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# Two examples of one-dimensional cold gases in new many-body regimes

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# New many-body regimes: **unique to cold atoms**

Features of interest, no prior analogue from solids:

A. Strong interaction. “High temperature physics” for ultracold atoms.

B. Large spin population and mass imbalance. Effective Zeeman splitting comparable with Fermi energy  $E_F$

C. Quantum particles (especially bosons) in the excited “higher orbital” bands of optical lattices. Beyond the s: p, d, ...

D. “Slow” quantum dynamics – learn the details from a slow motion of the physical process

E. ...

• This Talk



# Outline – two examples

## 1. Spin imbalanced Fermi gases

- A. Heuristic introduction to exotic pairing (FFLO, breached pair)
- B. The 1D problem – A heuristic model. Exact thermodynamic Bethe ansatz. Effective field theory and Phase diagram of FFLO. Effect of polarization: spin-charge mixing. Experimental progress.

## 2. Topological orbital ladder in optical lattices

- A. The p-band, Hamburg experimental system, etc.
- B. The sp-orbital ladder from Hamburg = Topological Insulator (indexing group  $Z$  class) -- Due to hybridization of opposite parity s and p orbitals. Fermion zero modes (edge states). Staggered sp tunneling  $\rightarrow$  equivalent of Rashba-like spin-orbit coupling. Topological phase transition to Mott insulator.



# Summary of Our Main Results – 1D Fermi gas part

- A. **A new effective field theory** of 1D crystalline superfluid (FFLO) phase  
[Zhao, WV, PRA 2008]
  - 1) Based on both Bethe ansatz and conformal field theory
  - 2) Constructed to match the exact Bethe ansatz spectrum (low energy part)
  
- B. **Zero Temperature ( $T=0$ ) phase diagram** for a system of weakly coupled tubes by **Renormalization Group (RG) analysis**
  - 1) Based on our effective field theory (EFT)
  - 2) New unpublished results: Finite T phase diagram derived by applying RPA to the above EFT
  
- C. **Breakthrough in exact solvable models:** Analytic, exact thermodynamic Bethe ansatz (TBA) solution of 1D imbalanced fermions at finite T and strong interaction by a simple set of 4 algebraic equations (pressures and chemical potentials). [Zhao, Guan, WV, et al, PRL 2009]
  
- D. **Proof of a new two-component Tomonaga-Luttinger liquid (TLL) with spin-charge mixing, outside the paradigm of standard spin-charge separation.**  
[This is for understanding crystalline superfluidity-FFLO.]
  - 1) By both EFT and analytical solution of TBA
  
- E. **Application: two-stage scheme to achieve accurate thermometry**

## *Topic 2:*

# Topological orbital ladder in optical lattices

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### *Work done (in collaboration) with:*

Sankar Das Sarma, Andreas Hemmerich, Xiaopeng Li, Kai Sun, Erhai Zhao.

### *References:*

- **Background and perspective (news & views):**  
[Nature Physics](#) 7, 101 (2011) [with M. Lewenstein]
- [Nature Physics](#) 8, 67 (2012)
- [arXiv:1205.0254](#)

# Outline – two examples

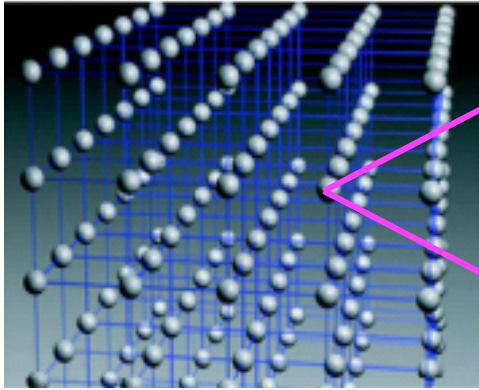
## 1. Spin imbalanced Fermi gases

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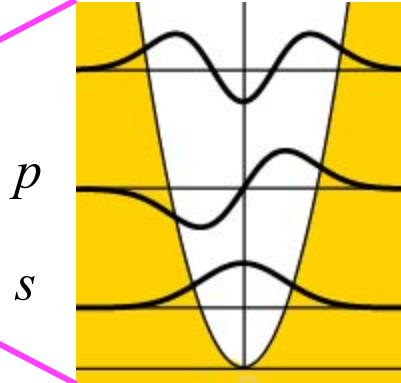
## 2. Topological orbital ladder in optical lattices

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# p-band of an optical lattice (illustration)

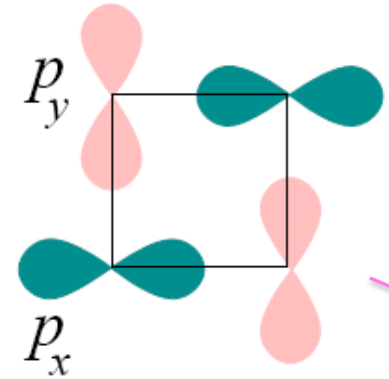


[optical lattice picture]

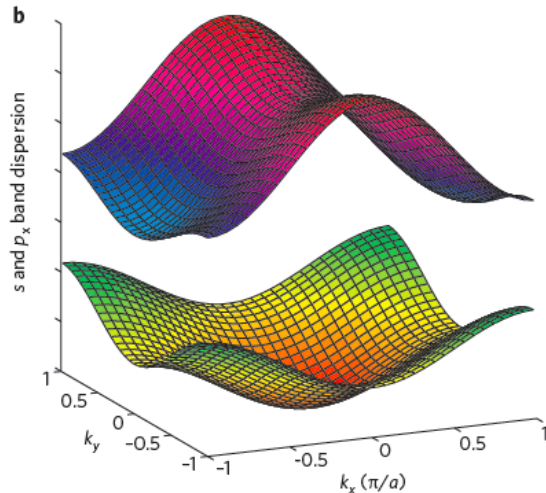


Lattice spacing  $\lambda_L \sim 400\text{nm}$   
 Level spacing  $\sim 1\text{MHz} \sim 50\mu\text{K}$

Top view



With tunneling, discrete levels become Bloch bands.

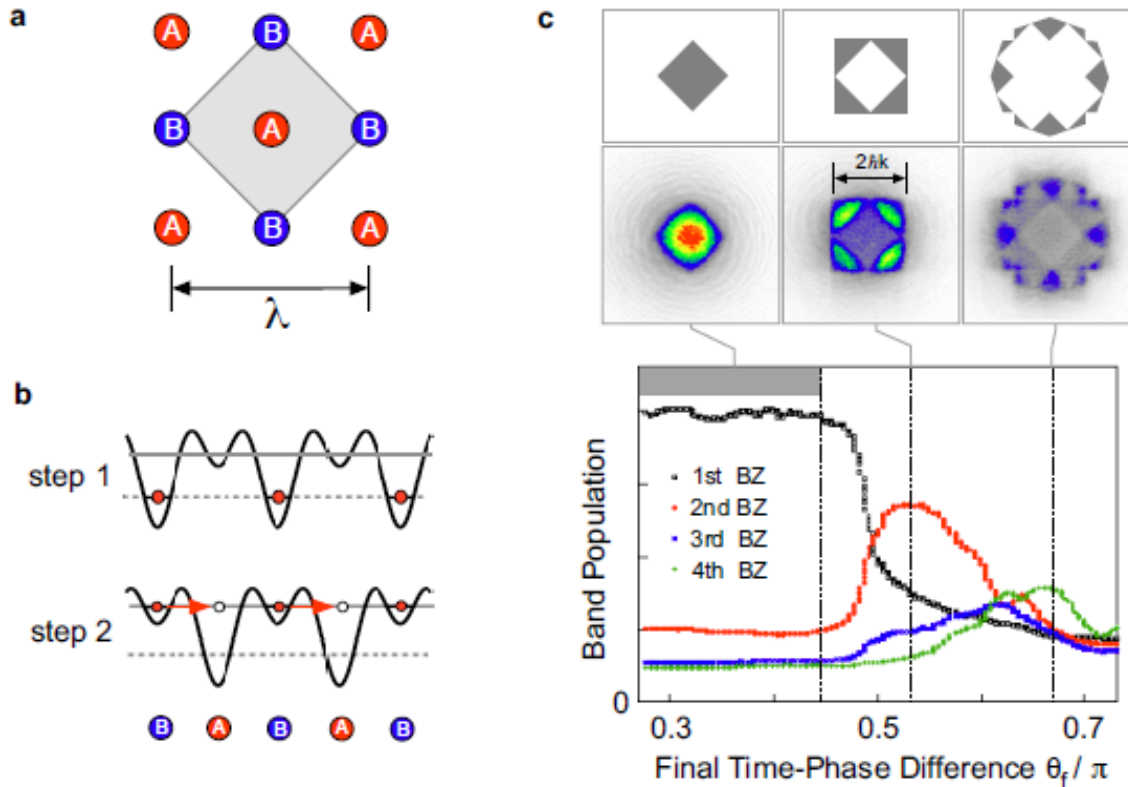


p-band

s-band

- This talk
- Compared with s: orbital degeneracy  $\rightarrow$  emergent symmetry

# New p- and f-band experiments – double well lattices



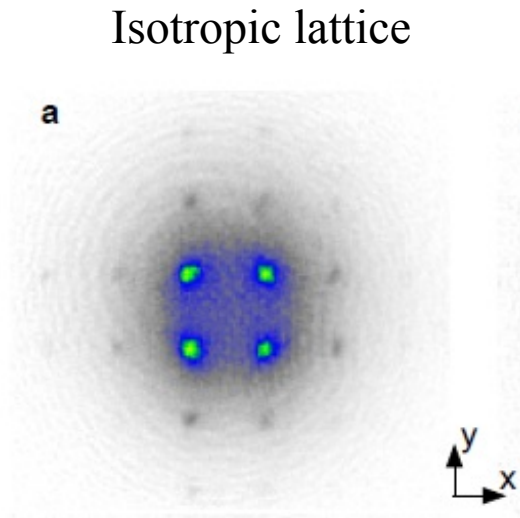
Hamburg/  
A. Hemmerich group

- “P-band superfluidity+orbital order in chequerboard (double well) lattice”, long life time [G. Wirth, M. Olschlager, A. Hemmerich, *Nature Physics* 2011]
- “F-band” [M. Olschlager, G. Wirth, A. Hemmerich, PRL 2011]

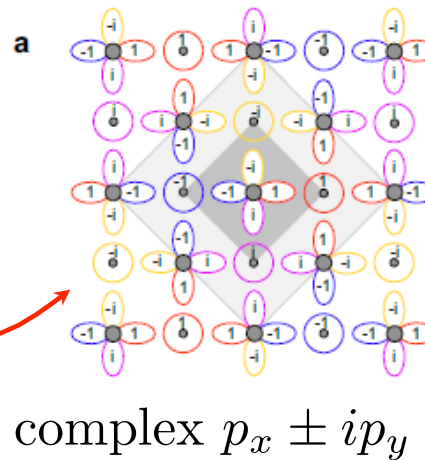
# Observation- Boson : complex p-orbital superfluids

G. Wirth, M. Olschlager, A. Hemmerich,  
Nature Phys. 2011

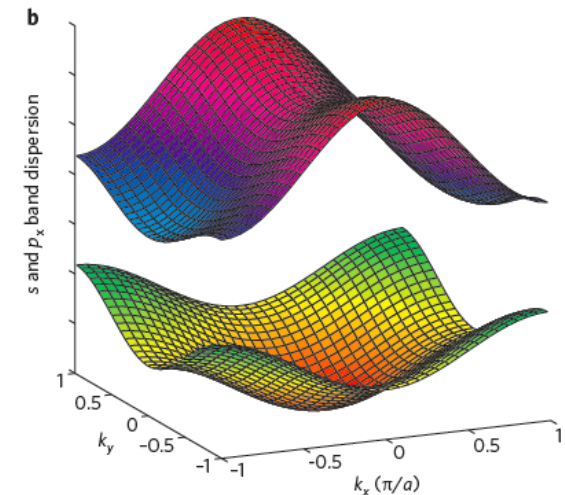
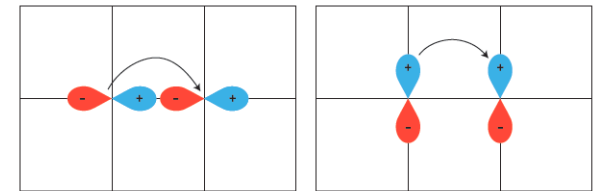
*Observed  
momentum  
distribution*



*Interpretation:  
Nature of orbital  
order*



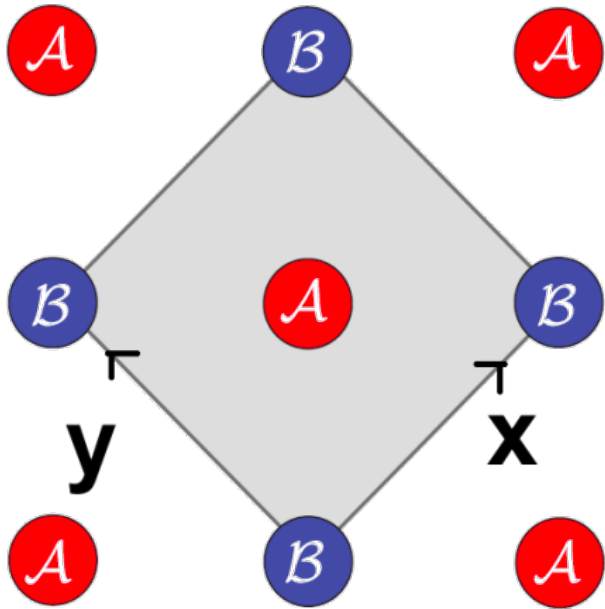
## Theoretical understanding



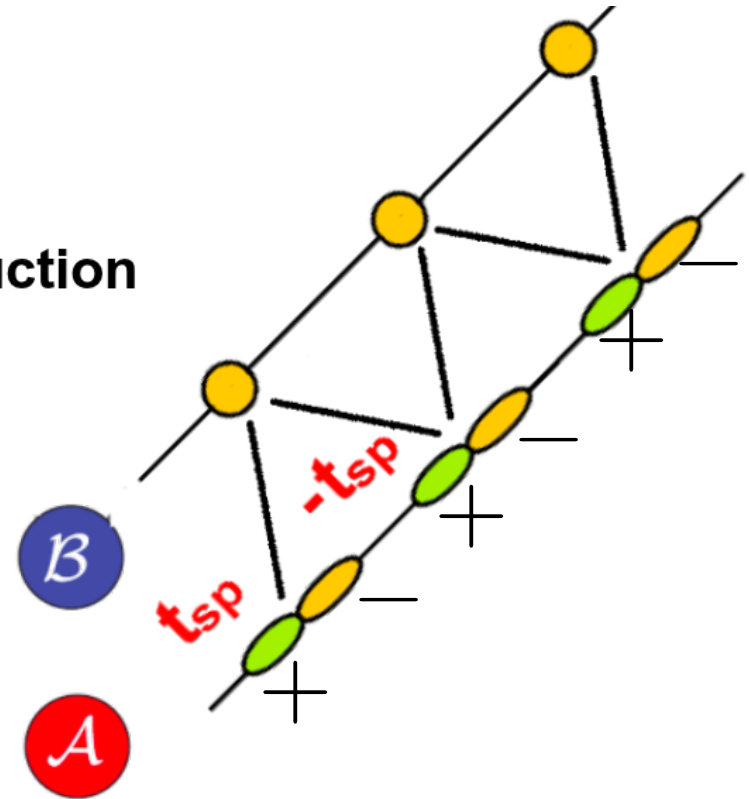
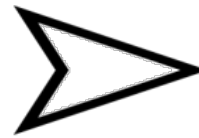
[M. Lewenstein & WV, Nature Phys. 7, 101 (2011)]



# Hamburg 2D double-well lattice $\rightarrow$ sp-orbital Ladder

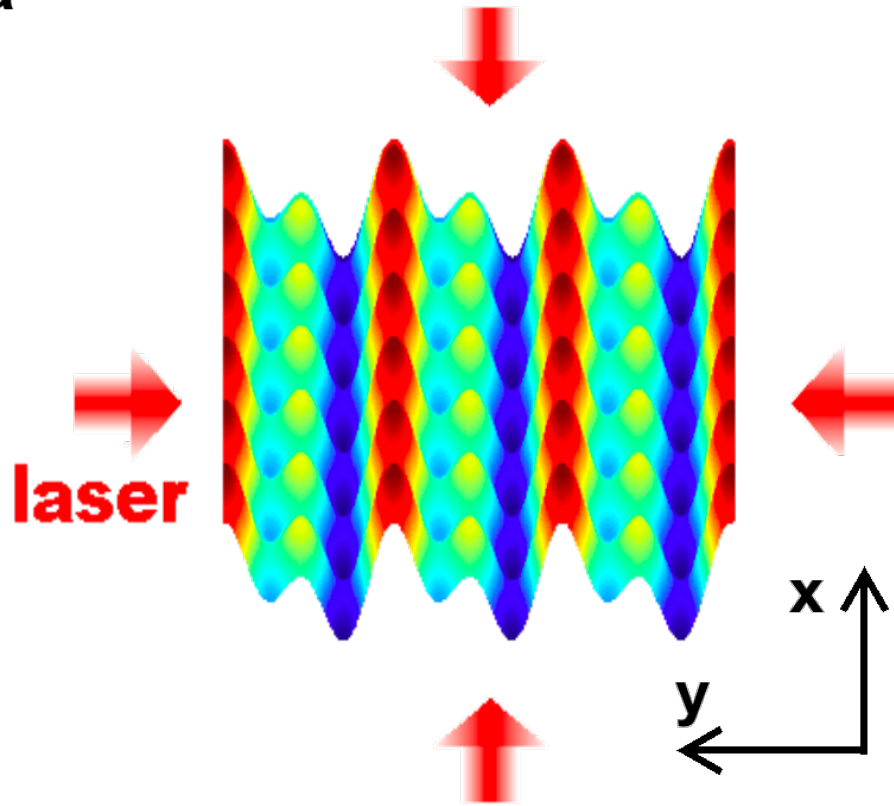


dimension reduction

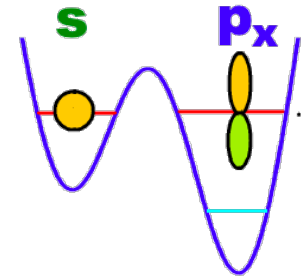


# Topological orbital ladders

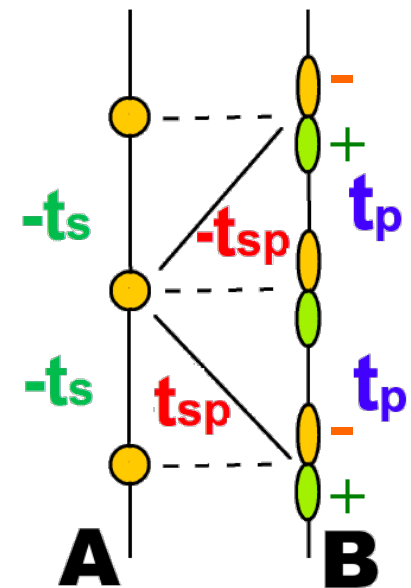
a



b

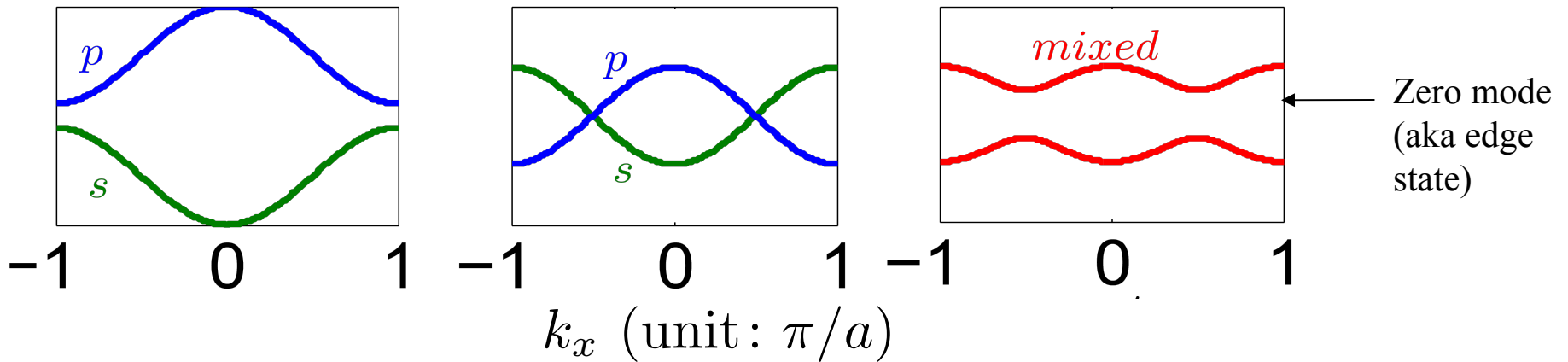


c



# sp-mixing and topological nature

## -sp-mixing

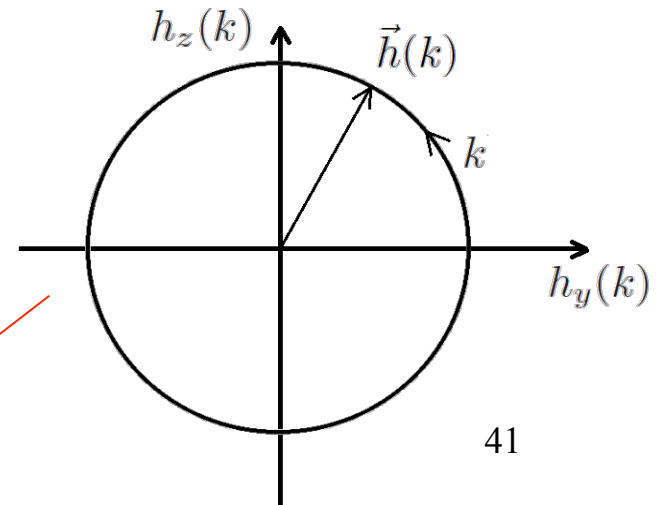


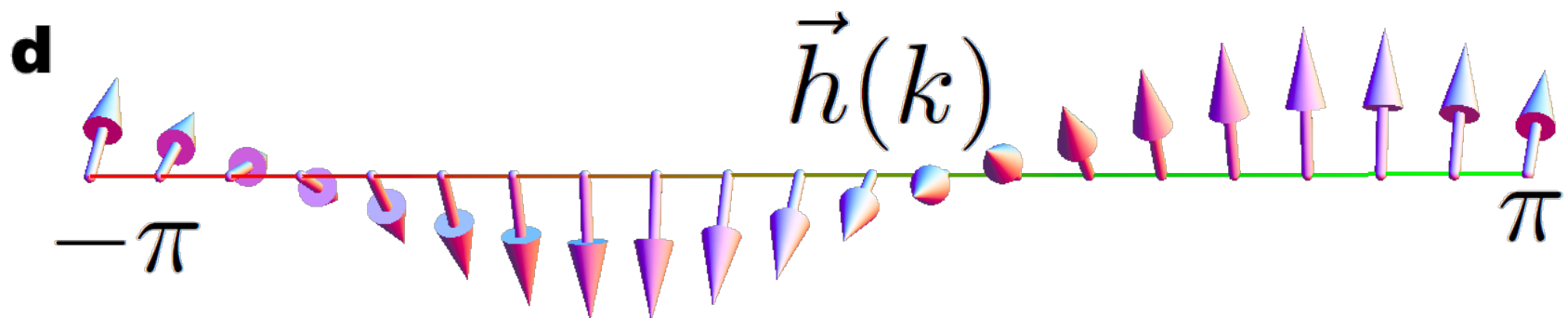
## -Hamiltonian in momentum space

$$\mathcal{H}(k) = \begin{bmatrix} -2t_s \cos k & -2it_{sp} \sin k \\ 2it_{sp} \sin k & 2t_p \cos k \end{bmatrix}$$

$$\mathcal{H}(k) = h_0(k)\mathbb{I} + \vec{h}(k) \cdot \vec{\sigma}$$

Berry phase is  $\pi$ .





*Momentum  $k$*  

*(lattice constant  $a = 1$ )*

# Edge states

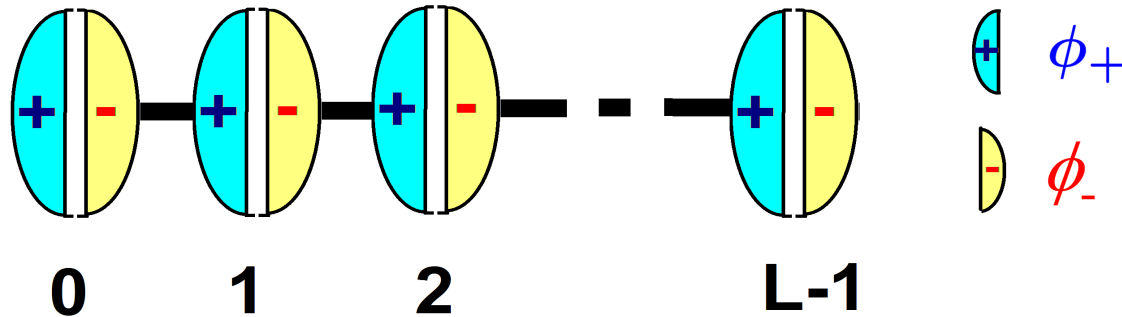
**-flat band limit (easy to show)**

$$t_s = t_p = t_{sp} = t \quad E(k) = \pm 2t$$

$$H_0 \rightarrow 2t \sum_j \phi_-^\dagger(j) \phi_+(j+1) + h.c.$$

$$\phi_\pm = [a_p \pm a_s] / \sqrt{2}$$

**Edge states are completely localized**



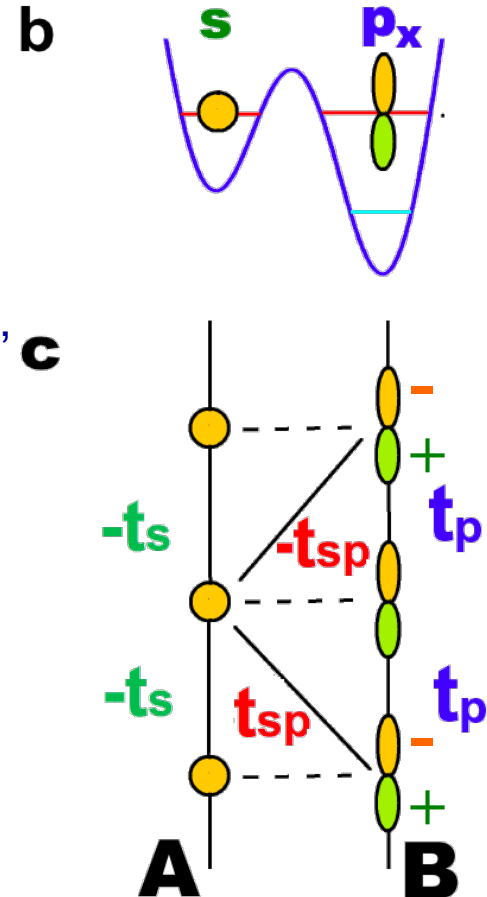
**-general case**

$$H_0 = \sum_j C_j^\dagger \left[ \frac{t_p - t_s}{2} \mathbb{I} - \frac{t_p + t_s}{2} \sigma_z - it_{sp} \sigma_y \right] C_{j+1} + h.c. \quad C_j = \begin{bmatrix} a_s(j) \\ a_p(j) \end{bmatrix}$$

Edge states decay with a width  $\xi = 2 / \log(|(\sqrt{t_s t_p} + t_{sp}) / (\sqrt{t_s t_p} - t_{sp})|)$

# Summary: topological insulator from odd parity

1. Topological insulator (index group  $Z$  class) at half filling.
2. Compare with spin-orbit coupling generated by artificial gauge field in cold atoms
  - A. Pioneer experiments: NIST (I. Spielman et al), ... MIT (M. Zwierlein)
  - B. First from China: USTC (S. Chen, Y. Deng, J. Pan, et al), Shanxi U (J. Zhang et al)
3. This model: No spin, but orbit. Resembles spin-orbit coupling if (s, p) space viewed pseudo-spin-1/2.
4. **New result: topological phase** not requiring any of previously known mechanisms: rotation, gauge field, p-wave pairing,...

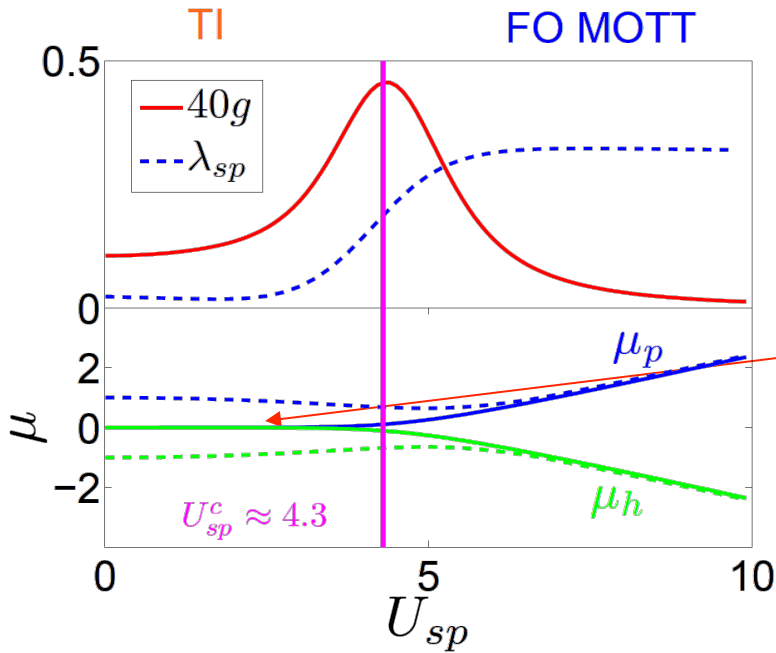




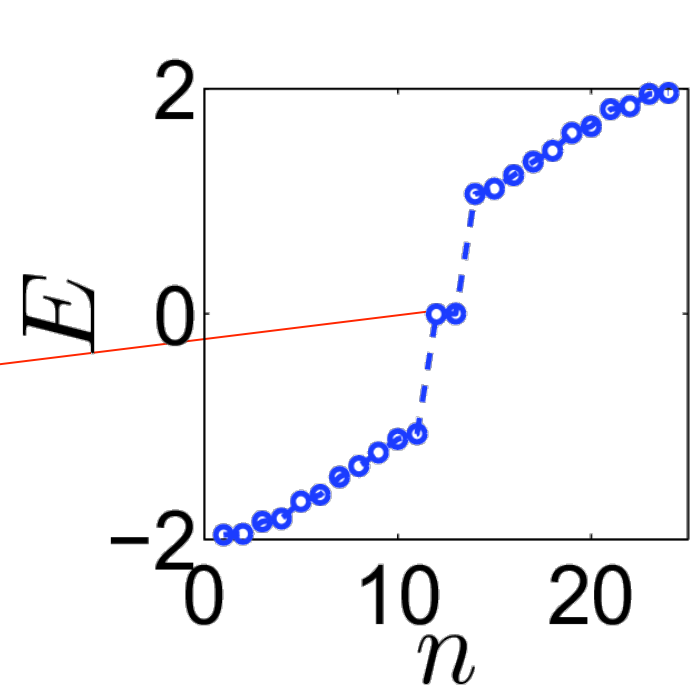
# Topological to Ferro-orbital Mott Insulator Transition

- With inter-orbital interaction

$$H_{\text{int}} = \sum_j U_{sp} \left[ n_s(j) - \frac{1}{2} \right] \left[ n_p(j) - \frac{1}{2} \right]$$



-spectra without interaction



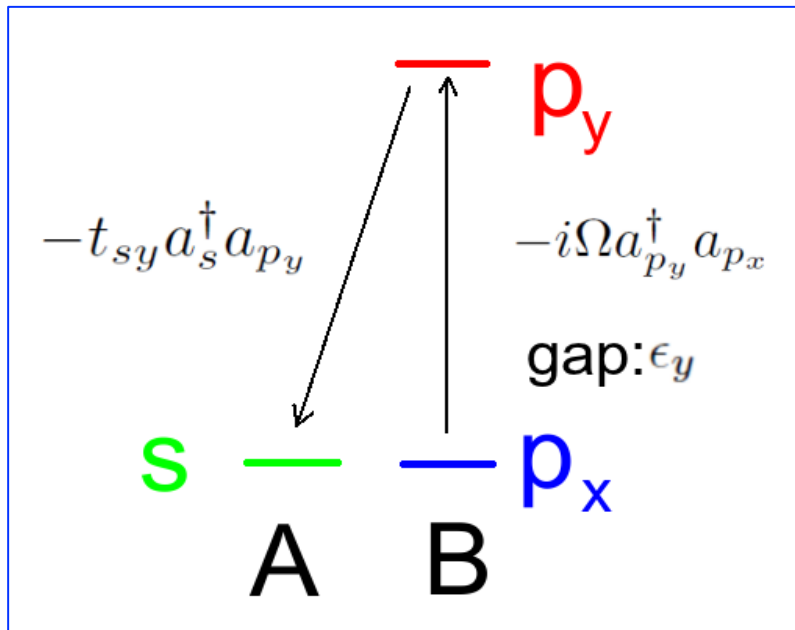
$$g \sim \frac{1 - |\langle \psi(U_{sp}) | \psi(U_{sp} + \delta U_{sp}) \rangle|}{(\delta U_{sp})^2} \quad \text{fidelity metric}$$

$$\lambda_{sp} \sim |\langle C_j^\dagger \sigma_x C_j \rangle|^2 \quad \text{Ferro-magnetic order (inter-orbital coherence)}$$

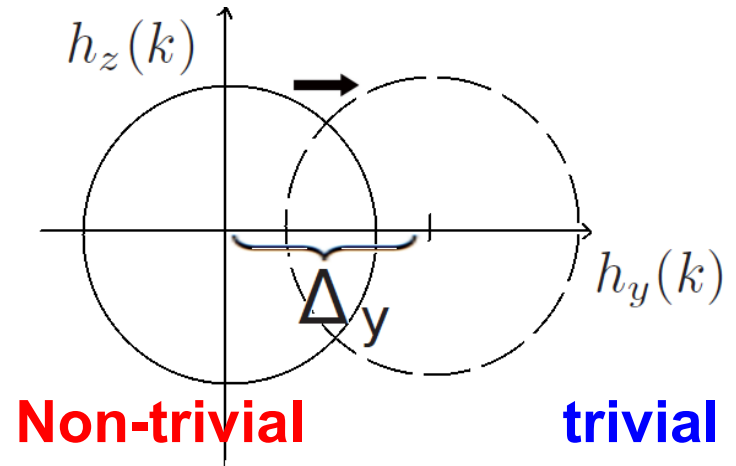
# Topological phase transition – driven by rotation

-rotating individual lattice sites

$$\delta H = \frac{\Omega^2}{\epsilon_y} C_j^\dagger \sigma_y C_j \quad \Delta_y = \frac{\Omega^2}{\epsilon_y}$$



-phase transition



$$\Delta_y^c = 2t_{sp}$$

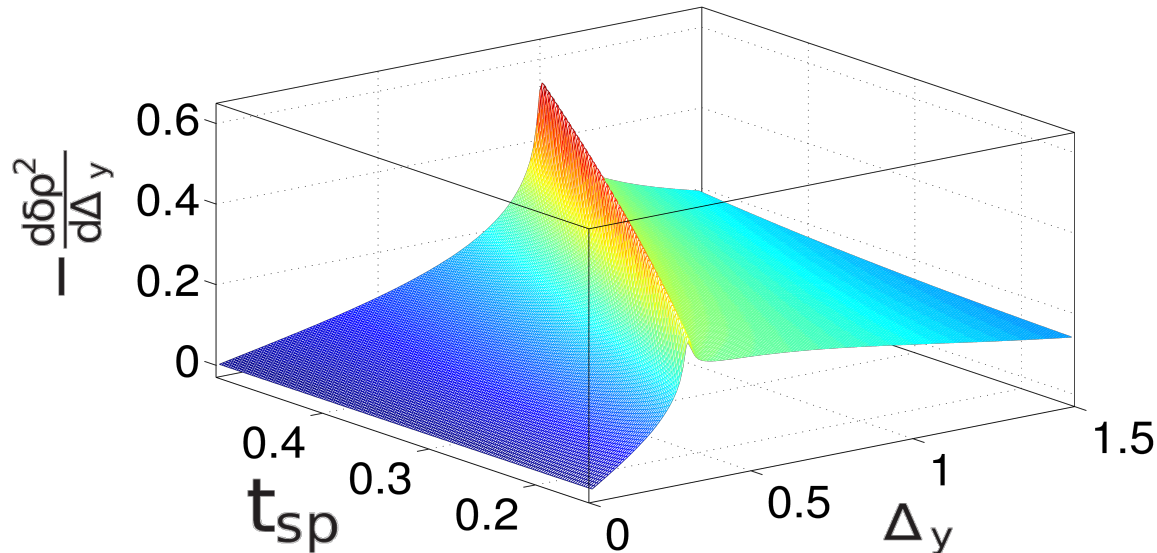
# Density fluctuations as a probe

$$\rho(j) = C_j^\dagger C_j - 1$$

$$\delta\rho \equiv \frac{1}{\text{system length}} \sum_j \sqrt{\langle \rho(j)\rho(j) \rangle}$$

$$\delta\rho^2 = \frac{1}{2} \left[ 1 - \left( \oint \frac{dk}{2\pi} \sin(\theta(k)) \right)^2 \right]$$

$$\sin(\theta) = \frac{h_y(k)}{\sqrt{h_y^2 + h_z^2}}$$



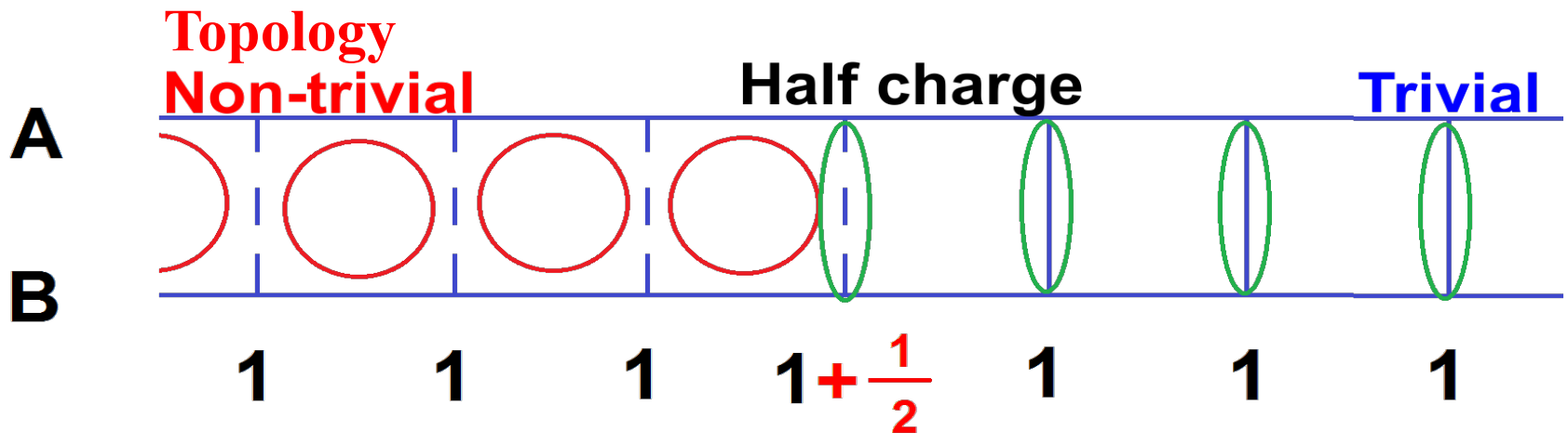
**The peaks coincide with the critical line.**

# Domain Wall Fractional Charge

$$H_\eta = H + \frac{\Delta_y}{2} \sum_j [1 - \cos \eta(j)] C_j^\dagger \sigma_y C_j$$

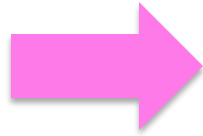
$$\eta(j = -\infty) = 0$$

$$\eta(j = +\infty) = \pi$$



# Firm Computation of Fractional Charge: Background (auxiliary) gauge field method

- Introduce background gauge field:  $(A_\tau, A_x)$ ,  $\tau = it$



$$\begin{aligned}
 D_\tau &= \partial_\tau + iA_\tau(j, \tau), \\
 \mathcal{T}_{j j+1} &= e^{iA_x(j+1/2, \tau)} T_{j j+1}, \\
 T_{j j+1} &= \begin{bmatrix} -t_s & -t_{sp} \\ t_{sp} & t_p \end{bmatrix}, \\
 \mathcal{T}_{j j+1} &= \mathcal{T}_{j+1 j}^\dagger.
 \end{aligned}$$

Berry phase

- Effective action  $\tilde{S}_{\text{eff}}[A_\mu] = \int dxdt (A_x \partial_t \eta - A_t \partial_x \eta) \frac{1}{2\pi} \partial_\eta \gamma(\eta)$

- Charge

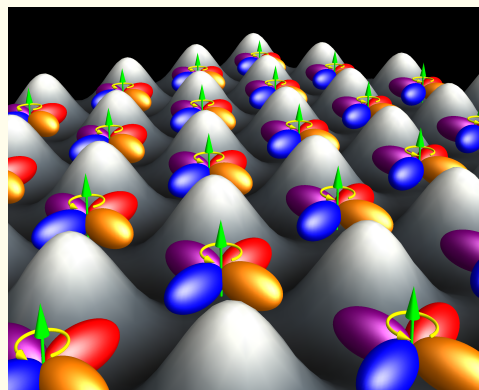
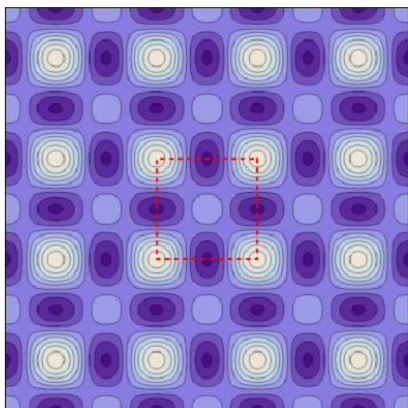
$$Q = \int \frac{\tilde{S}_{\text{eff}}}{\delta A_t} = -\frac{1}{2\pi} \int dx \partial_x \eta \partial_\eta \gamma(\eta) = -\int \frac{d\eta}{2\pi} \partial_\eta \gamma(\eta)$$

Find:  $Q = \frac{1}{2} \text{ mod } 1$

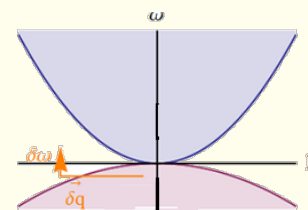
# Topological semimetal in a fermionic optical lattice

Kai Sun<sup>1</sup>, W. Vincent Liu<sup>2,3,4\*</sup>, Andreas Hemmerich<sup>5</sup> and S. Das Sarma<sup>1</sup>

System: **p-orbital** fermions  
on Double-well lattice



Ordered (topological  
insulator)



Normal(topological  
semimetal): winding  
number =2, quadratic  
dispersion, different than  
Graphene

0

$T_c$

T

## Ordered phase

- Breaks Time-reversal symmetry
- Topological Insulator
- New mechanism – interaction driven -- Differs from the previously known's: artificial gauge field, rotation, spin-orbit coupling, ...

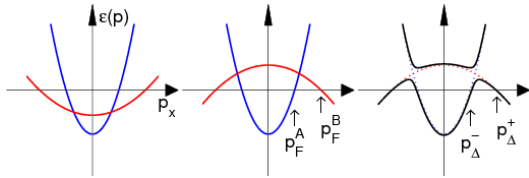


# Conclusion

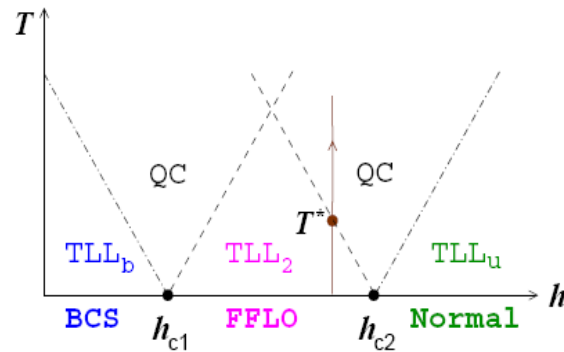
## 1. Spin imbalanced Fermi gas

### Sarma/ Breached Pair

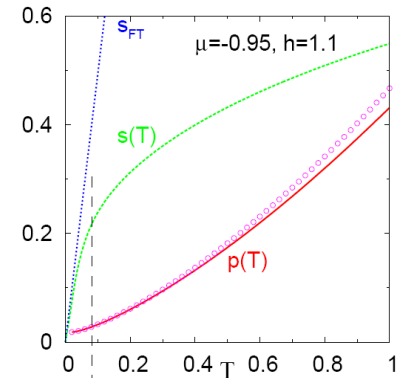
(a homogeneous polarized superfluid)



### Phase diagram of 1D polarized Fermi gas

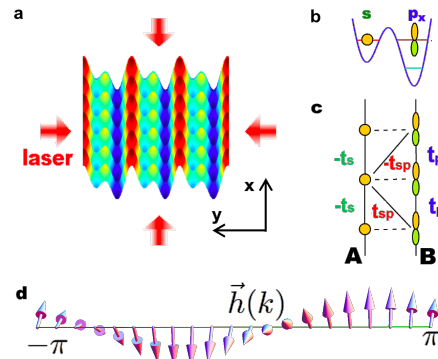
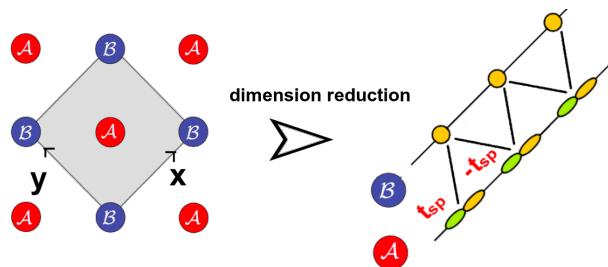


### Thermo Bethe Ansatz compared to Field Theory

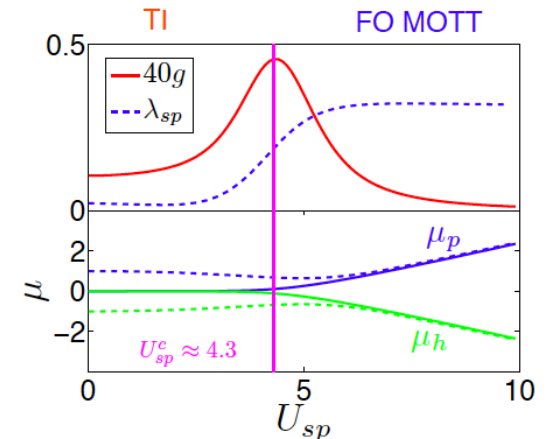


## 2. Topological sp-orbital ladder

**Prediction:** Hamburg double-well lattice, when dimension reduced, is topological insulator with fermion zero modes



Interaction drives transition to topologically trivial Mott



# Acknowledgement

## *Collaborators*

### *Imbalanced fermions:*

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M. T. Batchelor (Australian NU)  
*Experiment:* R. Hulet (Rice)

### **Student/Postoc:**

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EZ (also on the right)

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Philipp Hauke (ICFO Spain)  
Maciej Lewenstein (ICFO Spain)  
Joel Moore (Berkeley)  
Kai Sun (Maryland-> U Michigan)  
Congjun Wu (UCSD)  
*Exp:* A. Hemmerich (Hamburg)

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