Quantum Field Theory

A. Zee
Tsinghua University, Beijing

July 15, 2019

Arguably the greatest monument to the human intellect

In any case, the most accurately tested theory in physics

2

A couple of millenia before we realized that the universe is pervaded with quantum fields, that we are surrounded by quantum fields

What is Quantum Field Theory?

Who “needs” Quantum Field Theory?

These days, one doesn’t even need to know quantum mechanics to be a professor of theoretical physics. (I do not speak of experimental physicists, for whom I have only admiration since they can do things I cannot even dream of doing.)

Chapter I.1 of my textbook on quantum field theory

WHO NEEDS IT?

Quantum field theory arose out of our need to describe the ephemeral nature of life.

“clouds & smoke passing
in front of our eyes”

No, seriously, quantum field theory
is needed when we confront simultaneously
the two great physics innovations
of the last century of the previous millennium:
special relativity and quantum mechanics

	BIG	SMALL
FAST	Rocketship near lightspeed, no need for QM	The marriage of quantum mechanics & special relativity
SLOW	Classical physics	Slow moving electron scattering off a proton, no need for special relativity

In the peculiar confluence of special relativity & quantum mechanics a new set of phenomena arises: particles can be born & particles can die.

A new subject in physics, quantum field theory, is needed to describe birth & death, & some kind of life in between.

Two gold-plated equations of physics

QM: uncertainty principle

$$\Delta E \sim 1/\Delta t$$

"Accounting errors could be tolerated for a short time"

Special relativity: energy = matter

$$E = mc^2$$

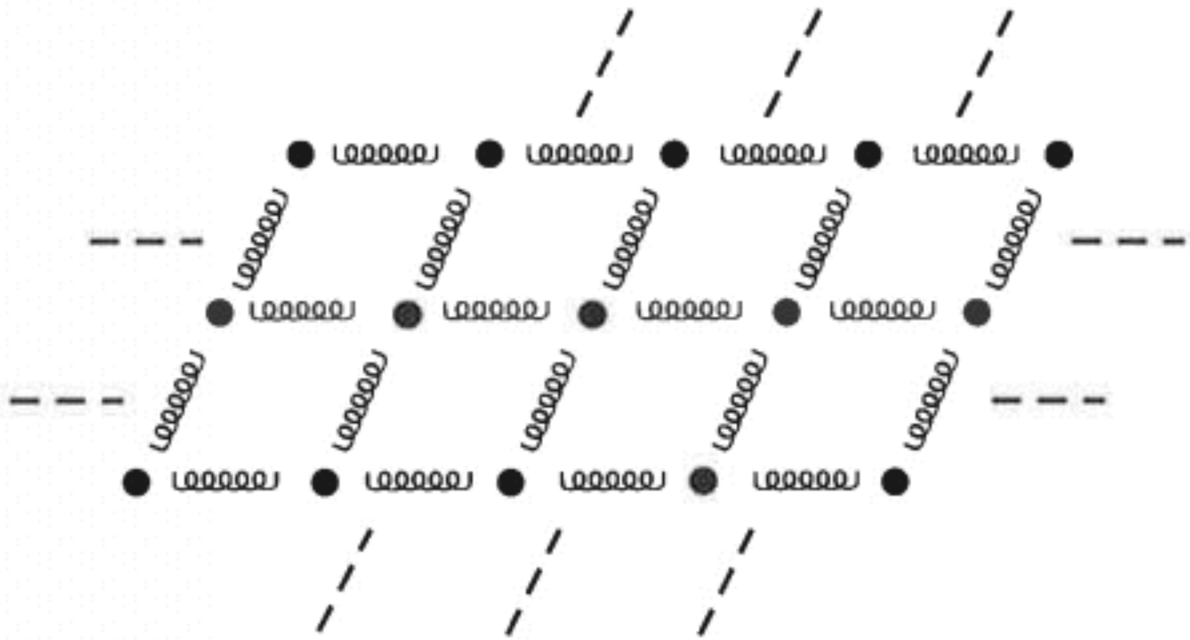
"Accounting errors could be turned into stuff"

If both of these principles, then for an instant, particles could pop out of and pop back into the vacuum

You need both quantum physics and special relativity!

No matter how you shake and bake the Schrödinger equation of one electron, always one electron

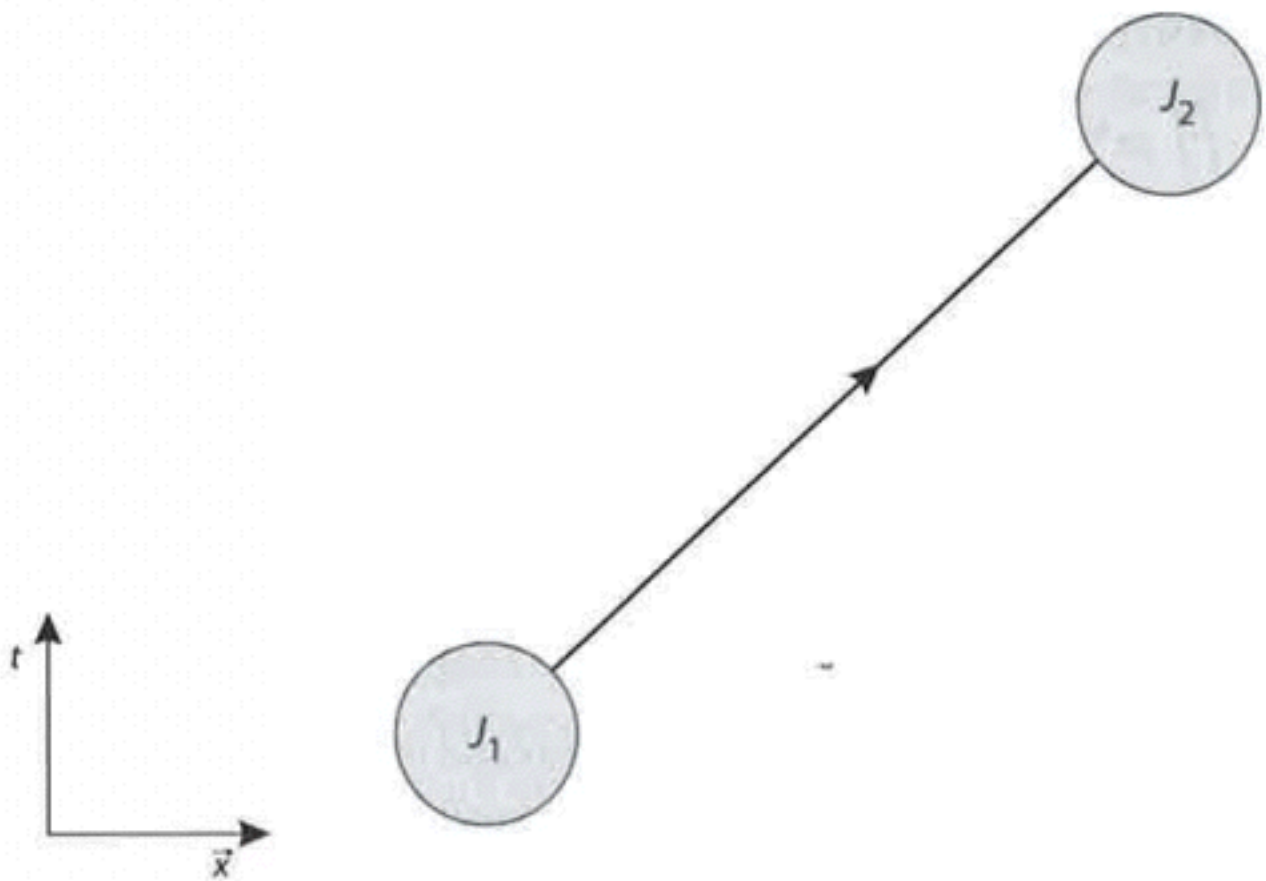
From mattress to field



Form wave packets of the normal modes
Anharmonicity causes them to scatter and decay

From field to particle

I.4. From Field to Particle to Force | 27



From particle to force 1

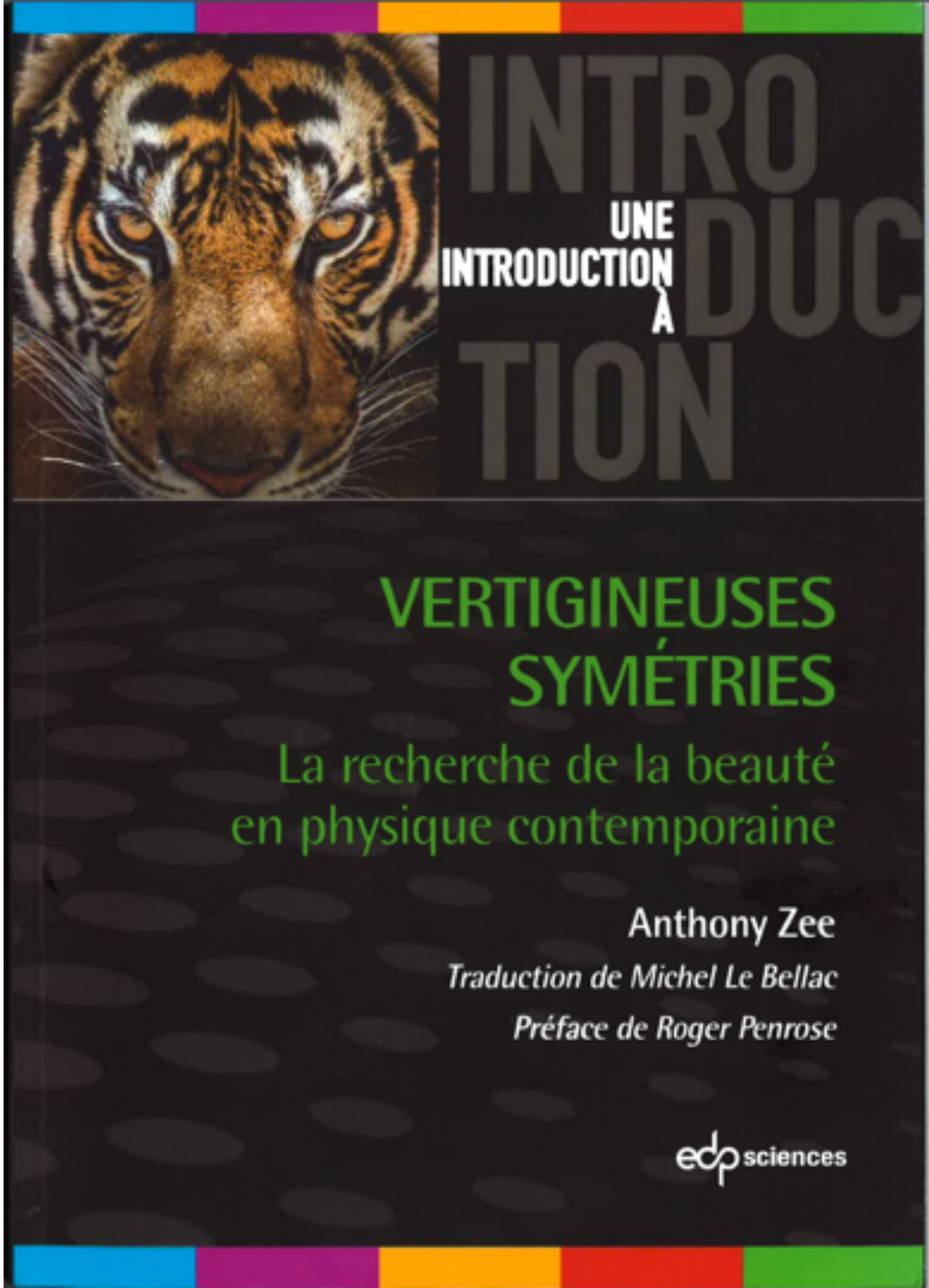
a nucleon?

Figure 11.3. The pion as a marriage broker of old, a corpulent lady whose inclination to travel is limited by her weight.

Taken from ……

Weinberg's advice 1984:
Write popular book before textbook on quantum field theory

>34 years after German, Spanish, Chinese, Japanese, Korean translations,
a French translation in 2018



The Germans invited me to India (Max Mueller) paid by the US State Department

Kolkata newspaper

Text of talk in Mumbai published in New Literary History 23 (1992) p815, illustrated by works of India art.)

Dr Anthony Zee

Physicist in search of God's plan

I want to know how God created this world. I am not interested in this or that phenomenon. I want to know His thoughts, the rest are details.
—*Albert Einstein*

To those who have been reared to believe that "rational" science is the very antithesis of "irrational" religion, it comes as a surprise to discover that the masters of modern physics are the soulmates of men of God. Both are seekers after the Ultimate Truth. And, more interestingly, both are reaching conclusions that are anything but dissimilar. Dr Anthony Zee, 43, a distinguished theoretical physicist from the US who was in the city recently, describes himself as a member of a small band of scientists who are the "intellectual descendants" of Einstein. And, like this century's greatest physicist, they are all engaged in the pursuit of God's plan.

These physicists are convinced that there is a grand design to creation, a single, simple law that governs all phenomena. They have not found that "unified" law, the Holy Grail of modern physics, but say that they are on the threshold of making the discovery. Of the four fundamental forces in the universe—the electromagnetic, the strong, the weak and the gravitational—physicists have managed to mathematically unite the first three. Gravity is proving to be the elusive force, though many fundamental physicists, including Dr Zee, believe that the superstring theory that is still being developed is capable of bringing gravity in line with the principles of quantum theory.

Allied to this reductionist quest for unity is the search for symmetry in nature. That is the other abiding concern of fundamental physicists. With Dr Zee, who has made landmark contributions to particle physics, symmetry is a passion. Born in China, raised in Brazil and educated at Harvard and Princeton, he holds a joint appointment as professor at the University of California and a permanent member of the Institute for Theoretical Physics, both at Santa Barbara.

He has authored nearly 150 scientific articles and two books. The first, *Unity of Forces in the Universe*, is a technical treatise for advanced graduate students and researchers. And the second, *Fearful Symmetry*, concerns his favourite subject and is written for the intelligent lay reader. The fervour with which he holds forth on the subject explains why he has sought to reach out to a wider audience. Published early last year, the book has already been an outstanding success. It will be no surprise if it becomes as big a bestseller as Fritjof Capra's *The Tao of Physics*, another book of the same genre of popular science.

In fact, the *New York Times* critic could not resist the temptation to compare the two works: "Mr Zee conveys the religious beliefs of theoretical

PASSING BY

physicists with far more accuracy than, for example, Fritjof Capra did in *The Tao of Physics*." Dr Zee himself resents the comparison: "As a practising physicist—Capra isn't one—and as an American with Chinese heritage who is familiar with Taoism and other Oriental faiths, I find him misleading and outdated in parts. He stretches the physics to accommodate the religion."

Dr Zee is clearly uncomfortable criticising a fellow author and prefers to talk about his own work. Ever since the publication of the book he has been receiving invitations to speak on the subject to lay and specialist audiences alike. Before coming to India, he addressed a congregation of American philosophers on "the search for beauty." And what has brought him to India is an international conference on symmetry.

We let him tell us more about science and symmetry: "Symmetry and beauty are universal notions in art and literature. But they are also the central organising principle in nature. And this symmetry in nature is intrinsic. Physicists like me are engaged not so much in exploring nature at progressively deeper levels by looking into the properties of fundamental particles, as in searching for the ultimate design. We believe that nature is not randomly put together. It has a design and a beautiful design at that."

The name of Dr Zee's book, of course, is taken from William Blake's famous poem which goes, "Tyger! Tyger! burning bright/ In the forests of the night,/ What immortal hand or eye/ Could frame thy fearful symmetry?" Not once does Dr Zee question the existence of the "immortal hand or eye." Says he, "I like to think of an Ultimate Designer defined by Symmetry." Note, not the other way round. He claims that physicists are close to achieving what Einstein sought to do, "knowing His thoughts" and adds, "We do not doubt that symmetry will light our way in our quest to know His mind."

But there is one thing that still bothers us and we cannot come away without asking Dr Zee about it. How do modern physicists solve the riddle of time? How do they reconcile chronological time, as man has devised, with cosmic timelessness? Time and consciousness, he confesses, have confounded scientists as nothing else. Mahakaal has had them: "Two of the great mysteries that physics has failed to address are time and consciousness. Western science developed, as compared to eastern science, because it drew a line at consciousness—between the physical and the non-physical. Eventually, that line will have to be crossed.

"Time is the other great mystery. Time could well be related to consciousness. It is very fundamental that 'time goes by,' but we do not understand why. The physical laws of nature are by and large reversible, but the flow of time is not. We call it 'time reversal invariance.' I do not understand it. But neither does anybody else." Will science ever be able to crack these two supreme mysteries? Only time can tell.

Vivek Sengupta

From particle to force 2

the other's intentions.
Since the early days of physics, the notion of force has been
among the most basic and the most mysterious. It was thus with
considerable satisfaction that physicists finally understood the ori-
gin of force as being due to the quantum exchange of a particle.
Given this understanding of the nature of force, in 1934
Yukawa decided that a "marriage broker" must be provided for
the strong interaction as well. Boldly, Yukawa hypothesized a new
particle, which became known later as the pi meson, or the pion
for short. Strikingly enough, he was able to predict the properties
of the pion, a feat for which he was awarded the Nobel Prize.
...itting more or less at rest inside an

From Fearful Symmetry (talk given at the National Center for the Performing Arts in Mumbai)

From particle to force 3

Another rough analogy: Two lumps on a mattress

Attract due to their distortion of the “field”

The physical world results from a web of interactions, some attractive, some repulsive.

Quantum field theory answers why:

The sign (+ or - attraction or repulsion between like objects) depends on whether the spin of the mediating particle is odd or even,

and ultimately could be traced to the difference between time and space

Spin 0: attraction (strong nuclear force; protons and neutrons coming together)

Spin 1: repulsion (electromagnetism; like charges repel)

Spin 2: attraction (gravity; the apple and Newton's head)

A lightning history of the universe

Universal gravitational attraction, due to the spin 2 particle, produces the instability that drives the formation of structure in the early universe. Dense regions became denser still.

The rich becomes richer! Free streaming photons and neutrinos impose taxation.

Attraction between the protons, due to the spin 0 particles, ignites the stars and eventually form the rich variety of nuclei.

The attraction between positively charged nuclei and the negatively charged electrons, due to the spin 1 particle, produce the atoms.

Concept of field goes back to Faraday

Without a proper education, the mathematics of the "continental philosophers" were beyond Faraday, who was compelled to invent his own pictorial language

A side remark about basic versus applied research:
Faster communication for the Royal Navy
Messing around with wires and frogs
versus breeding stronger pigeons

- Maxwell's theory of the electromagnetic field contains two hidden symmetries that will rock 20th century physics (relativistic & gauge)
- Is it conceivable that present theory also contains hidden structure?
- In hindsight, the terrible notation of the 19th century physics (magnetic field = H?)
- But now also, Yang-Mills theory terrible notation: redundancy in description (twistor perhaps first step in fixing this!)

Already, in non-relativistic QM, photon (electromagnetic field) treated as a field but not the electron => Jordan, Heisenberg, Dirac,… "half-assed treatment"

Fermi: photon can created, electron can also be created

All particles are excitations in some field

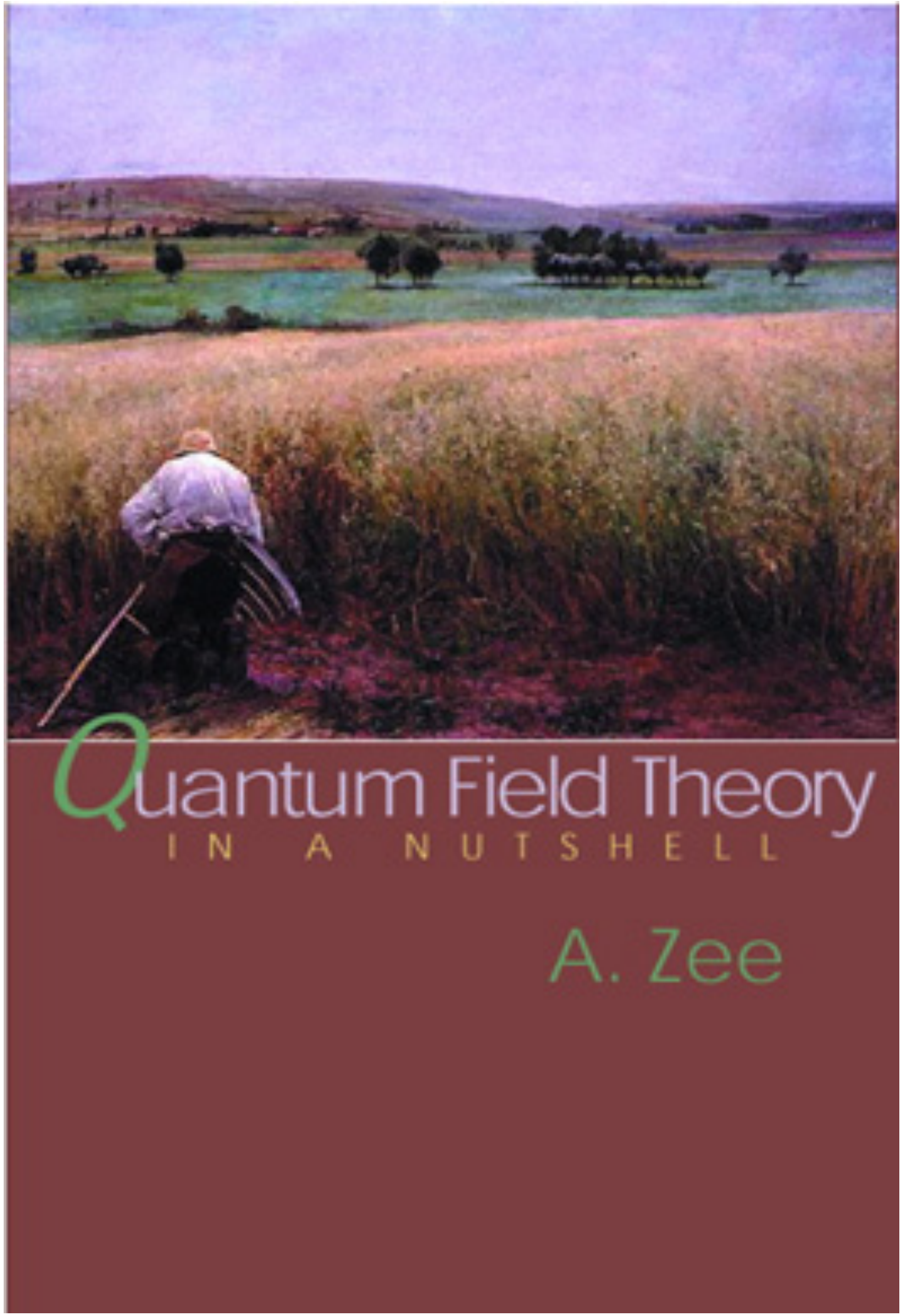
(graviton just a particle like any other, an excitation in the gravitational field (e.g. S. Weinberg's textbook on gravity) but somehow also responsible for the spacetime arena in which all fields work & play —

It is somehow different?

Quantum gravity?

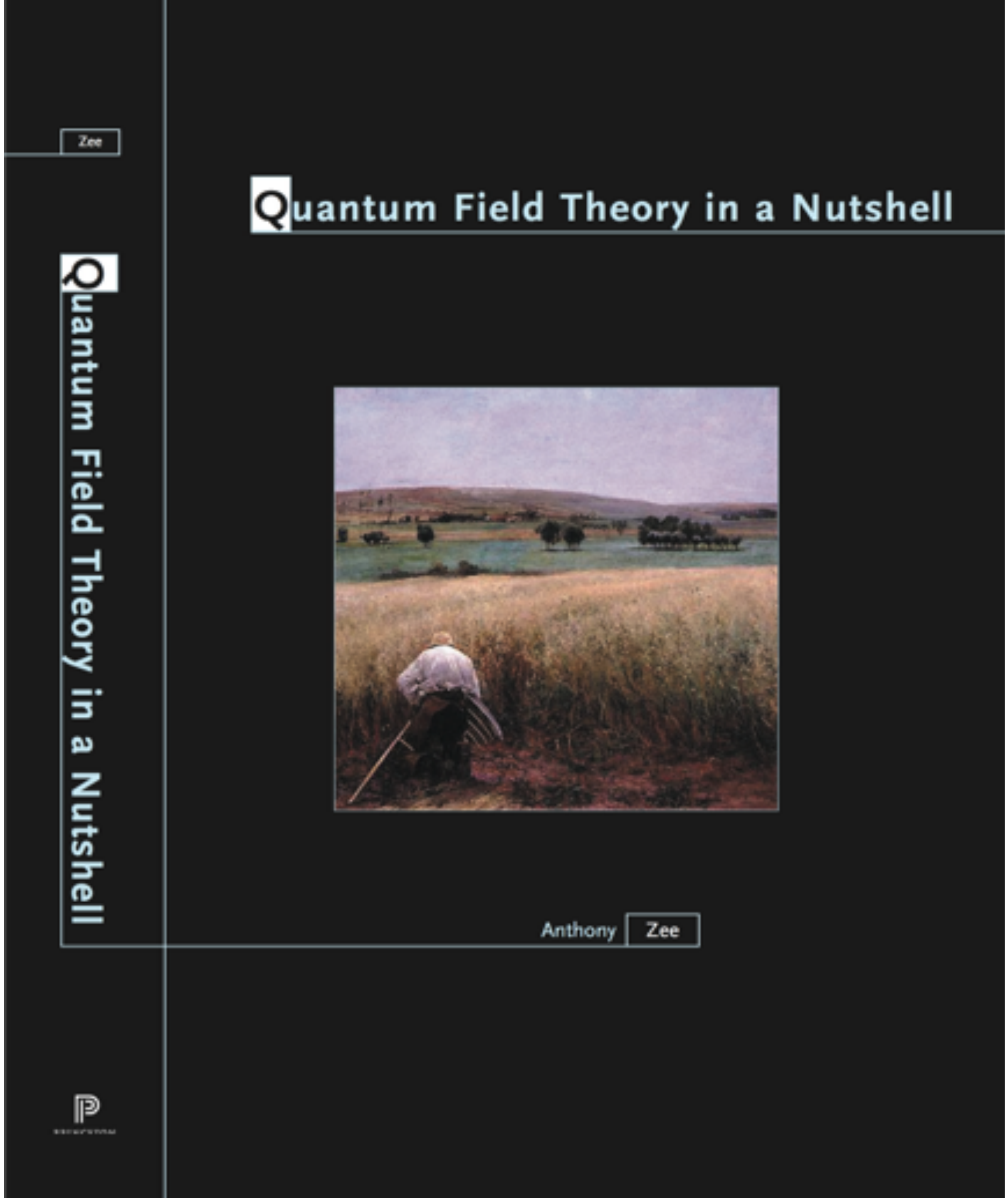
Cosmological constant?

(With gravity, cube of physics)



2003

2010


Zee
Quantum Field Theory in a Nutshell
Quantum Field Theory in a Nutshell
Anthony Zee

Translations into Russian

with logically inconsistent title!

and now into Japanese (in press)

— the rise of field theory in condensed matter physics (as distinct from earlier use of pertubative field theory,e.g. Fetter & Walecka, Abrikosov, et al)

e.g. quantum Hall fluid microscopic degrees of freedom = electrons but long distance physics = Chern-Simon gauge field with fractionally charged excitations

e.g. surface growth & renormalization group

e.g. replica & supersymmetry

— gravity introduced as early as possible

****

— Try to be much lighter in formalism, stories, jokes, fictitious characters (Confusio) a la Galileo

— Eight Parts of the Celestial Dragon

kitp.ucsb.edu/~zee/JingYong.html

Quantum field theory has had two near-death experiences

Late 1940s: inability to produce Lorentz covariant results & divergences

-The young people were the (revolutionary) conservatives

Late 1960s: S-matrix school, inability to deal with the strong interaction

-The triumph of field theory = “a victory parade” that made “the spectator gasp with awe & laugh with joy”

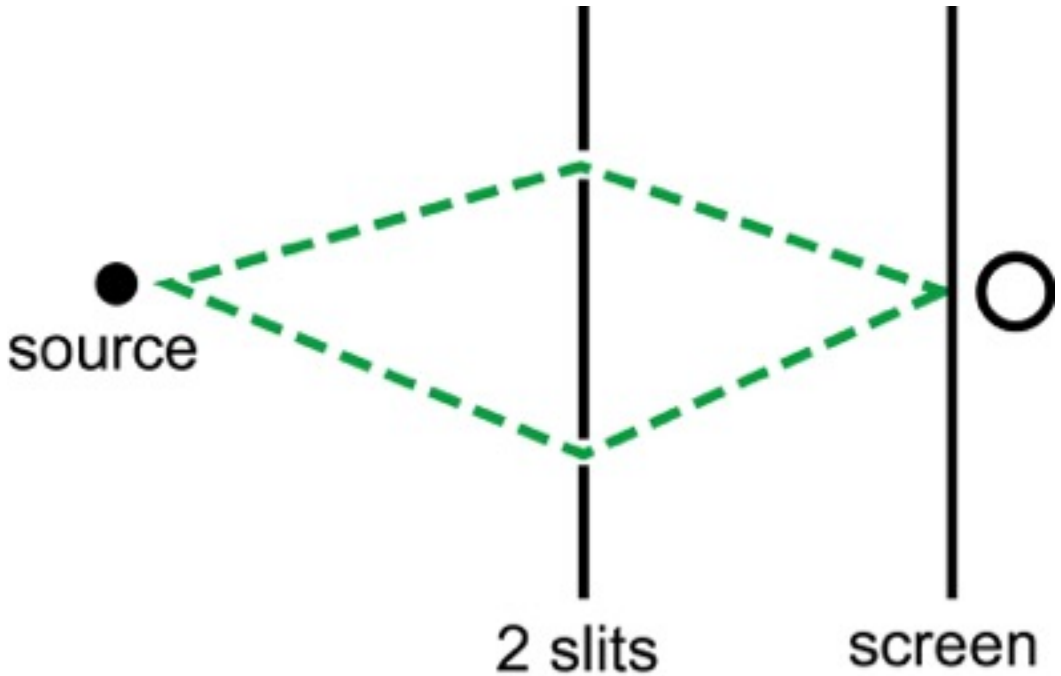
Quantum mechanics = (0 + 1) - dimensional field theory

String theory = (1 + 1) - dimensional field theory

When we teach (or learn) QM

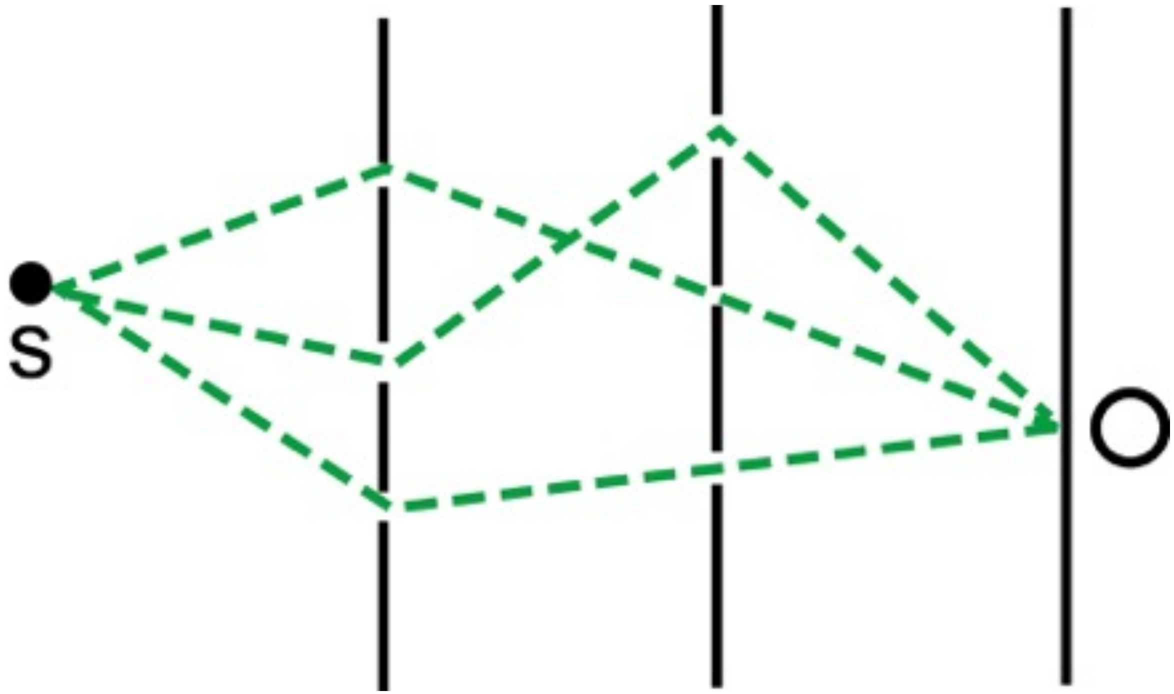
Schrodinger or Heisenberg formalism

conceptually more profound (& leading naturally to QFT): Dirac-Feynman path integral or Schwinger functional integral formalism



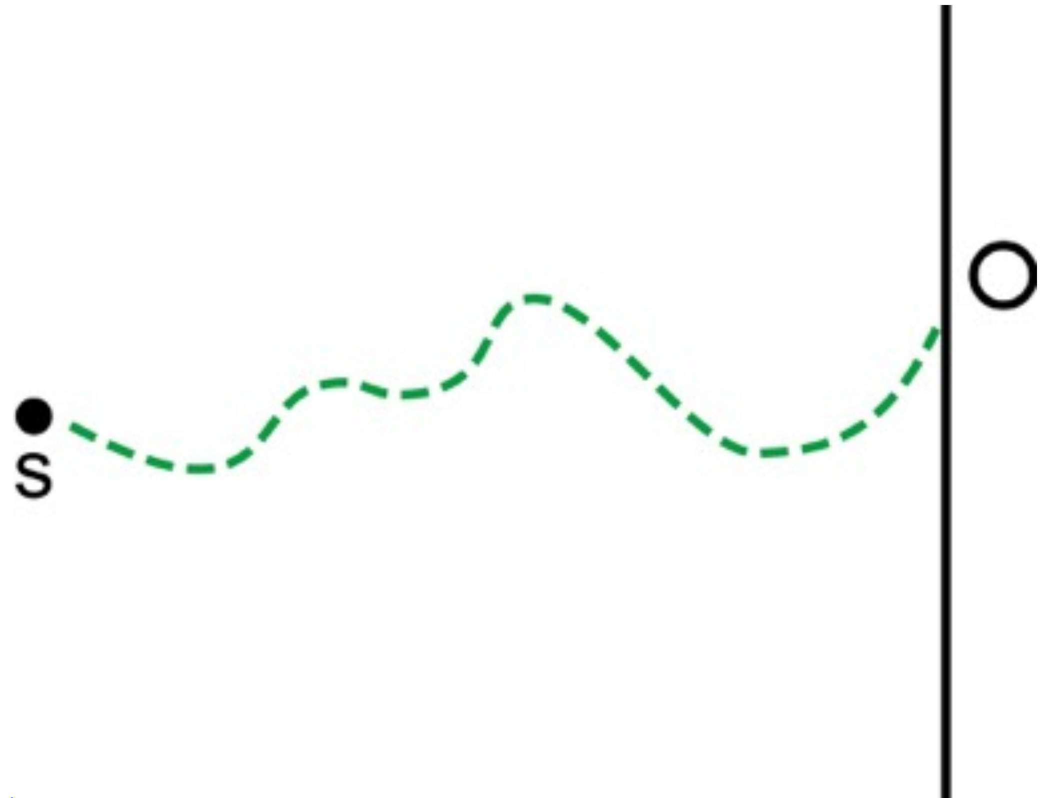
add amplitudes

Students almost never ask: What if 3 holes?



add amplitudes

The most* Zen of all: add ∞ number of screens & drill ∞ number of holes in each screen



Add amplitudes over all paths

Integral over all paths

*The sutra of Hui Neng (Liu Zhu Tan Jing)

Quantum field theory one great big fancy sum (integral)

↓ $\int d\varphi e^{-S(\varphi)}$ $\varphi(x,t)$

↑D dimensional

Quantum mechanics

↓ $\int d\varphi e^{-S(\varphi)}$

Call it coordinate or position $\varphi(t) \equiv q$

0+1 dimensional field theory

Random matrix theory

A matrix φ

↓ $\int d\varphi e^{-S(\varphi)}$

0+0 dimensional field theory

"Ordinary" integral in calculus $\int d\varphi e^{-S(\varphi)}$

~80 years of QFT finding methods or tricks to do the (functional) integral

e.g. lattice gauge theory, replace $\int$ by Σ & use computer

Simplest method

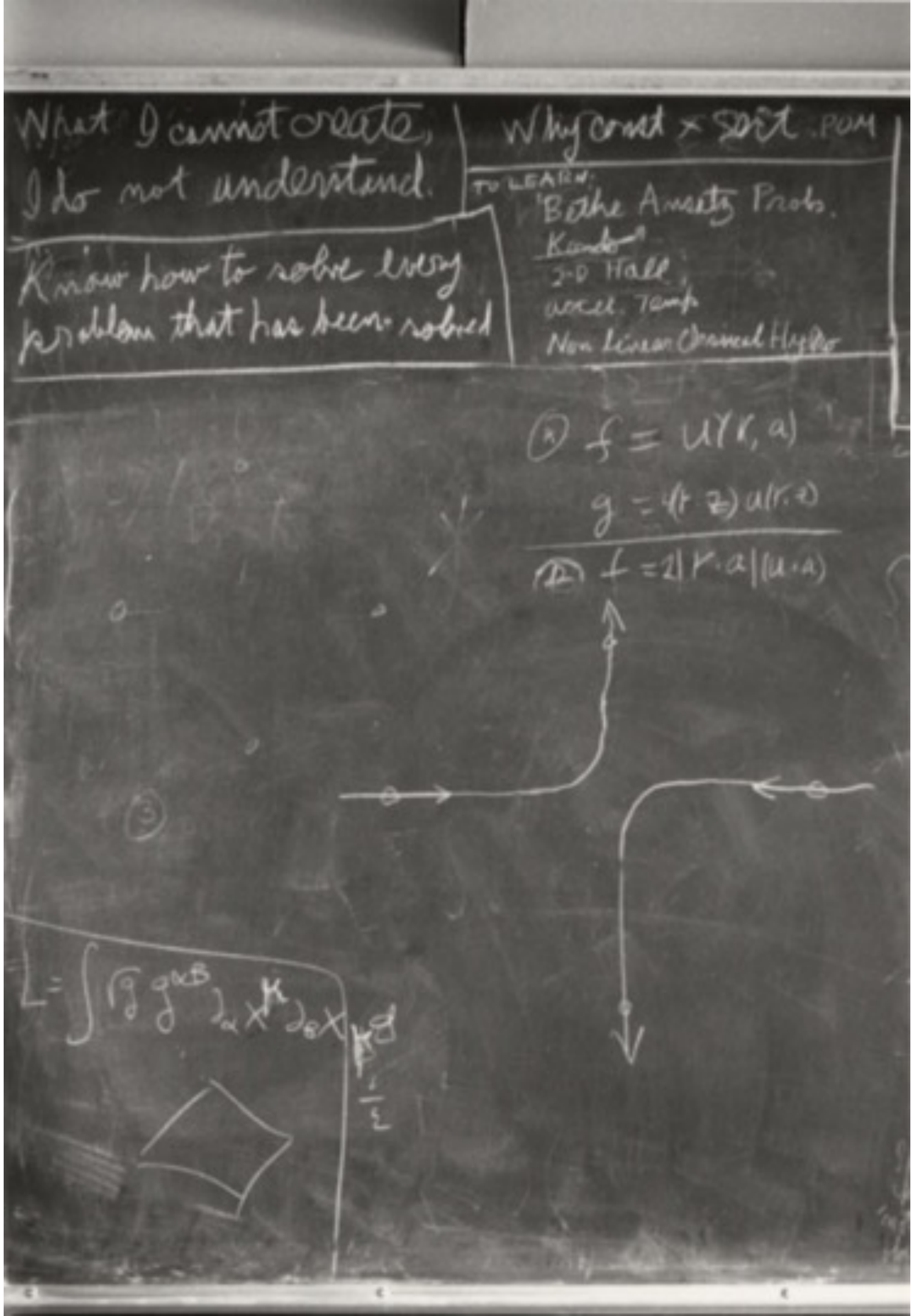
Expand

$$\int d\varphi e^{-\frac{1}{2}m^2\varphi^2 - g\varphi^4} = \sum_{n=0}^{\infty} \frac{(-g)^n}{n!} \int d\varphi e^{-\frac{1}{2}m^2\varphi^2} \varphi^{4n}$$

A fabulous idea:

Draw little diagrams to keep track of the terms

When I was a student, I was really eager to learn about Feynman diagrams which I had heard so much about



Feynman's last blackboard

sets impossible standards (two upper left boxes),
but note "to learn list" (second box on right)

Schwinger in 1948 calculated the first term of the anomalous (that is, deviation from Dirac's value) magnetic moment of the electron, beating Feynman! Expression carved on his tombstone.

Current value for the anomalous magnetic moment of the muon = 116,587,705.7 (2.9) x 10^(-11)

(Schwinger got the 116)

To put the accuracy in rough pictorial perspective, say the distance between Shanghai and Beijing is ~1200 km, then the present uncertainty corresponds approximately to ~6 X 10^(-2) millimeter (width of human hair ranges from 0.2 times this to about twice

From my QFT book

Schwinger's triumph

Let us now calculate $F_2(0)$ to order $\alpha = e^2/4\pi$. First draw all the relevant Feynman diagrams to this order (fig. III.6.1). Except for figure 1b, all the Feynman diagrams are clearly proportional to $\bar{u}(p', s')\gamma^\mu u(p, s)$ and thus contribute to $F_1(q^2)$, which we don't care about. Happy are we! We only have to calculate one Feynman diagram.

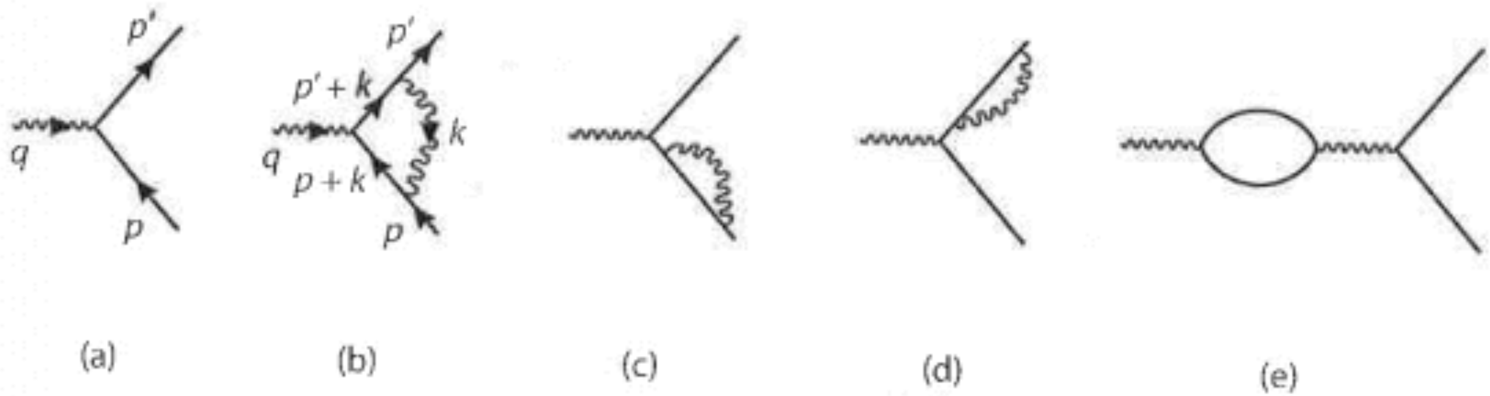


Figure III.6.1

Schwinger: "Feynman brought quantum field theory to the masses"

"In the dark recesses of the sub-basement of Lyman, where theoretical students retired to decipher their tablets, and where the ritual taboo on pagan pictures could be safely ignored, and…"

Paul Martin

Physica 96 (1979), p70

Landmarks

Internal space: Heisenberg, isospin, …

Gauge symmetry: Weyl …

Non-abelian gauge: Yang-Mills, …

The influence of Feynman diagrams on quantum field theory for many years

QFT = Σ Feynman diagrams

"Breaking the shackles of Feynman diagrams"

mid 1970's: 't Hooft & Polyakov + ….

$$e^{-\frac{1}{g^2}}$$

- non-perturbative structures (such as magnetic monopoles) that cannot be seen by Feynman diagrams…
- duality… topology …

What is the purpose of studying physics?

The purpose of physics
is to understand Nature

Intellectual completeness of different areas of physics

For example, Pauli exclusion & the spin-statistics connection (integer spin = Bosons, ½ integer spin = Fermions)

"There is no one fact in the physical world which has a greater impact on the way things are" Duck + Sudarshan

From atoms to neutron stars to lasers much of condensed matter physics e.g. band structure, superconductivity, etc.,etc.

Just a rule in non-relativistic quantum mechanics

Quantum field theory is more complete than quantum mechanics

Pauli exclusion can be derived: spin-statistics theorem

How do you know that all the electrons are identical?

They could have been made in a factory somewhere…

We need QFT to explain this elegantly simple explanation

Why is the electron charge exactly equal in magnitude to the proton charge?

Quantum electrodynamics less complete than grand unified theory

Why is spacetime (3+1) – dimensional?

Quantum field theory (can be written in any spacetime dimension)

Less complete than string theory

One of the deepest mysteries of physics:

Dynamical evolution in the quantum world

$$e^{-iHt/\hbar}$$

& Boltzmann factor

$$e^{-\beta H}$$

Imaginary time & inverse temperature

Just a coincidence or deeper?

In any case, can be applied, e.g. Hawking temperature of black hole

Partition function

$$\mathrm{tr}\, e^{-\beta H}$$

represented as Euclidean field theory (in contrast to Minkowskian field theory)

Many other problems in physics can be represented as field theories

Two Great Marriages

20^{th} century

quantum physics + special relativity

→ quantum field theory

Late 20^{th} century – early 21^{st} century

quantum physics + general relativity

→ string theory or ?