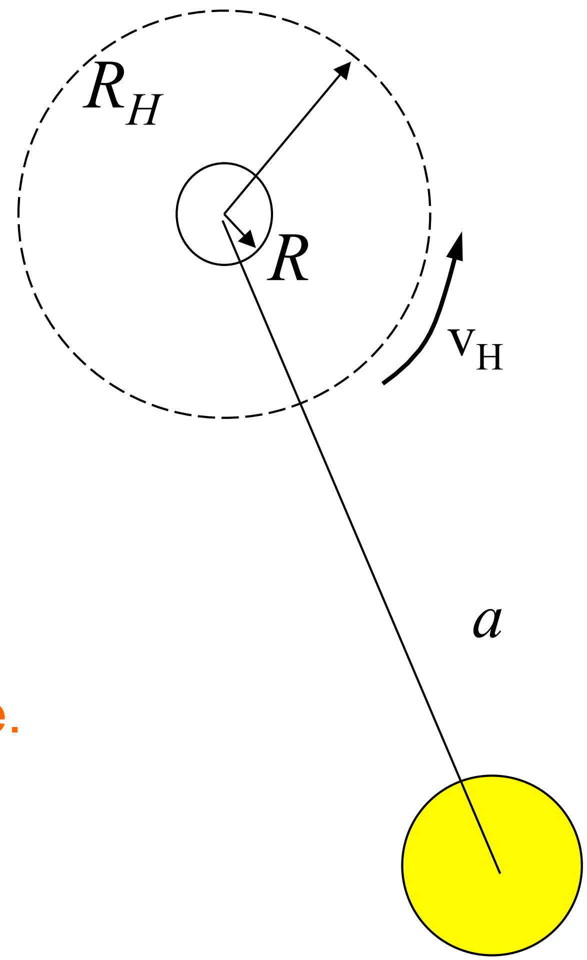
(for constant u)

Need a "cold" population of planetesimals for efficient growth.

Effect of differential rotation

Hill radius: $$R_H \sim \left(\frac{M}{M_*}\right)^{1/3} a$$

Hill velocity: $$v_H \equiv \Omega R_H = \sqrt{\frac{GM}{R_H}}$$



Dispersion dominated regime:

$$u > v_H$$ Simple two-body dynamics.

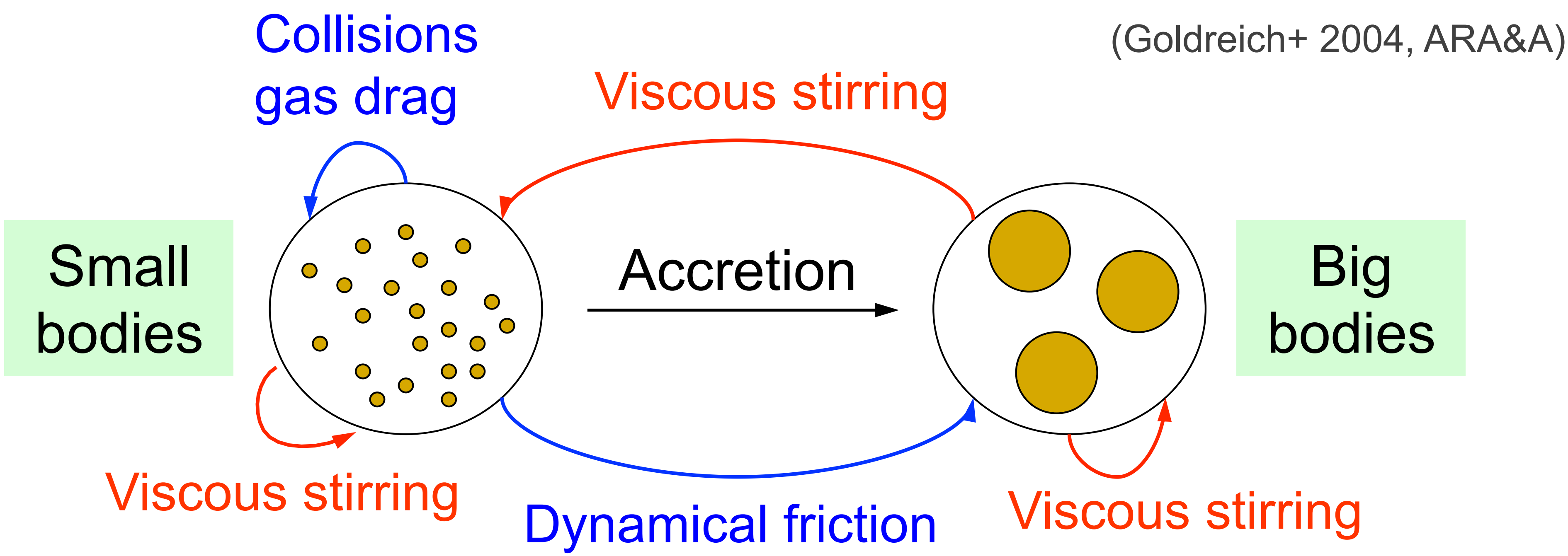
Previous analysis of runaway growth applies here.

Shear dominated regime:

$$u < v_H$$ Three-body effects must be considered.

Runaway growth proceeds further, but slower.

What determines velocity dispersion?

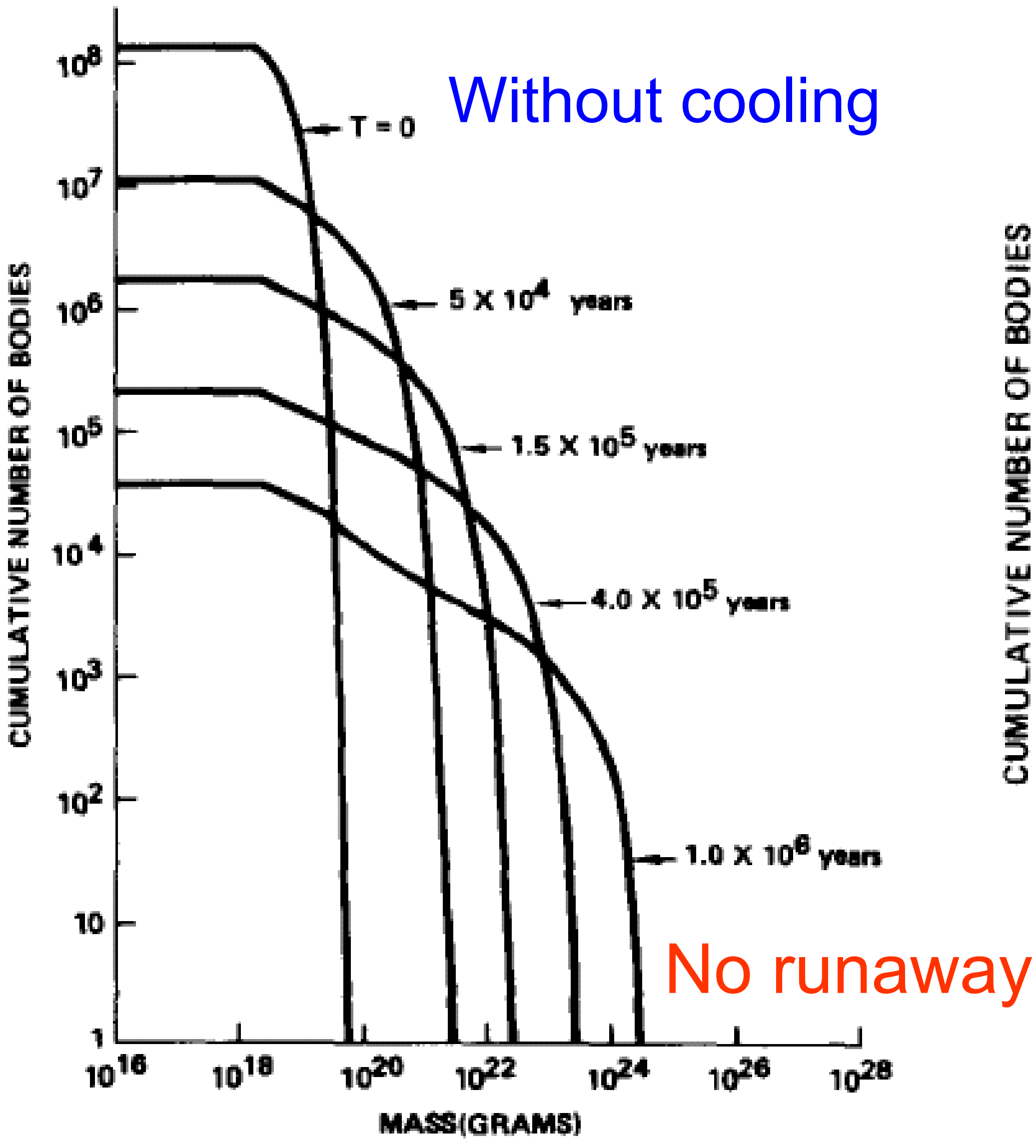


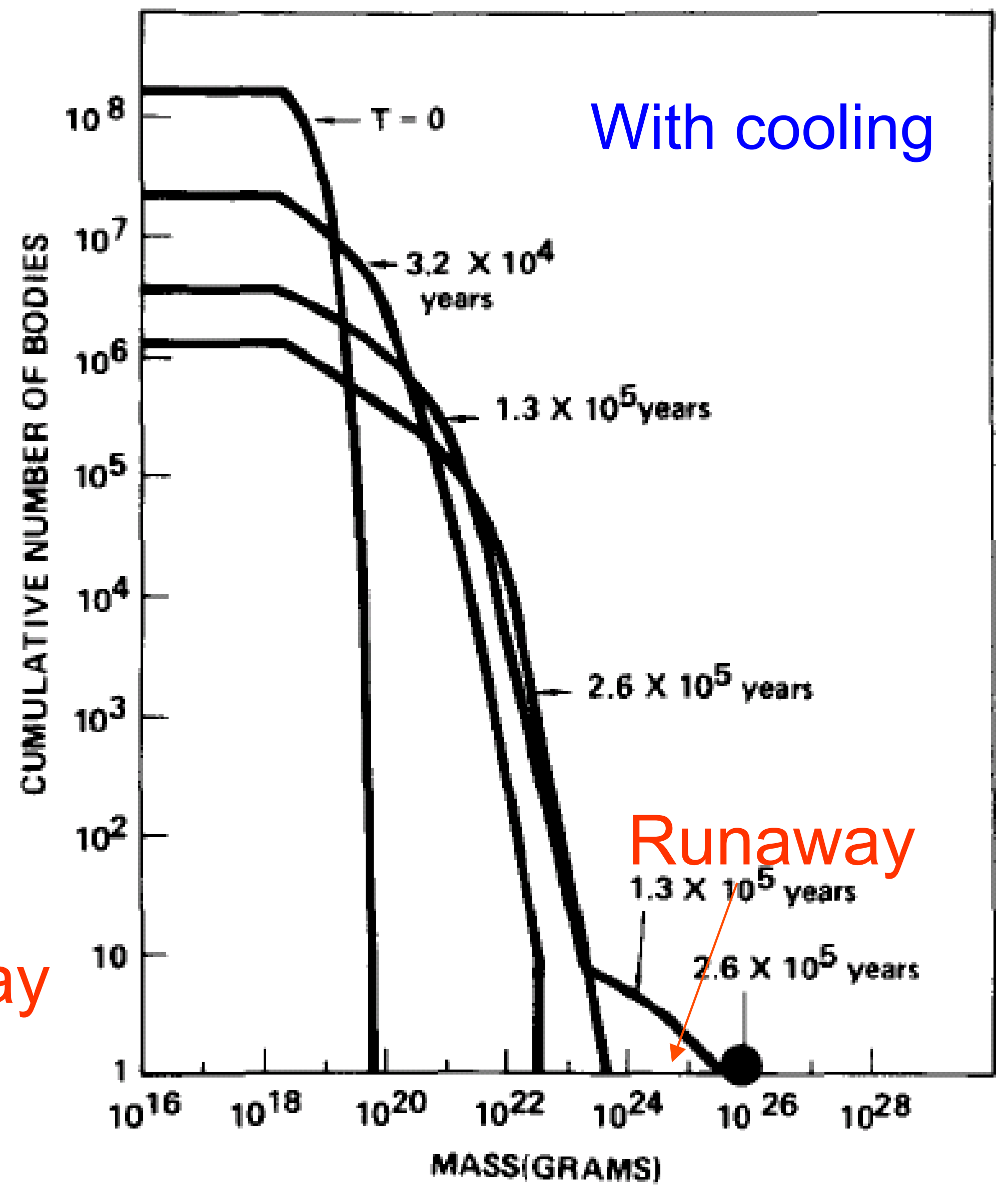
Small bodies are heated by big bodies, cooled by mutual collisions and gas drag.

Big bodies are heated by mutual viscous stirring, and cooled by dynamical friction.

Runaway growth

(Wetherill & Stewart 1989)





Oligarchic growth

(Kokubo & Ida 1998, 2000)

Once big bodies grow sufficiently massive, they can strongly stir up the eccentricities of neighboring small bodies (u increases with M).

$$\frac{1}{R}\frac{dR}{dt} \approx \frac{\sigma\Omega}{\rho_p R}\left(\frac{v_{\rm esc}}{u}\right)^2 \propto R/u^2$$

If u~R (empirical), then $$\frac{1}{R}\frac{dR}{dt} \propto R^{-1}$$

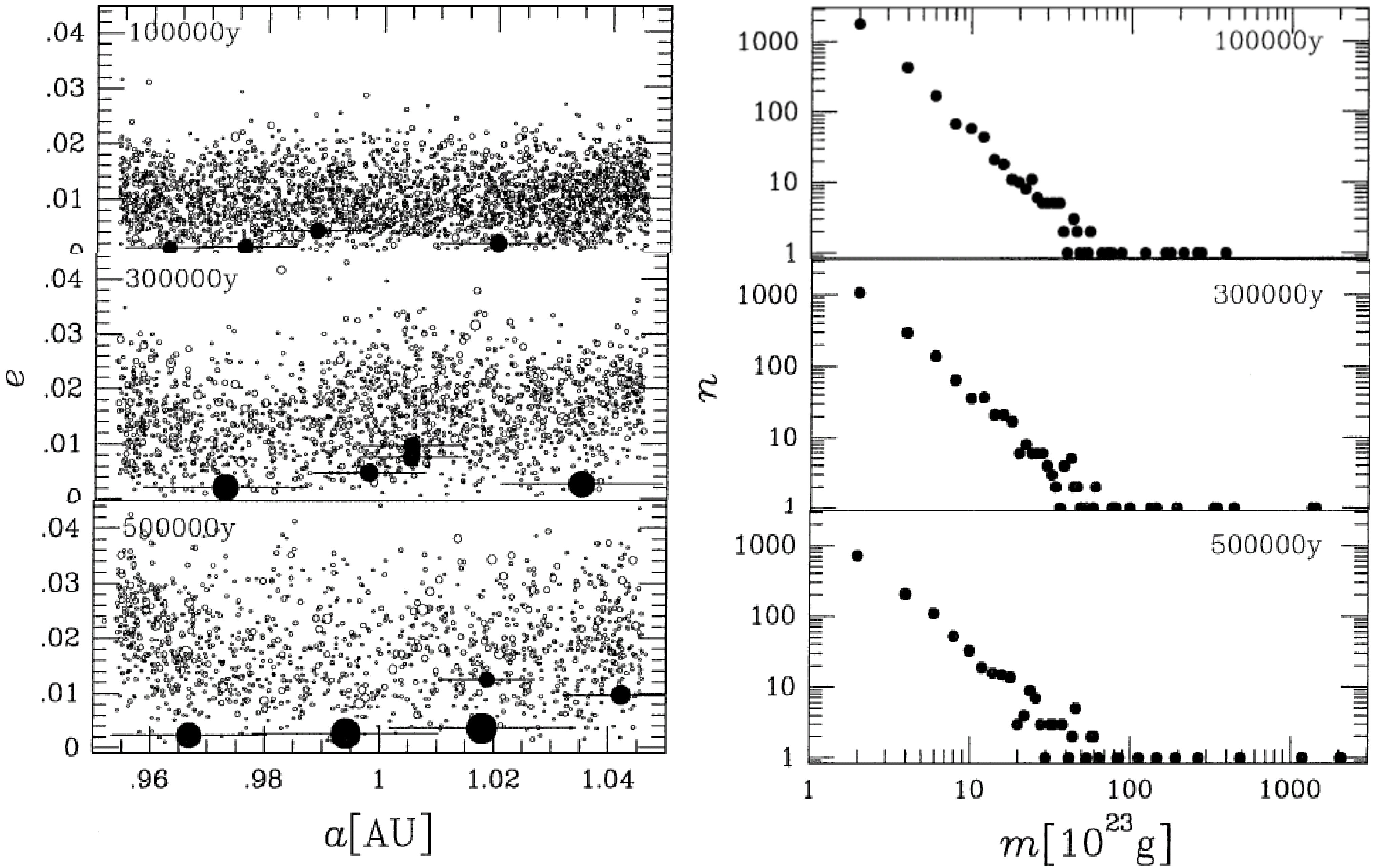
Bigger bodies now grow slower.

Self-regulated growth, a few oligarchs have comparable sizes and grow at similar rates.

The overall growth rate of oligarchs is still larger than planetesimals.

Oligarchic growth

(Kokubo & Ida 2000)



Feeding zone and isolation mass

(Kokubo & Ida 2000)

Every oligarch has its own domain of dominance, separated by

$$\Delta a \sim 10 R_H$$

The maximum mass of a planet embryo/core is limited by the amount of planetesimals available in its "feeding zone".

$$M_{\rm iso} \approx 2\pi a \Delta a \Sigma_p \approx 20\pi a^2 \Sigma_p (M_{\rm iso}/M_*)^{1/3}$$

At the location of the Earth:

$$M_{\rm iso} \approx 0.16 \left(\frac{\Sigma_p}{10 \text{ g cm}^{-2}} \right)^{3/2} \left(\frac{a}{1 \text{ AU}} \right)^3 M_\oplus$$

At the location of Jupiter:

$$M_{\rm iso} \approx 5 \left(\frac{\Sigma_p}{4 \text{ g cm}^{-2}} \right)^{3/2} \left(\frac{a}{5 \text{ AU}} \right)^3 M_\oplus$$

Timescale revisited

(Kokubo & Ida 2000)

In equilibrium, eccentricities of neighboring planetesimals excited by a protoplanet under gas drag is approximately

$$< e^2 >^{1/2} \sim 6 R_{H,\mathrm{Max}}/a$$

Recall: $\frac{1}{R}\frac{dR}{dt} = \frac{1}{3M}\frac{dM}{dt} \approx \frac{\Sigma_p \Omega}{\rho_p R}\left(\frac{v_{\mathrm{esc}}}{u}\right)^2$ and $u \sim e v_K$

The growth timescale in the oligarchic regime can be estimated to be

$$T_{\mathrm{grow}} \equiv \frac{M}{dM/dt} \approx 7\times 10^5 \left(\frac{e}{6R_H/a}\right)^2 \left(\frac{M}{10^{26}\mathrm{g}}\right)^{1/3} \left(\frac{\Sigma_p}{10\ \mathrm{g\ cm^{-2}}}\right)^{-1} \left(\frac{a}{1\ \mathrm{AU}}\right)^{1/2} \mathrm{yrs}$$

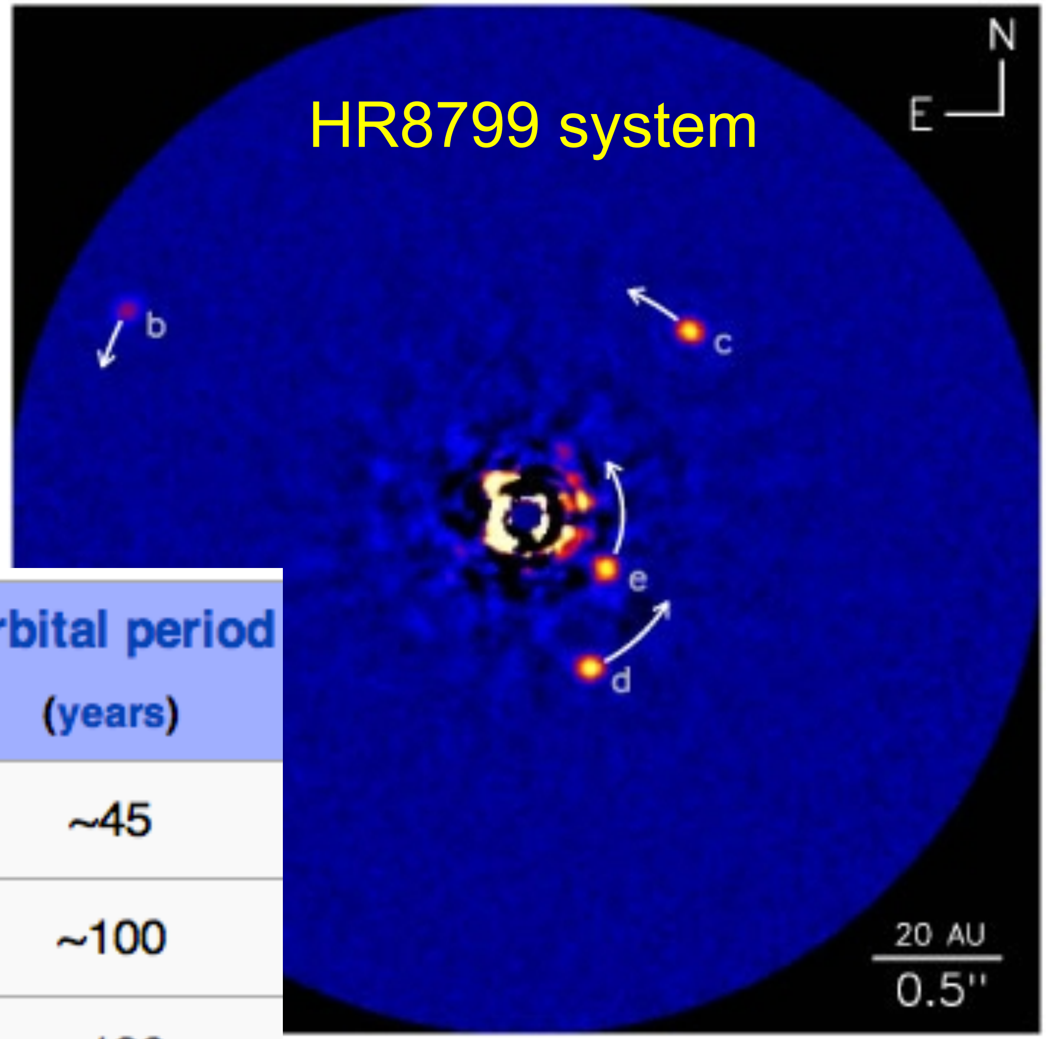
OK for terrestrial planets, but problematic toward outer region:

Timescale to build giant planet cores is excessively long at large separations.

(takes >10 Myr beyond 5 AU)

Challenges to core accretion theory

How to form giant planets at large separations?



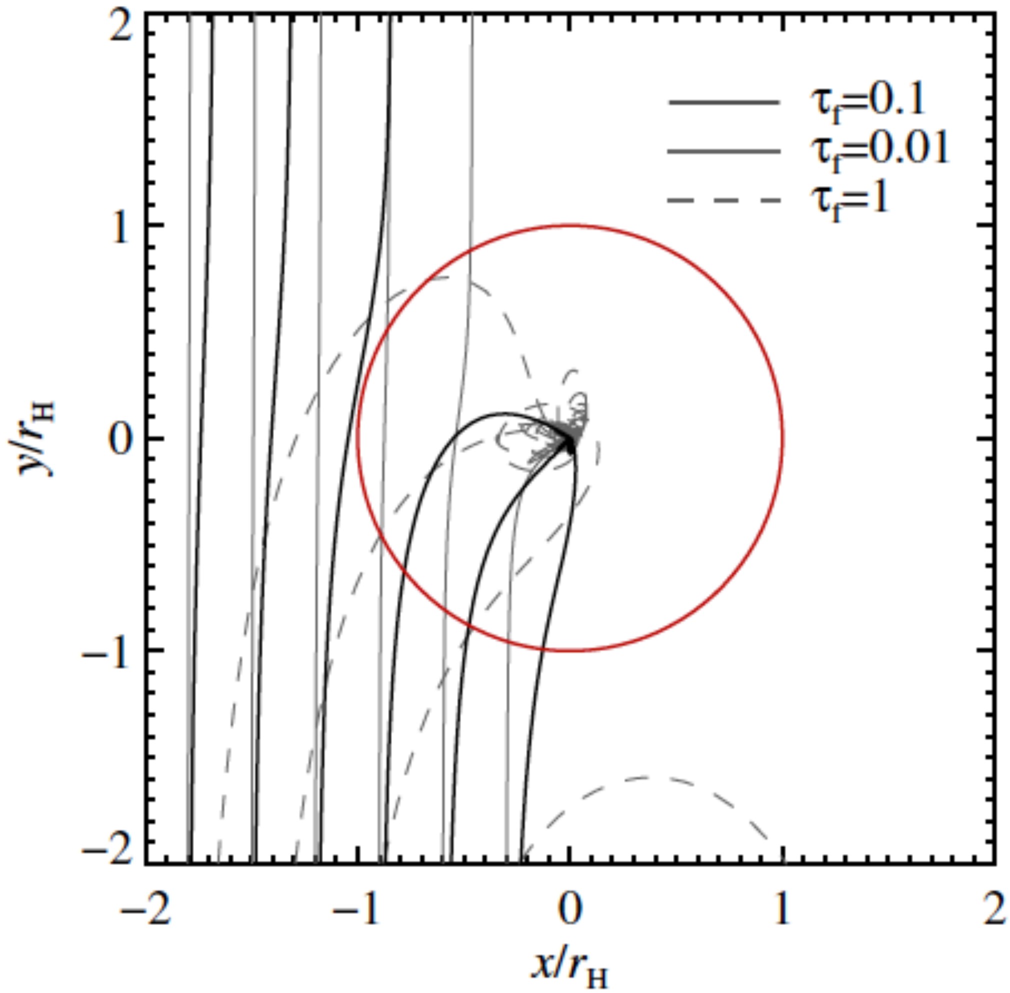
Companion (in order from star)	Mass	Semimajor axis (AU)	Orbital period (years)
e	$7^{+3}_{-2}\ M_J$	14.5 ± 0.5	~45
d	$7^{+3}_{-2}\ M_J$	24±0	~100
c	$7^{+3}_{-2}\ M_J$	38±0	~190
b	$5^{+2}_{-1}\ M_J$	68±0	~460

(Morois+ 2009, Nature)

New regime of pebble accretion

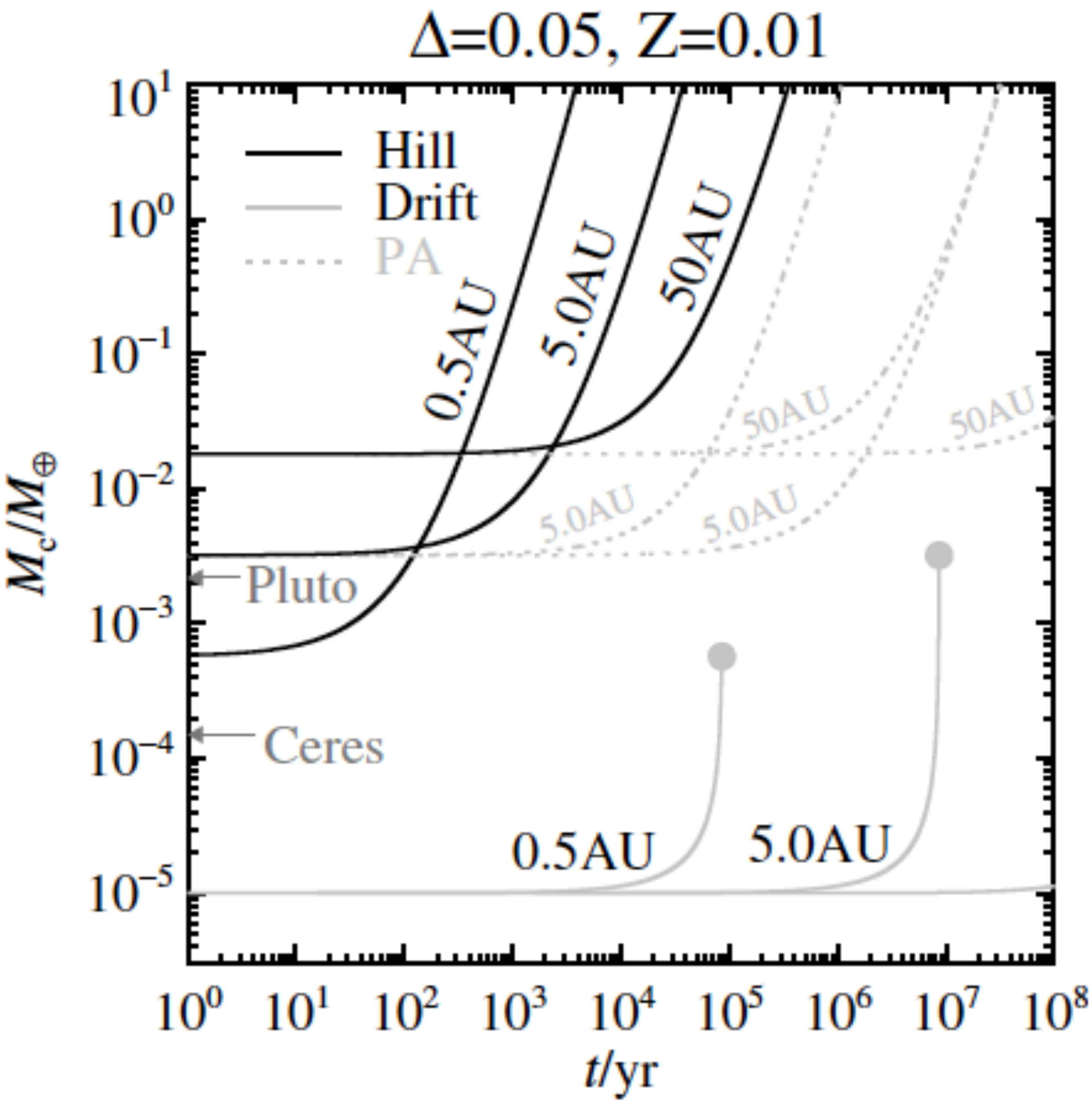
- Pebbles of mm-cm sizes are abundant in PPDs inferred from observations.
- These particles experience strong gas drag hence do not suffer from eccentricity excitations.
- At proper regime, all pebbles entering the Hillsphere spiral in, significantly enhancing the cross section.

$$\frac{dM}{dt} \sim 2R_H \Sigma_p v_H$$



(Lambrechts & Johansen, 2012)

New regime of pebble accretion



Timescale to reach critical core mass:

$$\Delta t \approx 4 \times 10^4 \left(\frac{M_{\rm crit}}{10 M_\oplus}\right)^{\frac{1}{3}} \left(\frac{a}{5 {\rm AU}}\right) {\rm yr}$$

weak dependence on a!

Most efficient for particles with

τ_s=0.1-1

independent of core size!

(Lambrechts & Johansen, 2012)

Summary

- Particles experience radial drift toward pressure maxima, most efficient for marginally coupled particles with τ_s~1.
- Grain growth typically proceeds to cm before bouncing/ fragmentation, but can reach bigger sizes if icy/fluffy.
- Planetesimal formation lies in the regime of strong dust-gas interaction. Contemporary theory favors streaming instability and passive concentration in zonal flows/vortices.
- Two modes of planetesimal growth: runaway and oligarchic growth by accreting planetesimals, effective only at inner disk; pebble accretion can be very efficient at outer disk.
- Better understanding the gas dynamics in PPDs is crucial for understanding early phases of planet formation.