Exotic Quantum Criticalities

Eun-Gook Moon

KAIST

Tsinghua, May 17, 2017

Main Questions

Novel physics in condensed matter?

How do we discover / measure it?

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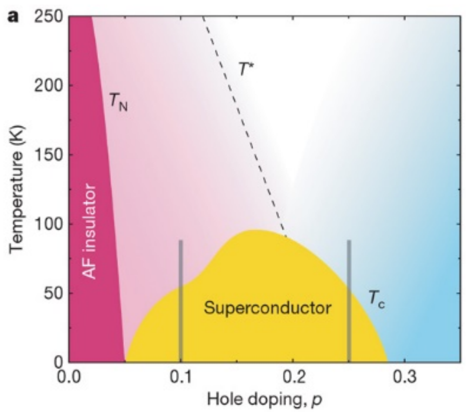
Novel physics in condensed matter?

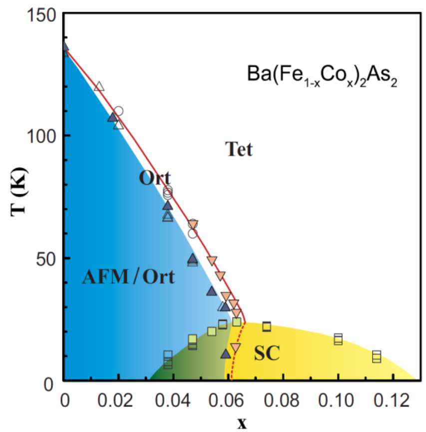
How do we discover / measure it?

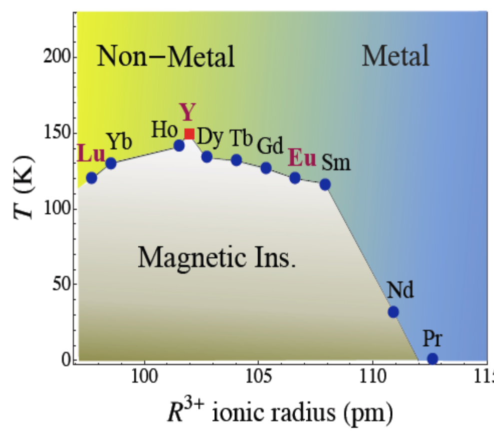
“exotic quantum criticalities”
(“exotic quantum phase transitions”)
to answer the questions.

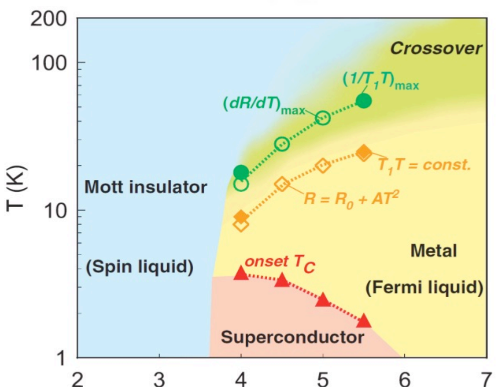
(Quantum) Phases

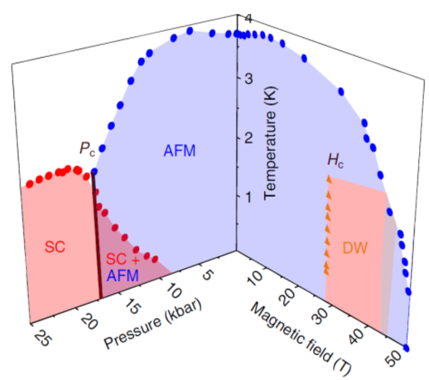
Various phases with tuning parameters
in strongly correlated systems.

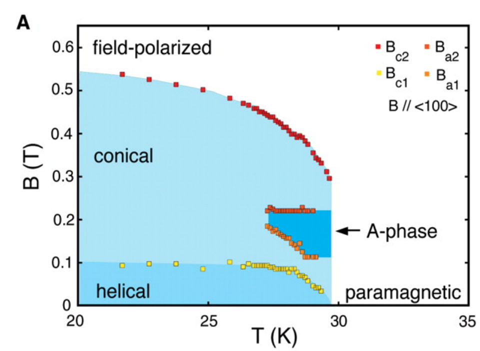






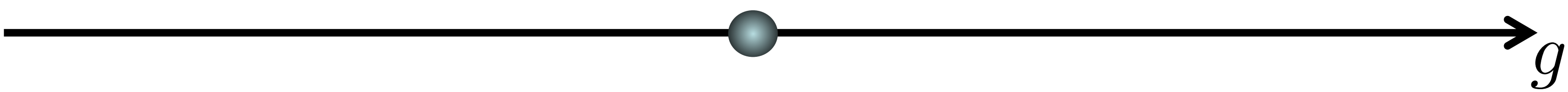






Colorful!!!

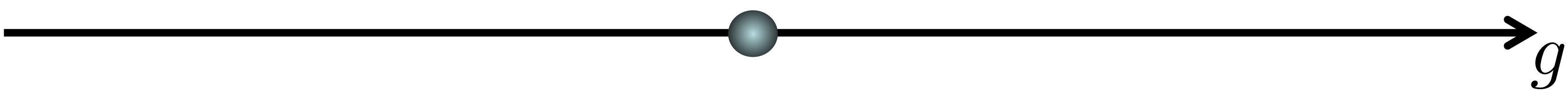
Conventional Phases and Phase Transitions



Symmetry broken phases
: magnetism, etc.

Symmetric phases :
(trivial) band insulator, etc

Conventional Phases and Phase Transitions



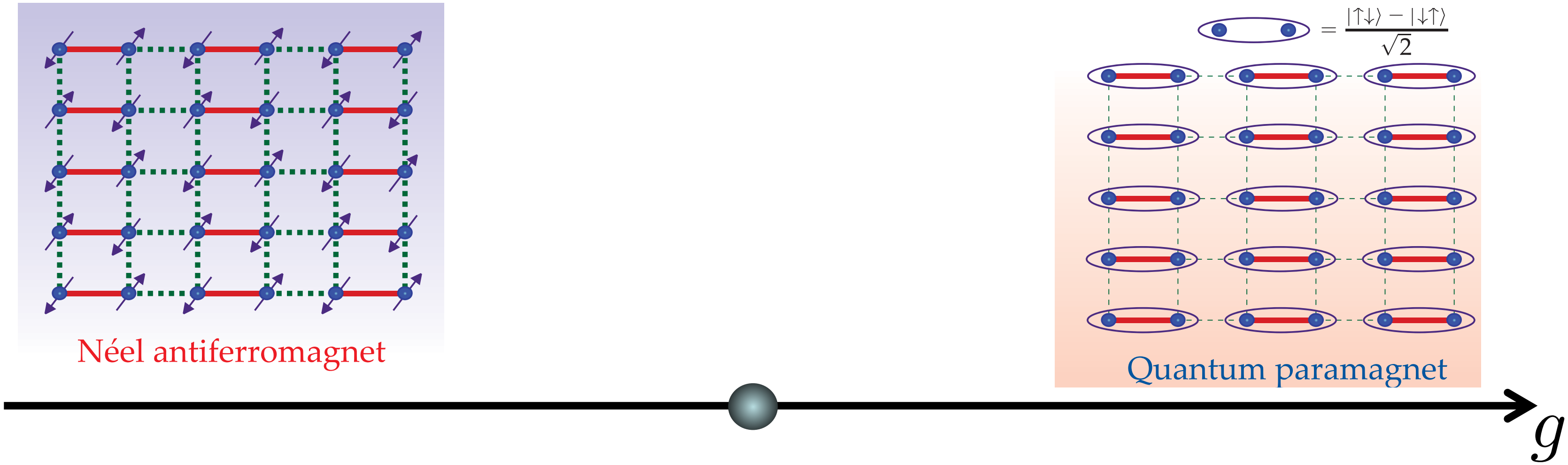
Symmetry broken phases
: magnetism, etc.

1. *Easy to detect*
2. *Theoretically well-understood*
3. *spin wave, vortex, etc..*

Symmetric phases :
(trivial) band insulator, etc

1. *Not difficult to detect*
2. *Theoretically well-understood*
3. *Ground state is fully gapped (for insulators)*

Conventional Phases and Phase Transitions



Excitations :
Goldstone modes (spin wave)

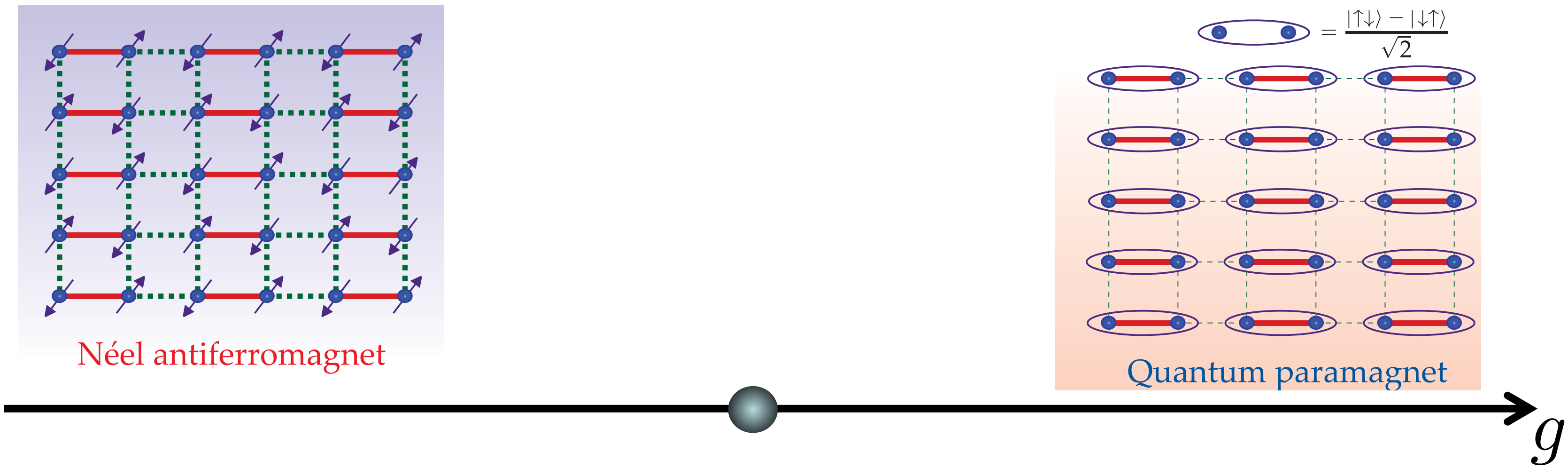
$$\langle \vec{\phi} \rangle \neq 0$$

Excitations :
Gapped triplons

$$\langle \vec{\phi} \rangle = 0$$

$$\mathcal{F}_{LGW} = \frac{1}{2}(\partial \vec{\phi})^2 + \frac{r}{2}(\vec{\phi})^2 + \frac{u}{4!}(\vec{\phi})^4$$

Conventional Phases and Phase Transitions



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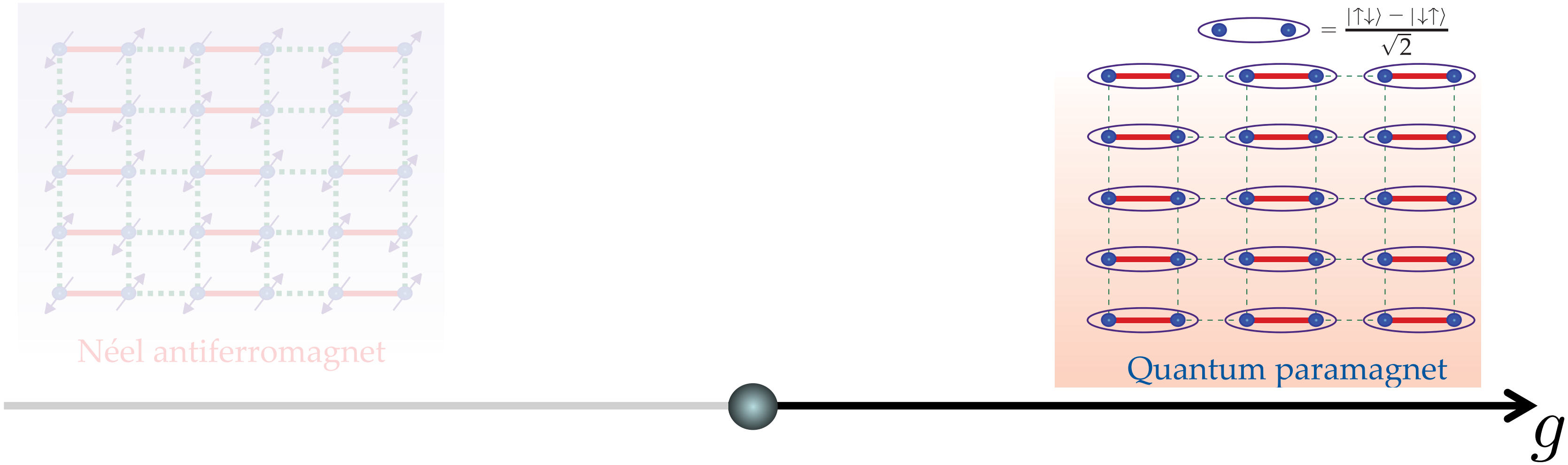
Landau-Ginzburg-Wilson criticalities

(def : conventional quantum criticalities in this talk)

Conventional Phases and Phase Transitions



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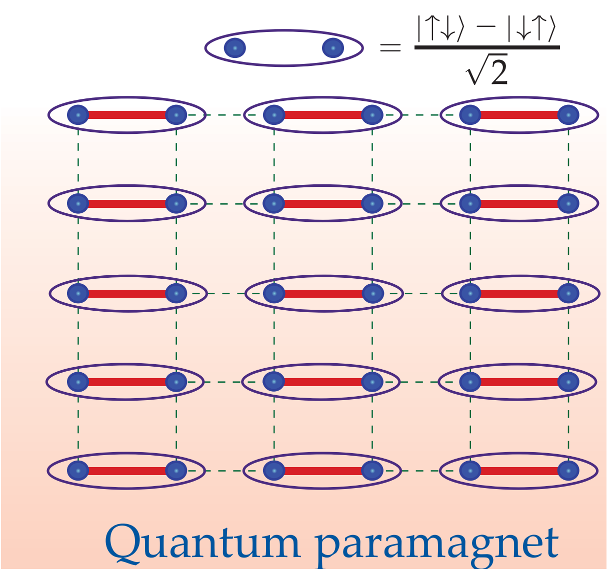
Conventional Phases and Phase Transitions

$$\mathcal{F}_{LGW} = \frac{1}{2}(\partial\vec{\phi})^2 + \frac{r}{2}(\vec{\phi})^2 + \frac{u}{4!}(\vec{\phi})^4$$

Symmetric phases in LGW :

Trivial product ground state



$$|G\rangle = \prod_i \left(|\uparrow\downarrow\rangle_i - |\downarrow\uparrow\rangle_i\right)$$

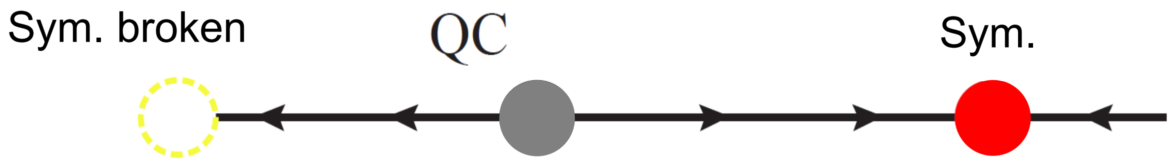
single-site physics
(trivial many-body ground state)

Conventional Phases and Phase Transitions

Symmetry rules!

Sym. Broken : SSB + Goldstone modes

Sym. : trivial product state + Gapped excitations



Search for Novel Physics

Difficulties :

1. Most materials : conventional ordered phases

2. Hard to find signals of exotic physics

Exotic Phases and Phase Transitions

Strategy

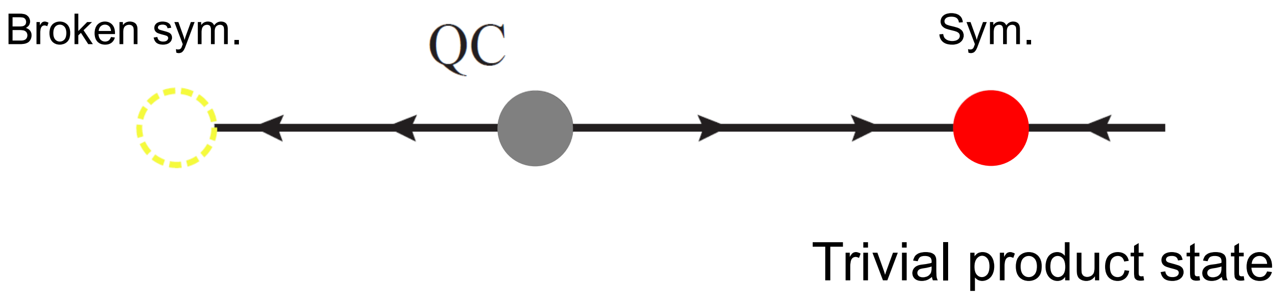
1. **Entangled symmetric** ground states

2. Quantum phase transitions near the entangled states

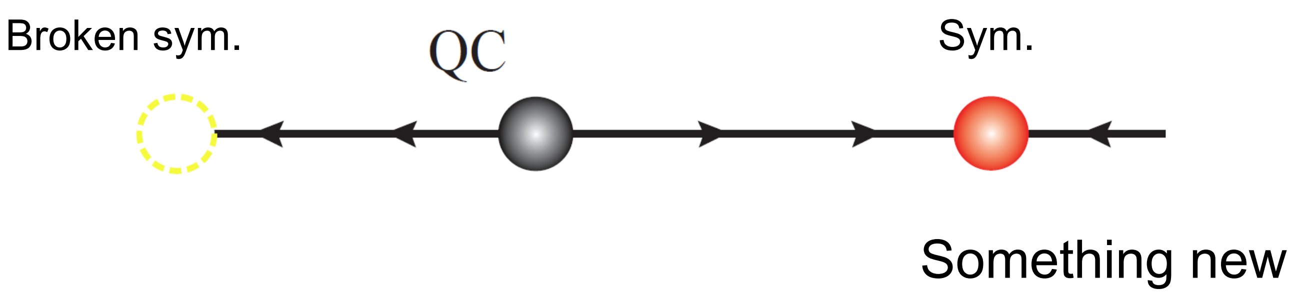
Beyond LGW criticalities : exotic physics!
(Physical properties near criticalities are characteristics!)

QPTs

Conventional LGW paradigm,



Exotic criticality



Search for Novel Physics

Entangled symmetric ground states ?

1. non-Fermi liquids

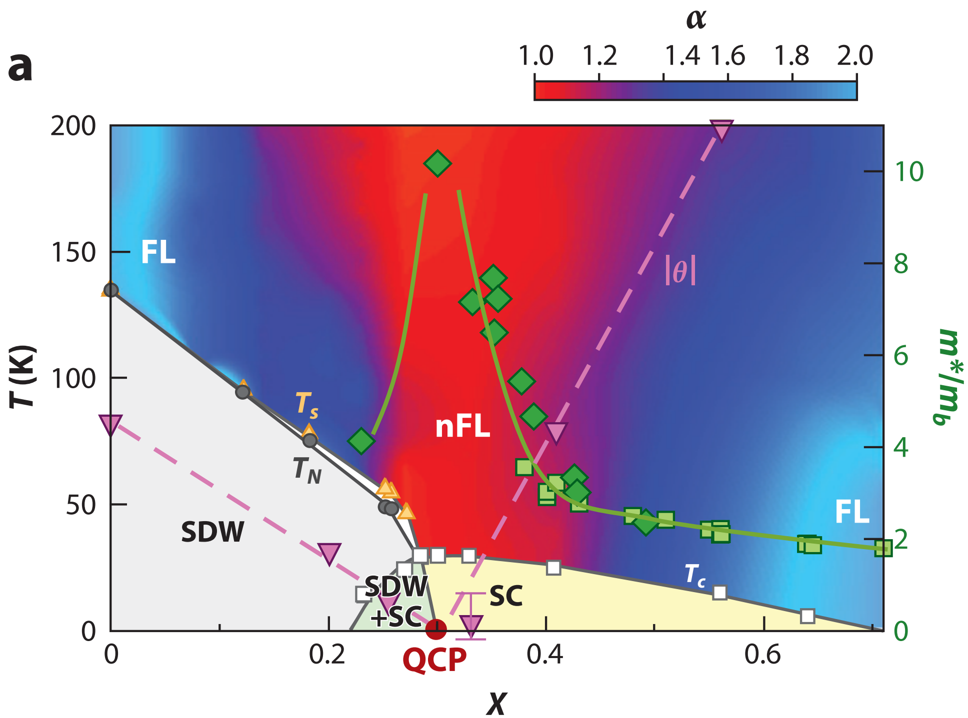
2. Topological phases

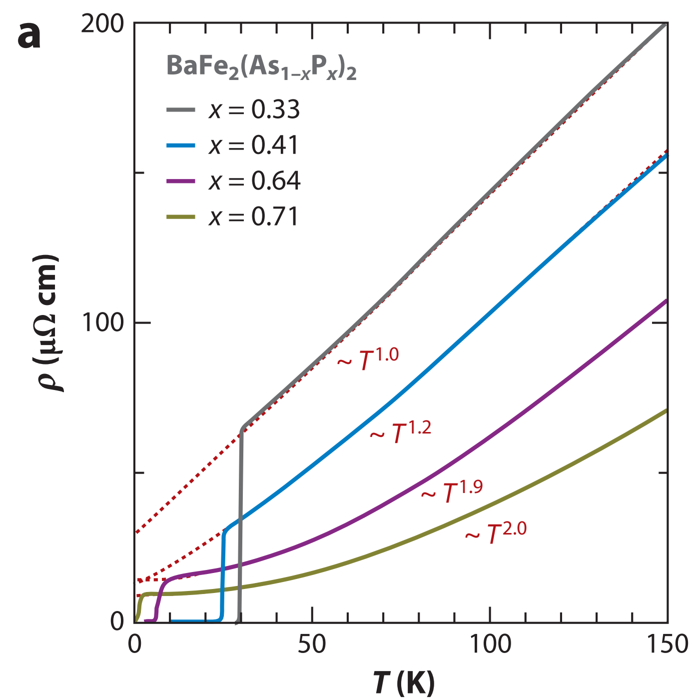
3. Phases with quantum anomalies

4. …

Non Fermi liquids

Resistivity does not show Fermi-liquid behaviors



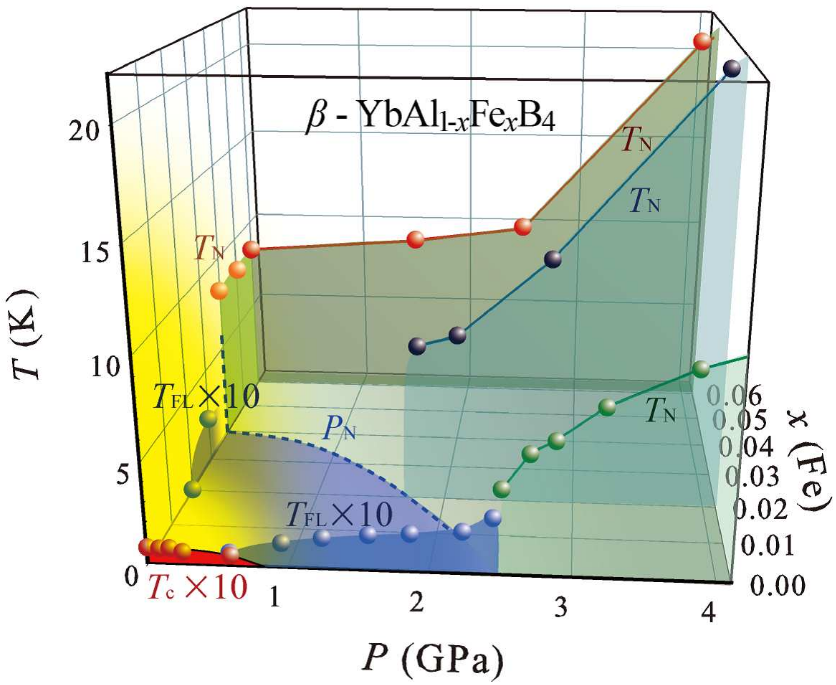


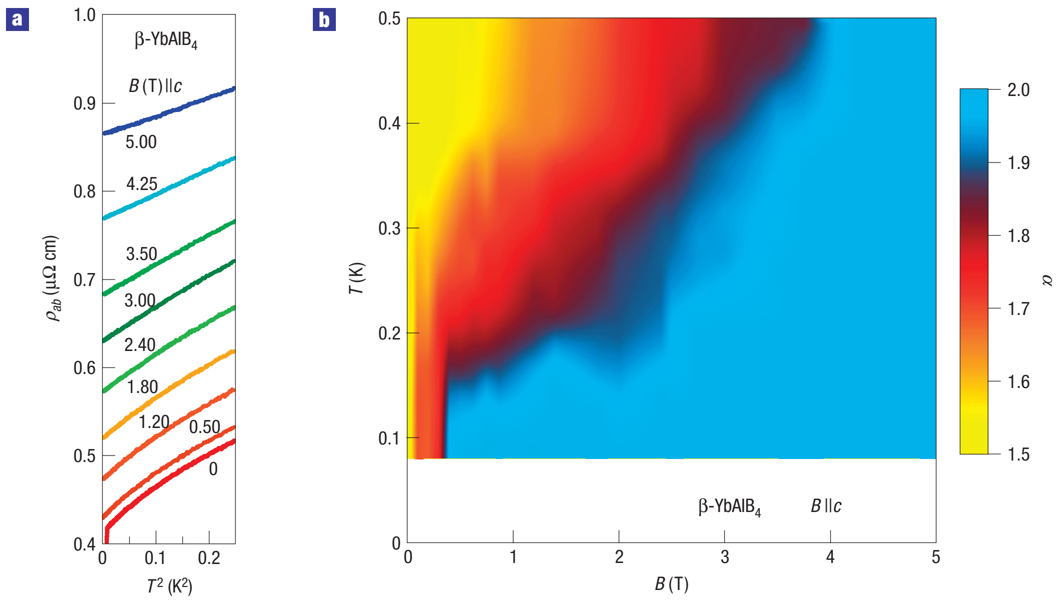
Non-Fermi liquid near QPT

Matsuda Group

Non Fermi liquids

Resistivity does not show Fermi-liquid behaviors



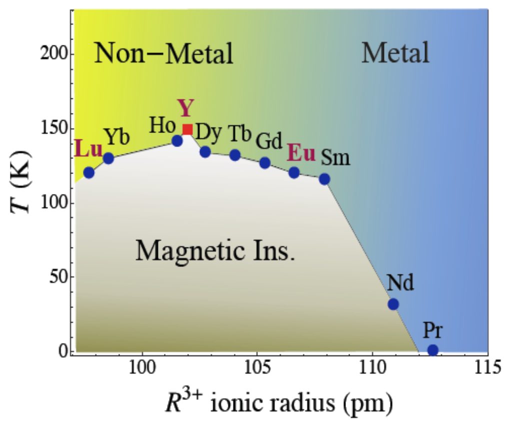


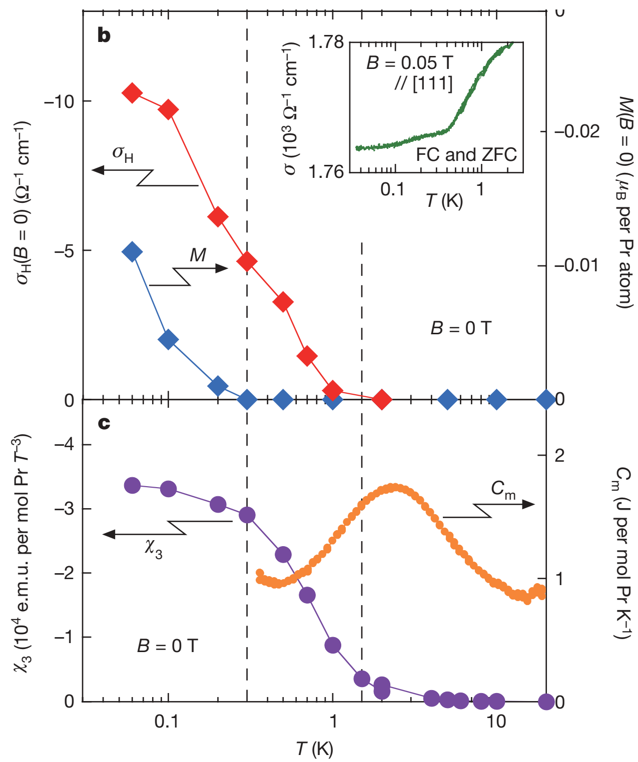
Quantum critical phase?

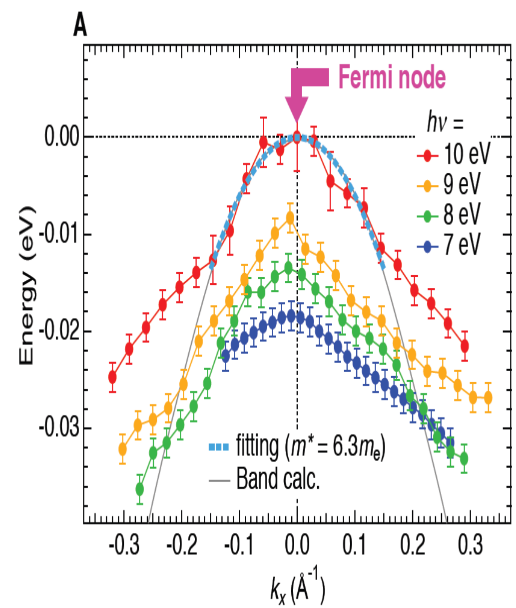
Nakatsuji group

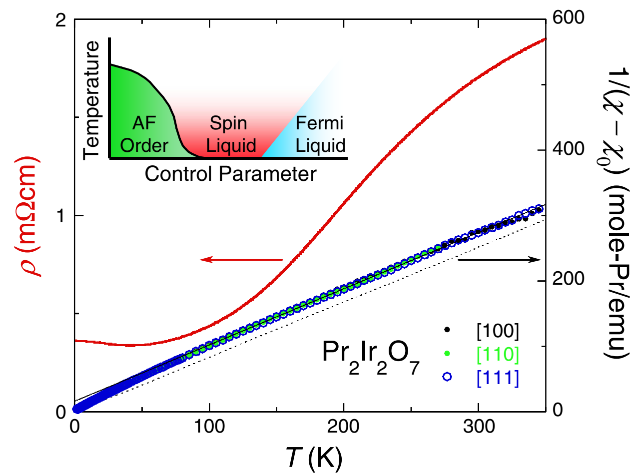
Non Fermi liquids

Strange behaviors : anomalous Hall, strange resistivity, etc…









Nakatsuji, Kondo, Shin groups

Fermi liquids VS non-Fermi liquids

Kinetic energy VS Coulomb energy

$E_{kin} \gg E_{Coulomb}$ Good metal : Fermi-liquid (perturbation works)

$E_{kin} \sim E_{Coulomb}$ Something new happens!

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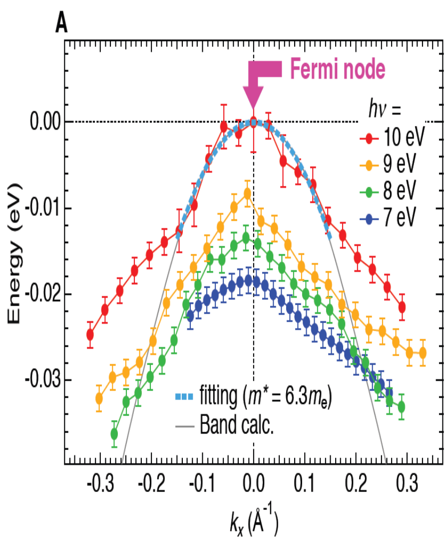

Kondo et. al. 2015

$$E_{kin}(k) \sim k^2 \qquad E_C \sim \frac{e^2}{r}$$

Heisenberg uncertainty $[r, k] = i\hbar$

$$E_{kin} \sim \frac{1}{mr^2} \quad , \quad E_C \sim \frac{e^2}{r}$$

$$E_{kin} \ll E_C \quad r \to \infty$$

Perturbation breaks down : NFL?

Fermi liquids VS non-Fermi liquids

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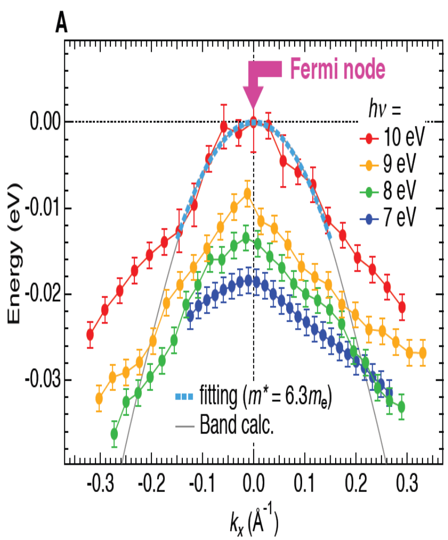


Kondo et. al. 2015

Exotic quantum critical phase!

PRL **111**, 206401 (2013) PHYSICAL REVIEW LETTERS week ending 15 NOVEMBER 2013

Non-Fermi-Liquid and Topological States with Strong Spin-Orbit Coupling

Eun-Gook Moon,[1] Cenke Xu,[1] Yong Baek Kim,[2] and Leon Balents[3]

Physical quantities show exotic critical behaviors

Scaling analysis

- RG setup

$$S_L = \int d\tau d^d x \left\{ \psi^\dagger \left[\partial_\tau - ie\,\varphi + \hat{\mathcal{H}}_0 \right] \psi + \frac{c_0}{2} (\partial_i \varphi)^2 \right\}$$

Scaling analysis in the spatial *d* dimension (z=2) :

$$k \to bk \quad \omega \to b^2 \omega \quad \psi \to b^{\frac{d}{2}} \psi \quad \varphi \to b^{\frac{d}{2}} \varphi$$

(No Fermi surface : Wilsonian scaling is well-defined.)

$[e^2] = 4 - d$ The electric charge is **relevant** below four spatial dimensions.

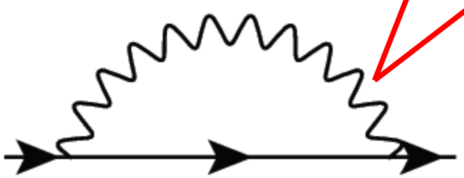
Two methods :
ε(=4-d) expansion, Large N_f expansion (*d*=3)

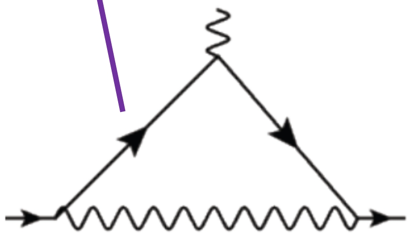
Renormalization group : ε expansion

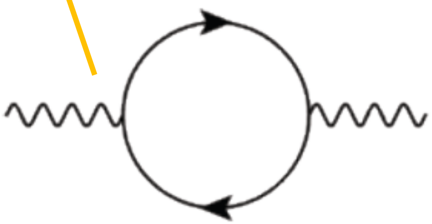
- RG setup

$$S_L = \int d\tau d^d x \left\{ \psi^\dagger \left[\partial_\tau - ie\varphi + \hat{\mathcal{H}}_0 \right] \psi + \frac{c_0}{2} (\partial_i \varphi)^2 \right\}$$

Quantum correction (ε expansion):







Renormalization group : ε expansion

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Quantum correction (ε expansion):

For general dimension (d) and fermion flavor number (N_f),
The RG equation is

$$\begin{aligned} \frac{d}{dl}(\tilde{e}^2) &= (4-d)\tilde{e}^2 - \frac{30N_f + 8}{15}\tilde{e}^4 + O(\tilde{e}^6) \\ &= +\tilde{e}^2 - \frac{38}{15}\tilde{e}^4 + O(\tilde{e}^6) \end{aligned}$$

Renormalization group : ε expansion

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$$= +\tilde{e}^2 - \frac{38}{15}\tilde{e}^4 + O(\tilde{e}^6)$$

Quantum correction :
Screening effect from virtual particle-hole excitation

Renormalization group : ε expansion

- RG setup

$$S_L = \int d\tau d^d x \left\{ \psi^\dagger \left[\partial_\tau - ie\,\varphi + \hat{\mathcal{H}}_0 \right] \psi + \frac{c_0}{2} (\partial_i \varphi)^2 \right\} \qquad \tilde{e}^2 = \frac{me^2}{8\pi^2 c_0 \Lambda^{4-d}}$$

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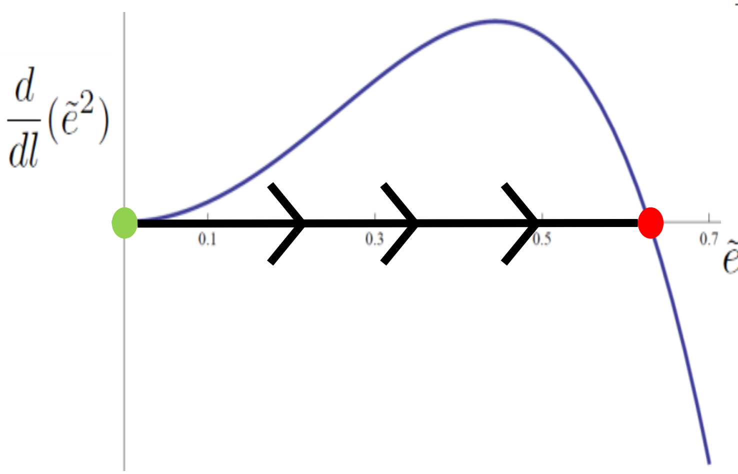


New stable fixed point :
LAB (Luttinger-Abrikosov-Beneslaevski)

Anomalous dimension in all physical quantities

$$\eta_b \to 1 \quad (\epsilon \to 1,\ N_f \to \infty)$$

Fermi liquids VS non-Fermi liquids

Kinetic energy VS Coulomb energy

$E_{kin} \gg E_{Coulomb}$ Good metal : Fermi-liquid (perturbation works)

$E_{kin} \sim E_{Coulomb}$ Something new happens!

Some Lessons :

Smaller Fermi volumes are useful (ex: semi-metal).

Symmetry protection (ex : cubic & TRS) is useful

Questions :

Thermal properties??

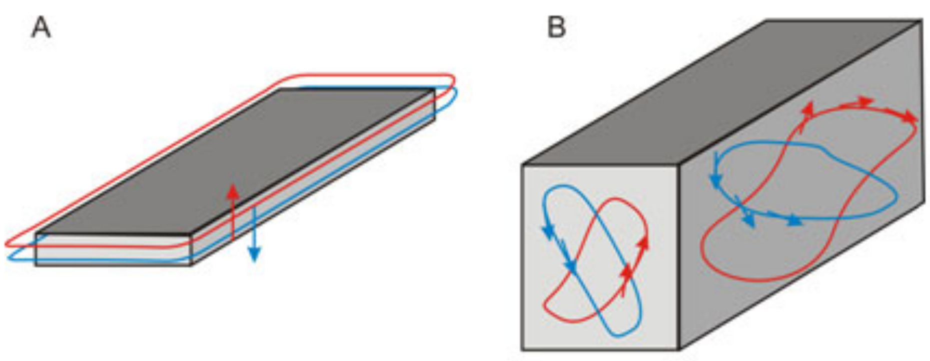
Take-home Message I

Non-Fermi liquids are interesting.

Strong interaction / correlation are necessary!

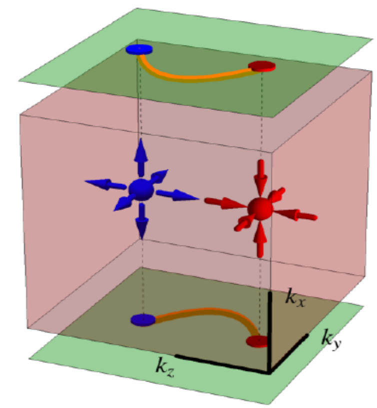
Topological Phases

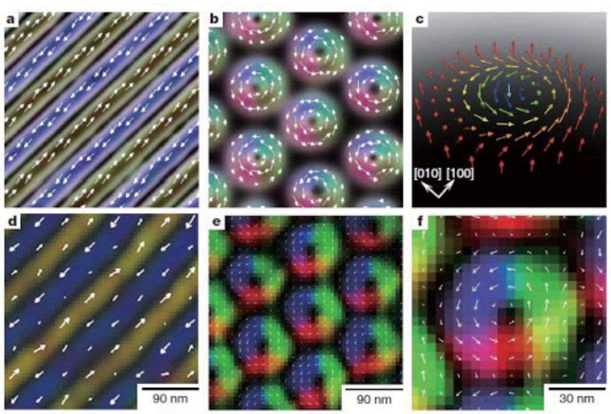
Beyond symmetry!

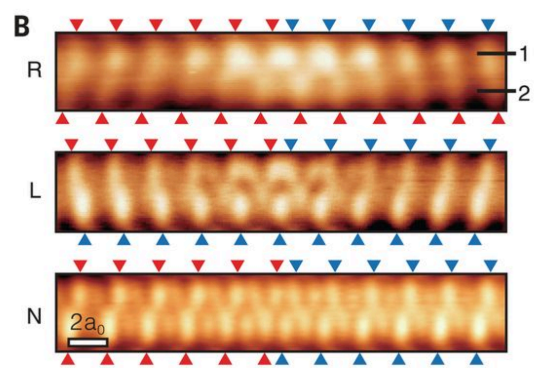


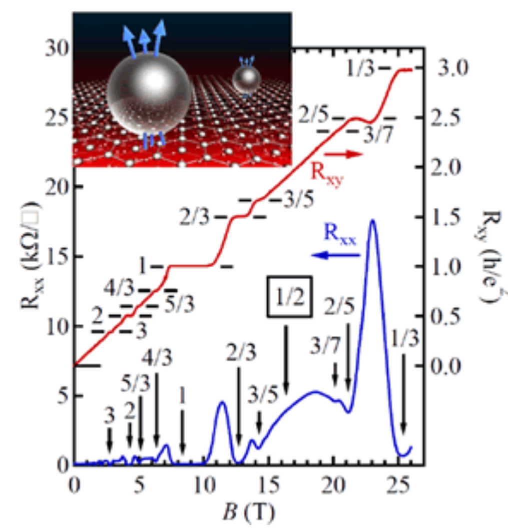
2D topological insulator

3D topological insulator



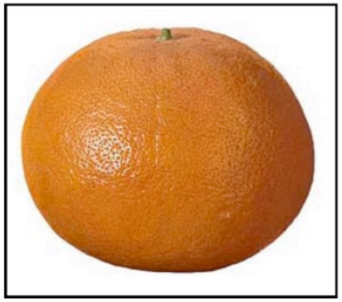






From google images with the key words “Topological matters”

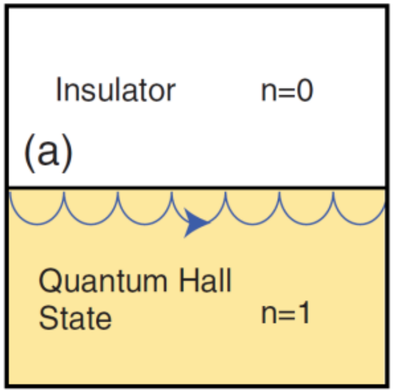
Topology

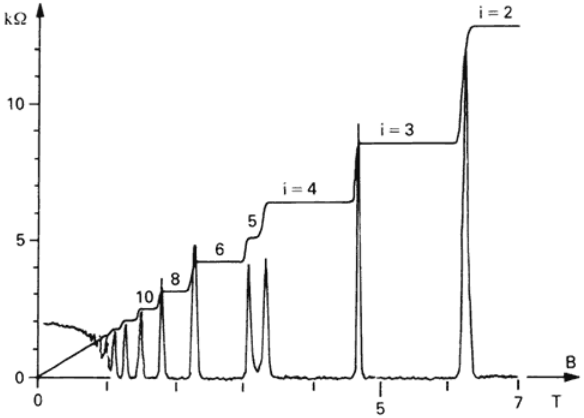


Two objects are topologically different.

: Continuous deformation cannot transform one to the other.

: Something happens between topologically different states.





Topology in condensed matter

Topological nature in insulators and gapped SC
: Well-understood!

REVIEWS OF MODERN PHYSICS, VOLUME 82, OCTOBER–DECEMBER 2010

Colloquium: Topological insulators

M. Z. Hasan*

Joseph Henry Laboratories, Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

C. L. Kane†

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

REVIEWS OF MODERN PHYSICS, VOLUME 83, OCTOBER–DECEMBER 2011

Topological insulators and superconductors

Xiao-Liang Qi

Microsoft Research, Station Q, Elings Hall, University of California, Santa Barbara, California 93106, USA and Department of Physics, Stanford University, Stanford, California 94305, USA

Shou-Cheng Zhang

Department of Physics, Stanford University, Stanford, California 94305, USA

Topology in condensed matter

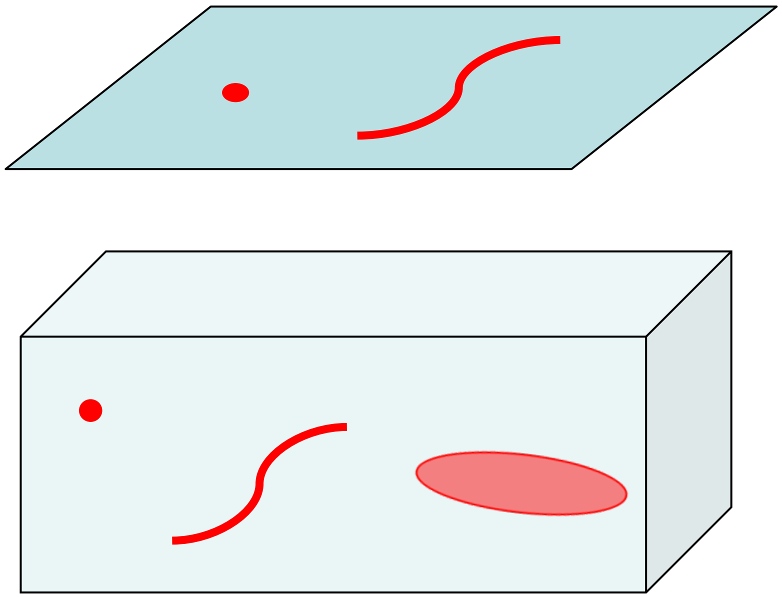
Topological nature in semi-metals and gapless SC

Spatial dimension : d=2

- point, line

Spatial dimension : d=3

- point, line, surface



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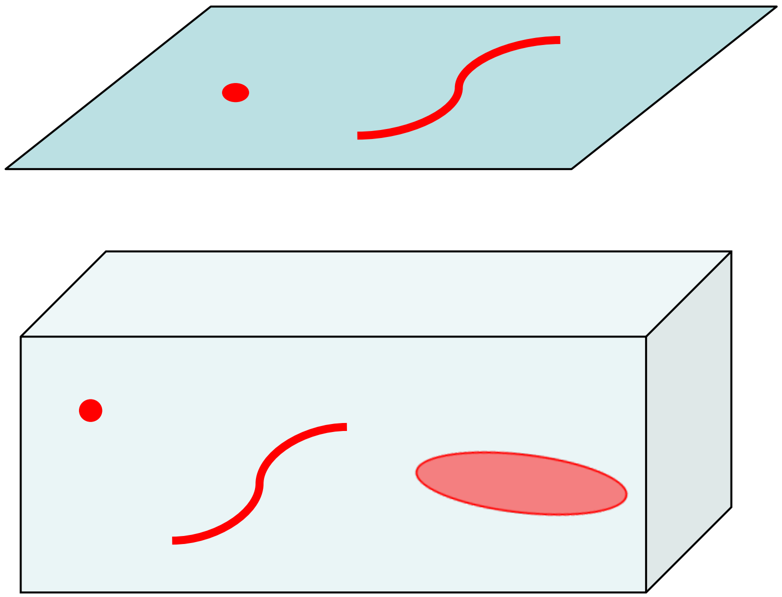

Topological invariants (ex : Chern number)

Topology in condensed matter

Topological nature in semi-metals and gapless SC

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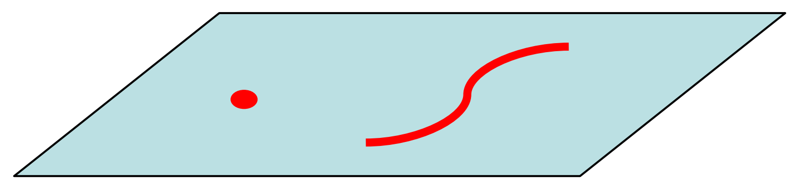

Spatial dimension : d=3

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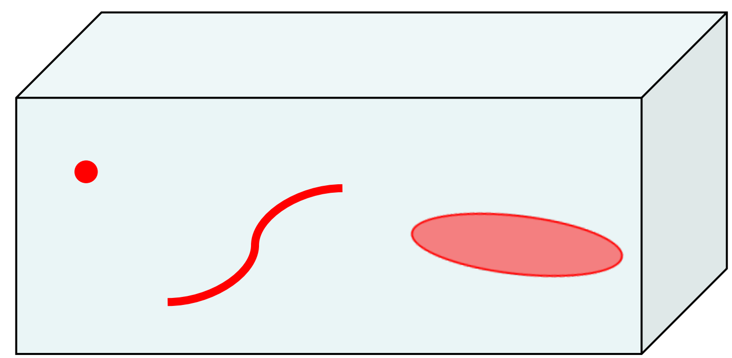


Most topological materials : weakly interacting (s and p orbitals)

How to observe strong correlation effects in topological matter?

Topology in condensed matter

Conventional phases : symmetry!

Topological phases : topology
(mostly, s and p orbitals)

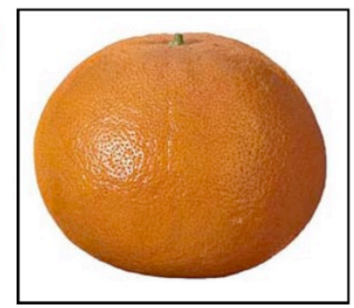
Next step : interplay between symmetry and topology
(d and f orbitals)

(see Pesin and Balents 2009)

Topological Phase Transitions

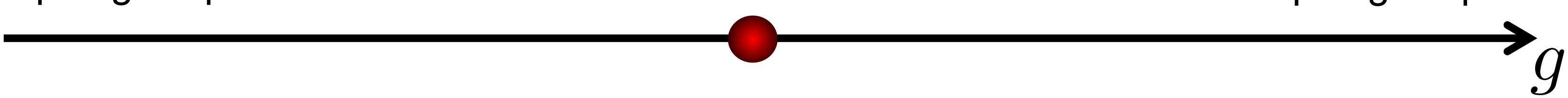
Strong correlation driven topological phase transitions

interplay between symmetry and topology



Topological phase

Non-topological phase



Beyond LGW criticalities

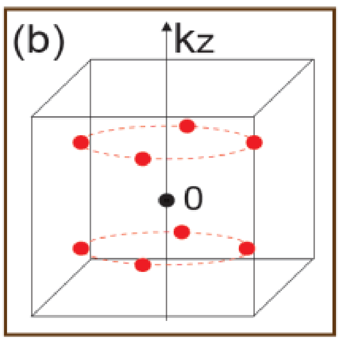
Physical quantities show exotic behaviors!
(ex: density fluctuations are highly anisotropic)

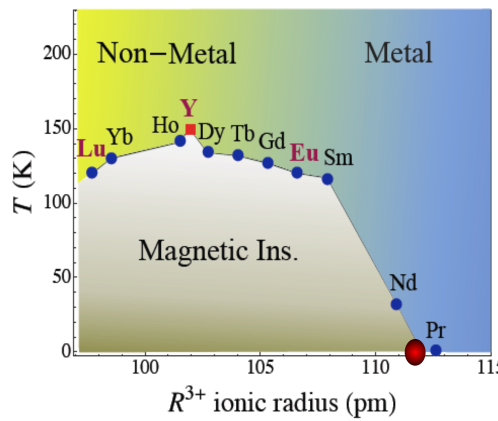
Topological Phase Transitions

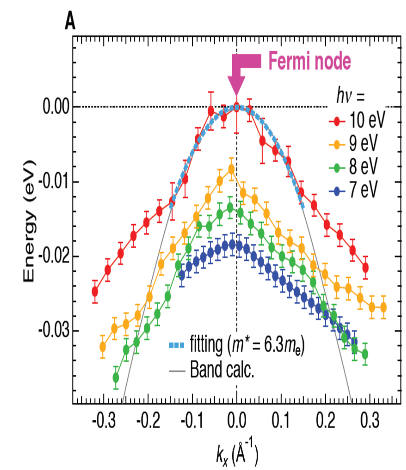
Strong correlation driven topological phase transitions

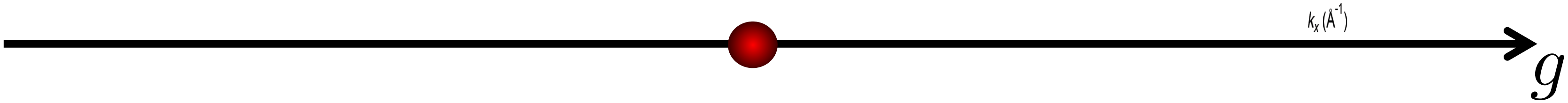
nodal points (Weyl semi-metals)

NFL









Beyond LGW criticalities

PHYSICAL REVIEW X **4,** 041027 (2014)

New Type of Quantum Criticality in the Pyrochlore Iridates

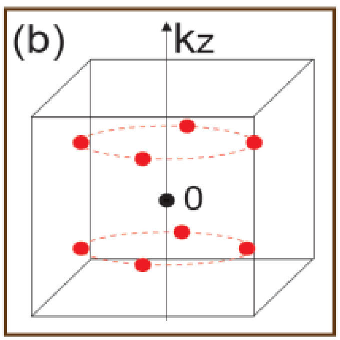
Lucile Savary,[1,*] Eun-Gook Moon,[1] and Leon Balents[2]

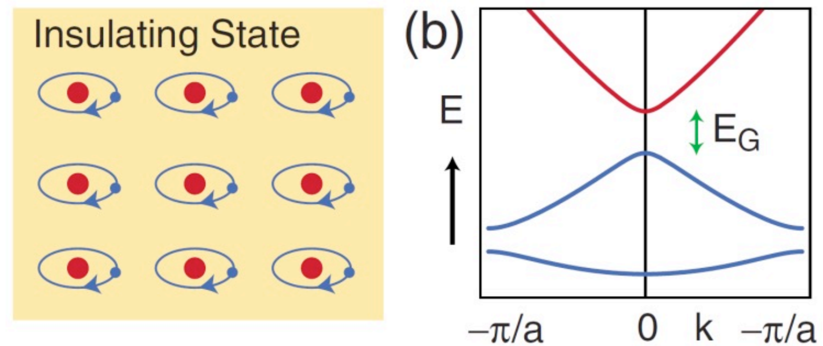
Topological Phase Transitions

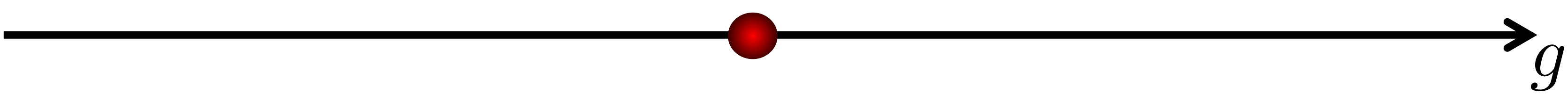
Weakly interacting topological phase transitions

nodal points (Weyl semi-metals)

insulators







Beyond LGW criticalities

ARTICLES
PUBLISHED ONLINE: 24 AUGUST 2014 | DOI: 10.1038/NPHYS3060
nature physics

Quantum criticality of topological phase transitions in three-dimensional interacting electronic systems

Bohm-Jung Yang[1]*, Eun-Gook Moon[2], Hiroki Isobe[3] and Naoto Nagaosa[1,3]*

SCIENTIFIC REPORTS

OPEN

Received: 02 September 2015

Novel Quantum Criticality in Two Dimensional Topological Phase transitions

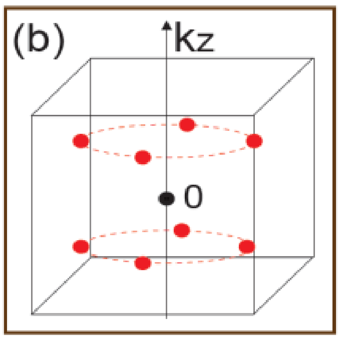
Gil Young Cho & Eun-Gook Moon

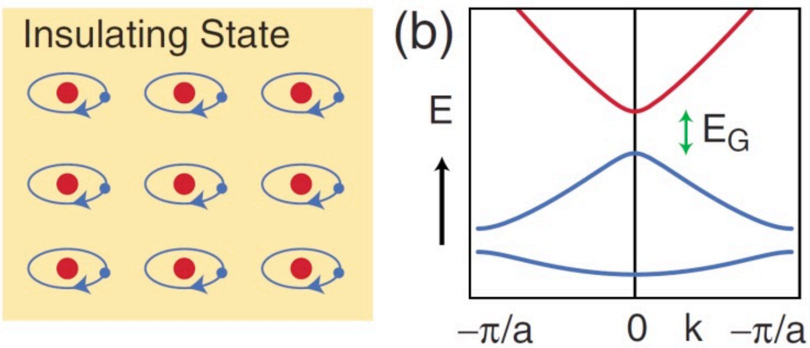
Topological Phase Transitions

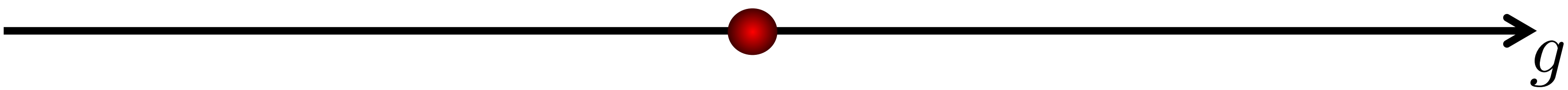
Strong correlation driven topological phase transitions

nodal points (Weyl semi-metals)

insulators







Beyond LGW criticalities

Chiral Symmetry Breaking with Long-range Coulomb Interaction in Topological Semi-metals

SangEun Han and Eun-Gook Moon
Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea

(Dated: May 15, 2017)

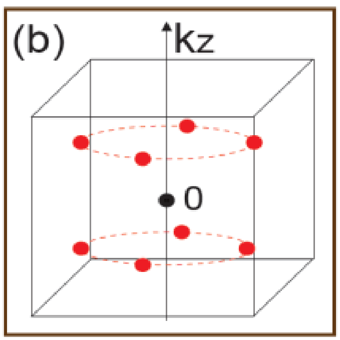
In preparation

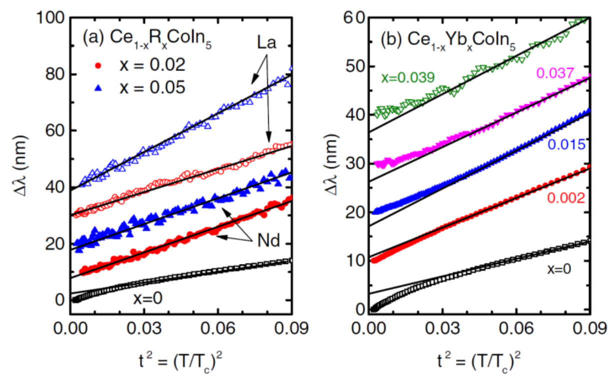
Topological Phase Transitions

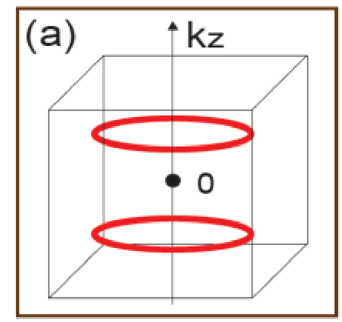
Strong correlation driven topological phase transitions

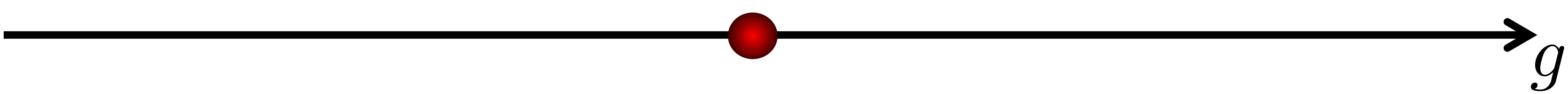
nodal point SC

Nodal line SC









Beyond LGW criticalities

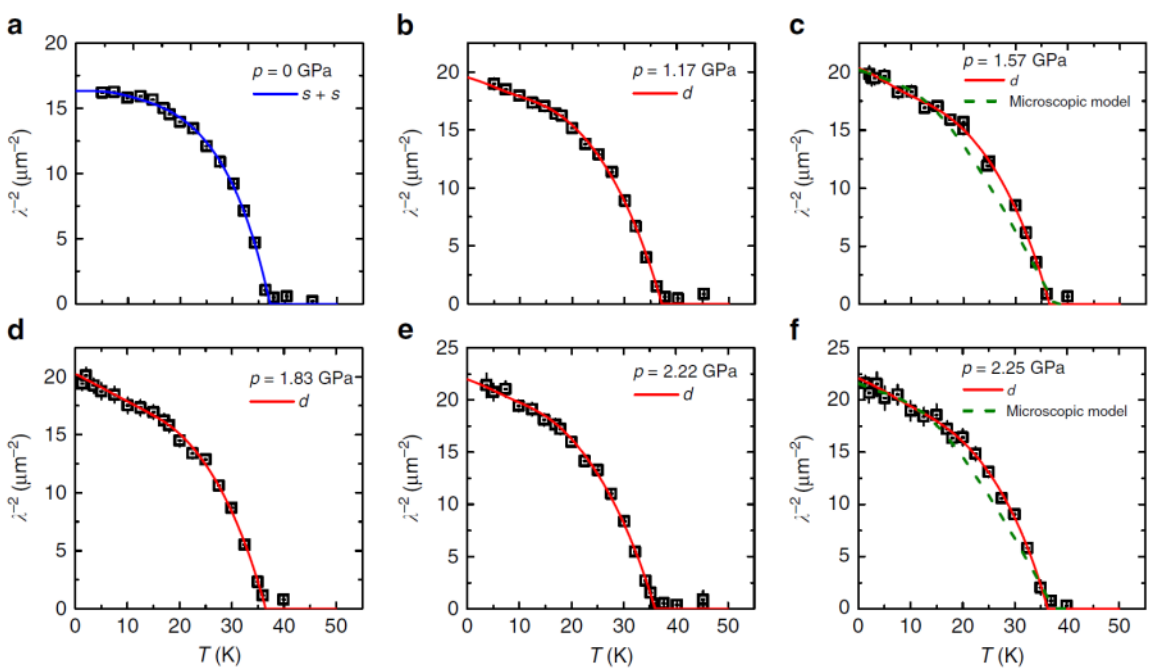
PHYSICAL REVIEW B **95**, 094502 (2017)

Topological phase transitions in line-nodal superconductors

SangEun Han, Gil Young Cho, and Eun-Gook Moon

Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea

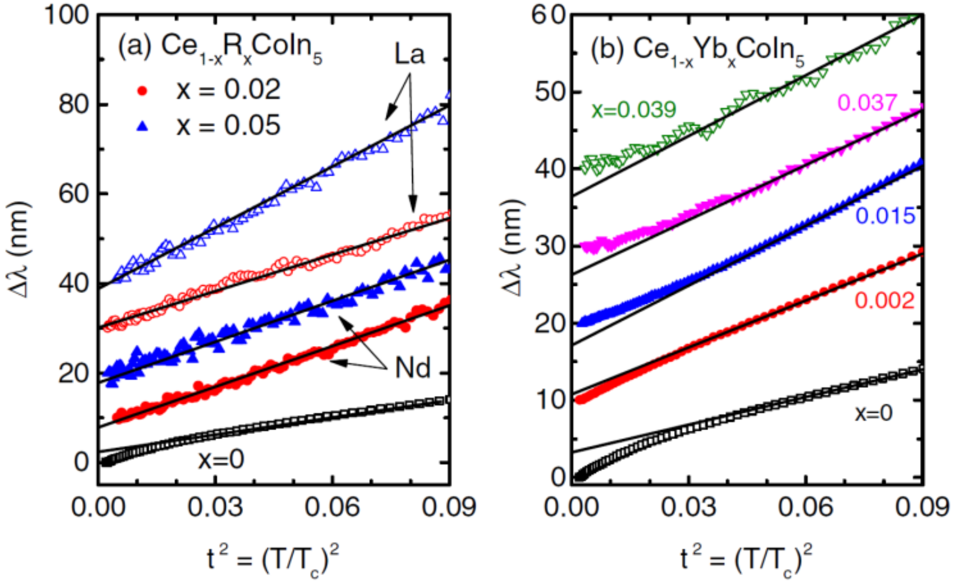
Topological phase transitions in SCs



Topological phase transitions in SCs

PRL **114**, 027003 (2015) PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2015

Nodal to Nodeless Superconducting Energy-Gap Structure Change Concomitant with Fermi-Surface Reconstruction in the Heavy-Fermion Compound $CeCoIn_5$



Line-nodal Superconductors

PRL **94,** 197002 (2005) PHYSICAL REVIEW LETTERS week ending 20 MAY 2005

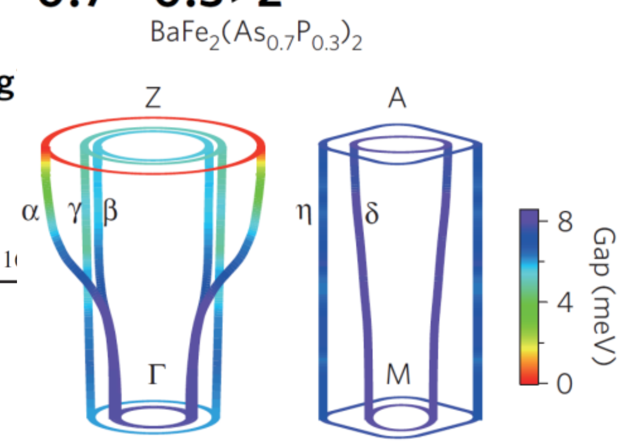
Line Nodes in the Superconducting Gap Function of Noncentrosymmetric $CePt_3Si$

K. Izawa,[1] Y. Kasahara,[1] Y. Matsuda,[1,2] K. Behnia,[1,3] T. Yasuda,[4] R. Settai,[4] and Y. Onuki[4]

nature physics LETTERS
PUBLISHED ONLINE: 4 MARCH 2012 | DOI: 10.1038/NPHYS2248

Nodal superconducting-gap structure in ferropnictide superconductor $BaFe_2(As_{0.7}P_{0.3})_2$

Y. Zhang, Z. R. Ye, Q. Q. Ge, F. Chen, Juan Jiang, M. Xu, B. P. Xie and D. L. Feng



PRL **115,** 165304 (2015) PHYSICAL REVIEW LETTERS

Polar Phase of Superfluid ^3He in Anisotropic Aerogel

V. V. Dmitriev,[1,*] A. A. Senin,[1] A. A. Soldatov,[1,2] and A. N. Yudin[1]
[1]*P.L. Kapitza Institute for Physical Problems of RAS, 119334 Moscow, Russia*
[2]*Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia*

(Received 10 July 2015; published 16 October 2015)

Unconventional Superconductors

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **18** (2006) R705–R752 doi:10.1088/0953-8984/18/44/R01

TOPICAL REVIEW

Nodal structure of unconventional superconductors probed by angle resolved thermal transport measurements

Y Matsuda[1,2], K Izawa[2,3] and I Vekhter[4]

excitations. The temperature dependence of the London penetration depth $\lambda(T)$, electronic part of the specific heat $C(T)$, thermal conductivity $\kappa(T)$, and nuclear magnetic resonance (NMR) spin–lattice relaxation rate T_1^{-1} all reflect the changes in the quasiparticle occupation numbers. In the fully gapped (s wave) superconductors the quasiparticle density of states

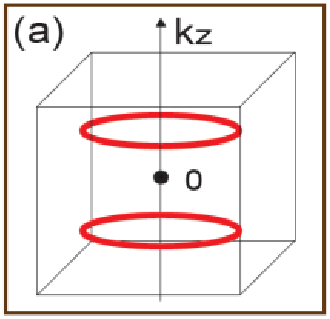
Unconventional Superconductors

Table 1. Superconducting gap symmetry of unconventional superconductors. TRS, AFMO and FMO represent time reversal symmetry, antiferromagnetic ordering and ferromagnetic ordering, respectively.

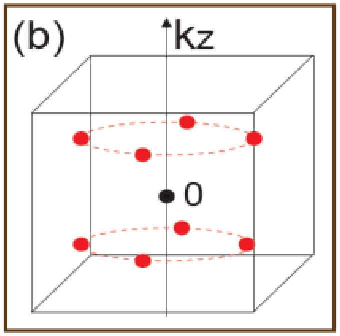
	Node	Parity	TRS	Proposed gap function	Comments
High T_c cuprates	Line (vertical)	Even [7]		$d_{x^2-y^2}$ [7]	
$Sr_2Ca_{12}Cu_{24}O_{41}$	Full gap [25]	Odd [25]			Spin ladder system
κ-$(ET)_2Cu(SCN)_2$	Line (vertical) [66]	Even [148]		d_{xy} [66]	
$(TMTSF)_2PF_6$		Odd [23]			Superconductivity under pressure
$(TMTSF)_2ClO_4$	Line [173] Full gap [181]				
Sr_2RuO_4	Line (horizontal) [65] (vertical) [73]	Odd [24]	Broken [32]	$(k_x + ik_y) \times (\cos k_z c + \alpha)$ [65] $(\sin k_x + i \sin k_y)$ [73]	
$Na_xCoO_2 \cdot yH_2O$	Line [174]	Even [175, 176], Odd [26, 27]			
$(Y, Lu)Ni_2B_2C$	Point-like [67]	Even [88]		$1 - \sin^4\theta\cos(4\phi)$ [87, 177]	Very anisotropic s wave
Li_2Pt_3B	Line [35]	Even + odd			No inversion centre
$CeCu_2Si_2$	Line [178]	Even [178]			Two superconducting phases [183]
$CeIn_3$	Line [179]				Coexistence with AFMO
$CeCoIn_5$	Line (vertical)	Even [61]		$d_{x^2-y^2}$ [53, 61, 136], d_{xy} [69]	FFLO phase
$CeRhIn_5$	Line [180]	Even [180]			Coexistence with AFMO
$CePt_3Si$	Line [34]	Even + odd [182]			No inversion centre
UPd_2Al_3	Line (horizontal)	Even [109]		$\cos k_z c$ [62]	Coexistence with AFMO
UNi_2Al_3	Line [19]	Odd [19]			Coexistence with SDW
URu_2Si_2	Line [184]	Odd [20]			Coexistence with hidden order
UPt_3	Line + point [185]	Odd [18]	Broken [31]		Multiple superconducting phases
UBe_{13}	Line [189]	Odd [22]			
UGe_2	Line [186]	Odd [28]			Coexistence with FMO
URhGe		Odd [29]			Coexistence with FMO
UIr		Even + odd [30]			Coexistence with FMO and no inversion centre
$PuCoGa_5$	Line [187]	Even [125]			
$PuRhGa_5$	Line [188]				
$PrOs_4Sb_{12}$	Point [68]	Odd [22]	Broken [33]		Multiple superconducting phases

Matsuda et. al., *2006*

Topological phase transition



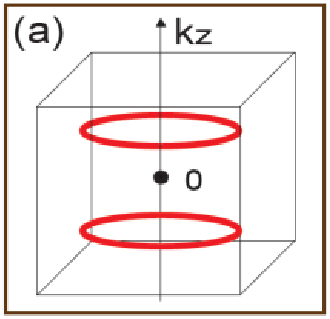
Nodal line SC



Nodal point SC

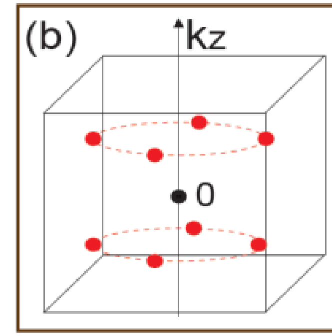
We focus on a special class
: symmetry protected topological line node.

Topological phase transition



Nodal line SC

Topological
QPT



Nodal point SC

We focus on a special class
: symmetry protected topological line node.

If protecting symmetry is broken, line nodal structure is modified.

Symmetry breaking and topological change are concomitant!

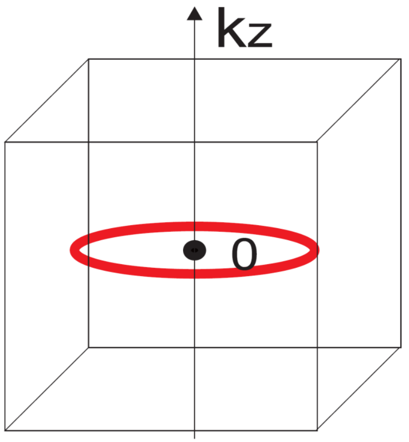
Line-nodal Superconductors

Toy model : p-wave pairing gap

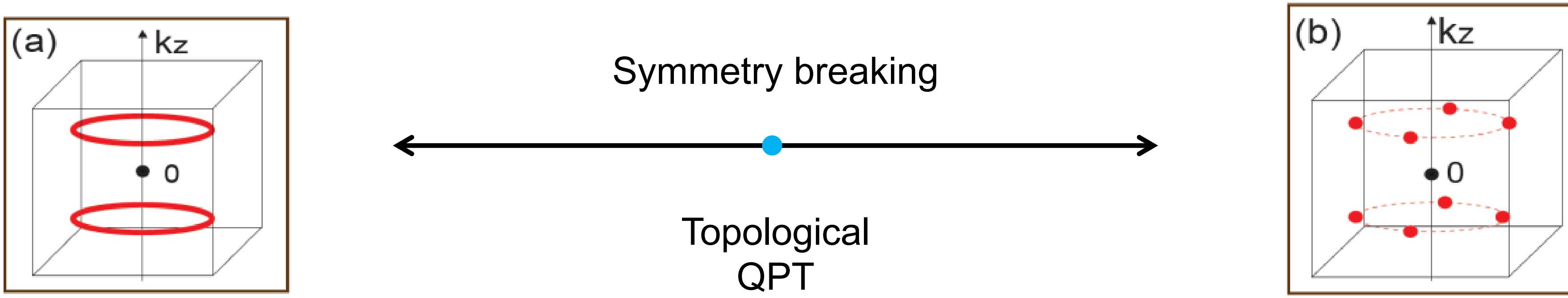
$$H_0 = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \Big(h(\boldsymbol{k})\tau^z + \Delta(\boldsymbol{k})\tau^x \Big) \Psi_{\boldsymbol{k}}$$

$$\mathcal{H}_0 = \frac{k_x^2 + k_y^2 - k_F^2}{2m}\tau^x + v_z k_z \tau^z \qquad E(\mathbf{k}) = \pm\sqrt{\frac{(k_x^2+k_y^2-k_F^2)^2}{4m^2} + v_z^2 k_z^2}$$

One line node exists in k_z=0 plane.



Topological phase transition



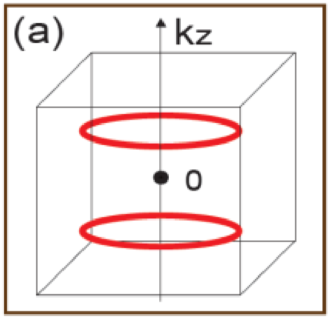
Nodal line SC

Nodal point SC

Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

$$H_0 = \sum_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^{\dagger} \Big(h(\boldsymbol{k}) \tau^z + \Delta(\boldsymbol{k}) \tau^x \Big) \Psi_{\boldsymbol{k}} \qquad + \qquad \phi \sum_{\boldsymbol{k}} \mathcal{F}(\boldsymbol{k}) \Psi_{\boldsymbol{k}}^{\dagger} \tau^y \Psi_{\boldsymbol{k}}$$

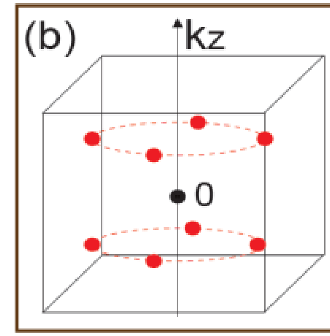
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



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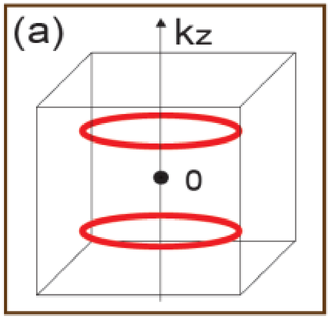
An order parameter exists.

Symmetric phase : line-node
Symmetry-broken phase : no line-node
(either point-node or fully gapped)

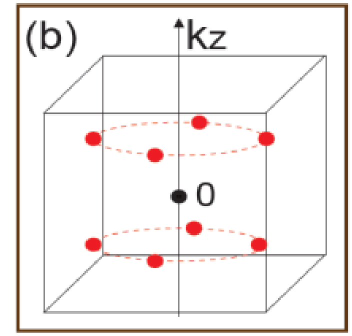
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



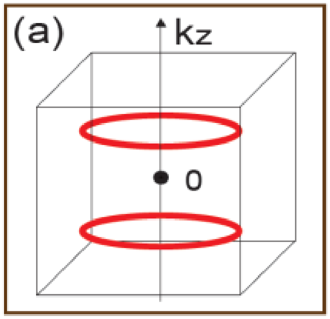
Nodal point SC

Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

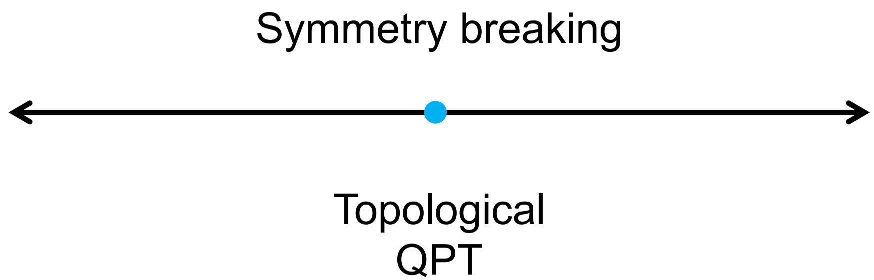
An order parameter exists.

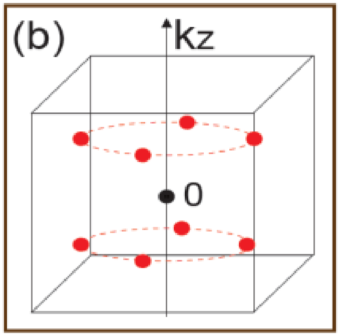
Landau-Ginzburg theory?

Topological phase transition



Nodal line SC





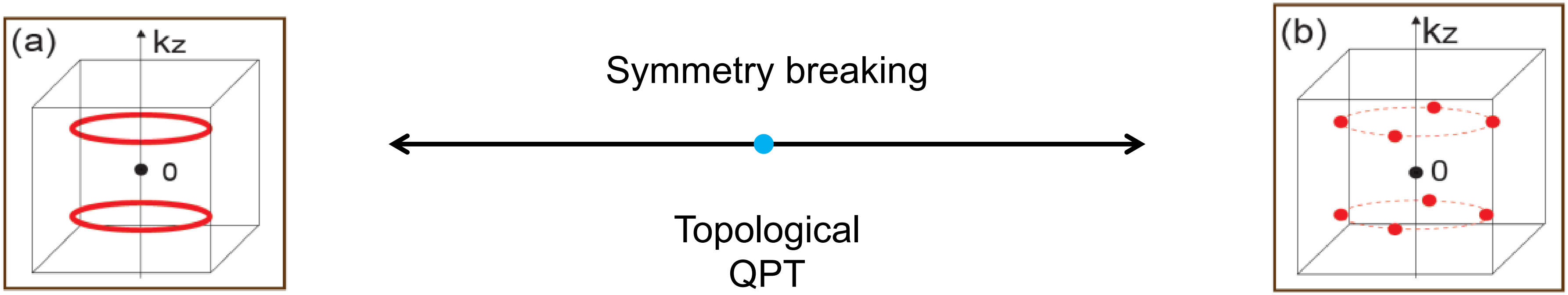
Nodal point SC

Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

Can the Landau-Ginzburg theory describe the transition?

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

Topological phase transition



Nodal line SC

Nodal point SC

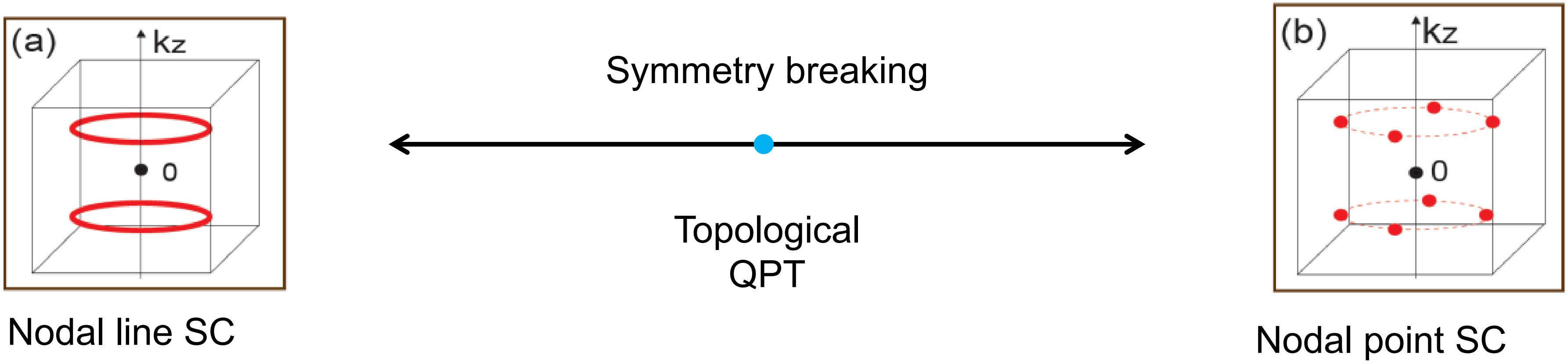
Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

Can the Landau-Ginzburg theory describe the transition?

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla \phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

NO! (no information about topological nature in the L-G theory)

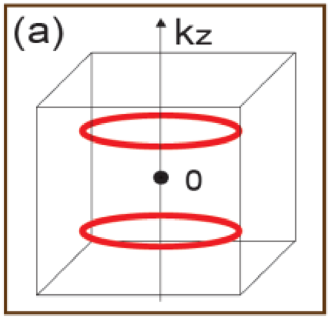
Topological phase transition



Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

How to incorporate the topological nature?

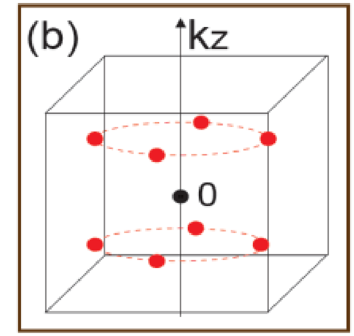
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



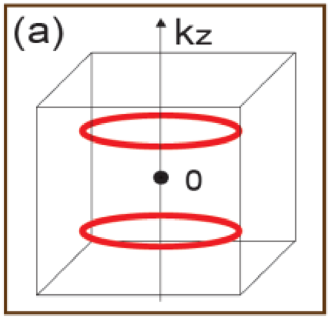
Nodal point SC

Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

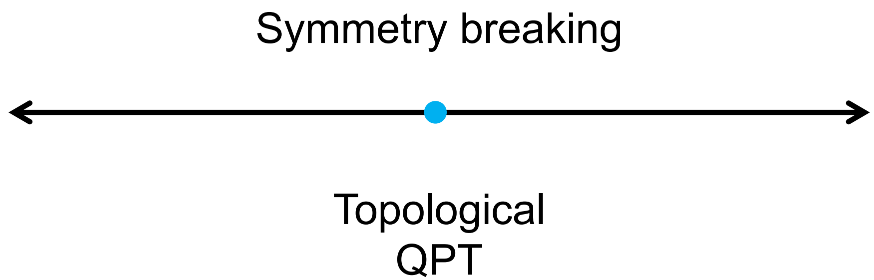
How to incorporate the topological nature?

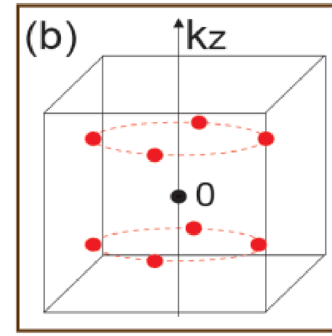
Fermions!
(Berry phase or curvature)

Topological phase transition



Nodal line SC





Nodal point SC

Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

$$S_c = S_\phi + S_\psi, \quad S_\psi = \int_{x,\tau} \Psi^\dagger(\partial_\tau + \mathcal{H}_0)\Psi + g\int_\tau H_{\psi-\phi}$$

Nodal line Hamiltonian $$H_0 = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \left(h(\boldsymbol{k})\tau^z + \Delta(\boldsymbol{k})\tau^x \right) \Psi_{\boldsymbol{k}}$$

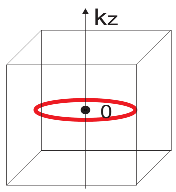
Topological phase transition

Example :

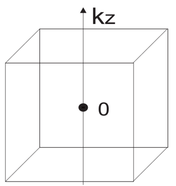
$$H_0 = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \Big(h(\boldsymbol{k})\tau^z + \Delta(\boldsymbol{k})\tau^x \Big) \Psi_{\boldsymbol{k}} \qquad H_{\psi-\phi} = \phi \sum \Psi^\dagger_k \tau^y \Psi_k$$

$$\mathcal{H}_0 = \frac{k_x^2 + k_y^2 - k_F^2}{2m}\tau^x + v_z k_z \tau^z$$

$$E(k) = \pm\sqrt{\frac{(k_x^2 + k_y^2 - k_F^2)^2}{4m^2} + v_z^2 k_z^2 + \phi^2}$$

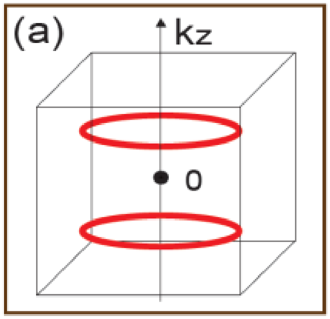


One line node exists in the symmetric phase.

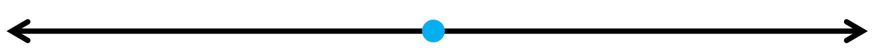


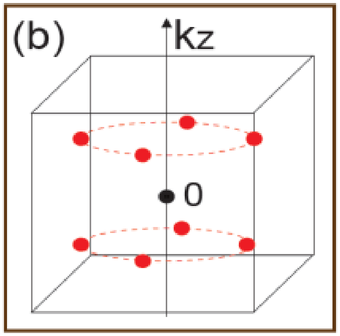
No line node exists in the symmetry-broken phase.

Topological phase transition



Nodal line SC





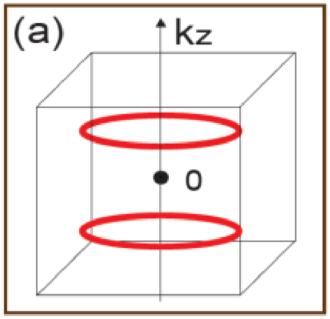
Nodal point SC

Can be generalized to general symmetry groups.

$$\mathcal{G} = C_{4v} \times \mathcal{T} \times \mathcal{P}$$

Rep.	Lattice $(\mathcal{F}_s(\boldsymbol{k})M^s)$	Continuum	#
A_1	τ^y	τ^y	0
A_2	$\sin(k_x)\sin(k_y)(\cos(k_x)-\cos(k_y))\tau^y$	$\sin(4\theta)\tau^y$	16
B_1	$(\cos(k_x)-\cos(k_y))\tau^y$	$\cos(2\theta)\tau^y$	8
B_2	$\sin(k_x)\sin(k_y)\tau^y$	$\sin(2\theta)\tau^y$	8
E	$\sin(k_x)\sin(k_z)\tau^y$, $\sin(k_y)\sin(k_z)\tau^y$	$\cos(\theta)\tau^y\mu^z$, $\sin(\theta)\tau^y\mu^z$	4

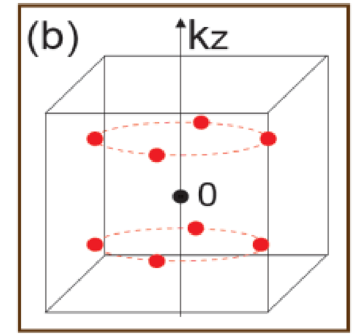
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



Nodal point SC

Mean field theory with

$$H = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} H_{\boldsymbol{k}} \Psi_{\boldsymbol{k}} - \frac{u}{2}\Big(\sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \mathcal{F}(\theta_{\boldsymbol{k}}) \tau_y \Psi_{\boldsymbol{k}}\Big)^2$$

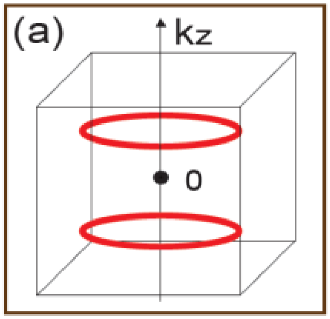
$$\phi = u \langle \Psi^\dagger_{\boldsymbol{k}} \mathcal{F}(\theta_{\boldsymbol{k}}) \tau_y \Psi_{\boldsymbol{k}} \rangle$$

$$H_{\mathrm{MF}} = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} (H_{\boldsymbol{k}} - \phi \mathcal{F}(\theta_k) \tau_y) \Psi_{\boldsymbol{k}} + \frac{\phi^2}{2u}$$

$$\mathcal{F}_{\mathrm{MF}}(T, \phi) = -\frac{T}{V} \ln(\mathrm{tr}(e^{-\beta H_{\mathrm{MF}}}))$$

$$\mathcal{F}_{MF}(\phi) = \Big(\frac{1}{u} - \frac{1}{u_c} + T\Big)\phi^2 + k_f |\phi|^3 + \cdots$$

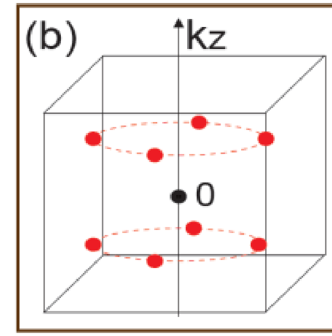
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



Nodal point SC

Mean field Free energy

$$\mathcal{F}_{MF}(\phi) = (\frac{1}{u} - \frac{1}{u_c} + T)\phi^2 + k_f|\phi|^3 + \cdots$$

The cubic term appears due to nodal line fermion excitation.

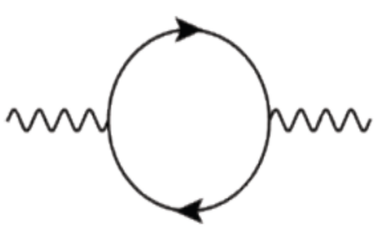
The MFT already shows the universality class is special!

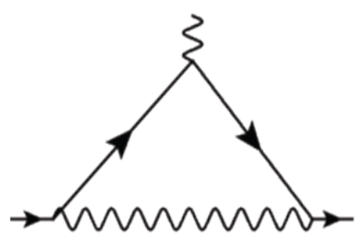
Critical theory

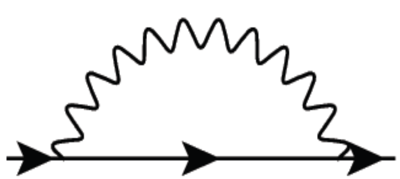
$$S_c = S_\phi + S_\psi, \quad S_\psi = \int_{x,\tau} \Psi^\dagger (\partial_\tau + \mathcal{H}_0)\Psi + g\int_\tau H_{\psi-\phi}$$

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

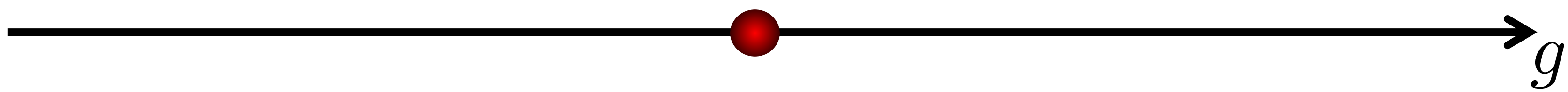
Quantum corrections





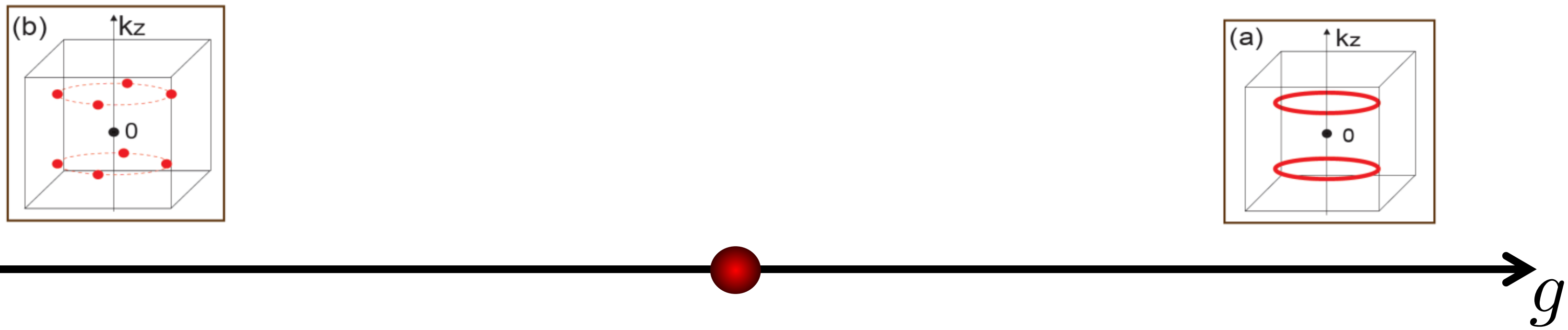


Conventional Phase Transitions



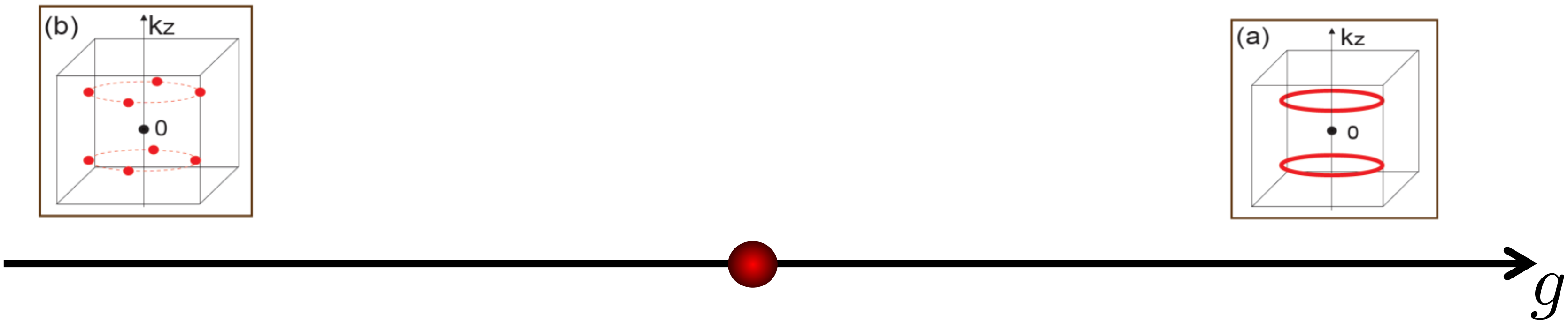
$$\mathcal{S}_{LGW} = \int_{\Omega,q} (\Omega^2 + q^2 + r)|\phi(q,\Omega)|^2 + \cdots$$

Topological Phase Transitions



$$\mathcal{S}_{exotic} = \int_{\Omega,q} (\sqrt{\Omega^2 + q^2} + r)|\phi(q,\Omega)|^2 + \cdots$$

Topological Phase Transitions

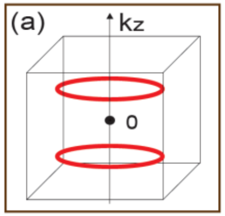


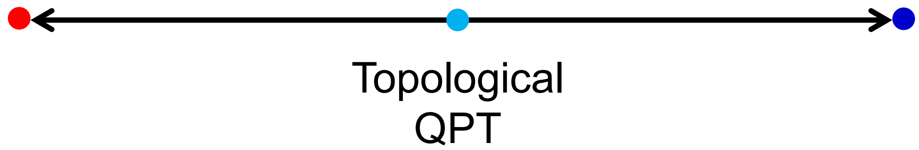
$$\mathcal{S}_{exotic} = \int_{\Omega,q} (\sqrt{\Omega^2 + q^2} + r)|\phi(q,\Omega)|^2 + \cdots$$

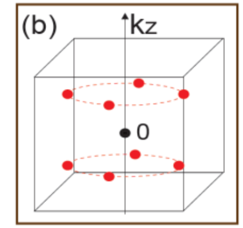
QCP in $3d$	z	ν	β	γ	η	HS
ϕ^4 theory[27]	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	O
Higgs-Yukawa[27, 28]	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	O
QBT-QCP[29, 30]	2	1	2	1	1	O
Hertz-Millis[31, 32]	2 or 3	$\frac{1}{2}$	$\frac{1}{2}$	1	0	X
Nodal line QCP	1	1	1	1	1	X

($\Omega \sim q^z$, $\xi^{-1} \sim |r-r_c|^\nu$, $\chi_\phi \sim |r-r_c|^{-\gamma}$, and $[\phi] = \frac{d+z-2+\eta}{2}$)

Critical theory







$$\mathcal{S}_{\phi}^{c} = \int_{\Omega,\boldsymbol{q}} N_f k_f \sqrt{\Omega^2 + v_z^2 q_z^2 + v_\perp^2 q_\perp^2}\, \mathcal{R}(\rho(\Omega,\boldsymbol{q})) \frac{|\phi|^2}{2}$$

$$\rho(\Omega, \mathbf{q}) = 1/\left(1 + \frac{\Omega^2 + v_z^2 q_z^2}{v_\perp^2 q_\perp^2}\right)$$

1. Large anomalous dimension.
2. Emergent Lorentz inv.
3. Hyper-scaling violation.

QCP in $3d$	z	ν	β	γ	η	HS
ϕ^4 theory[27]	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	O
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Nodal line QCP	1	1	1	1	1	X

$$(\Omega \sim q^z,\ \xi^{-1} \sim |r - r_c|^{\nu},\ \chi_\phi \sim |r - r_c|^{-\gamma},\ \text{and } [\phi] = \tfrac{d+z-2+\eta}{2}\ )$$

Comparison

In 3d,

1 ϕ^4 theory and Higgs-Yukawa theory (upper-critical dimension)

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla \phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

Mean-field + logarithmic correction

2. Hertz-Millis Theory

$$\mathcal{S}_{HM} = \int_{q,\omega} (\frac{|\omega|}{\Gamma_q} + q^2 + r)|\phi(k,\omega)|^2 + \int_{x,\tau} \frac{u}{4}\phi(x,\tau)^4$$

z=2,3 + hyperscaling violation

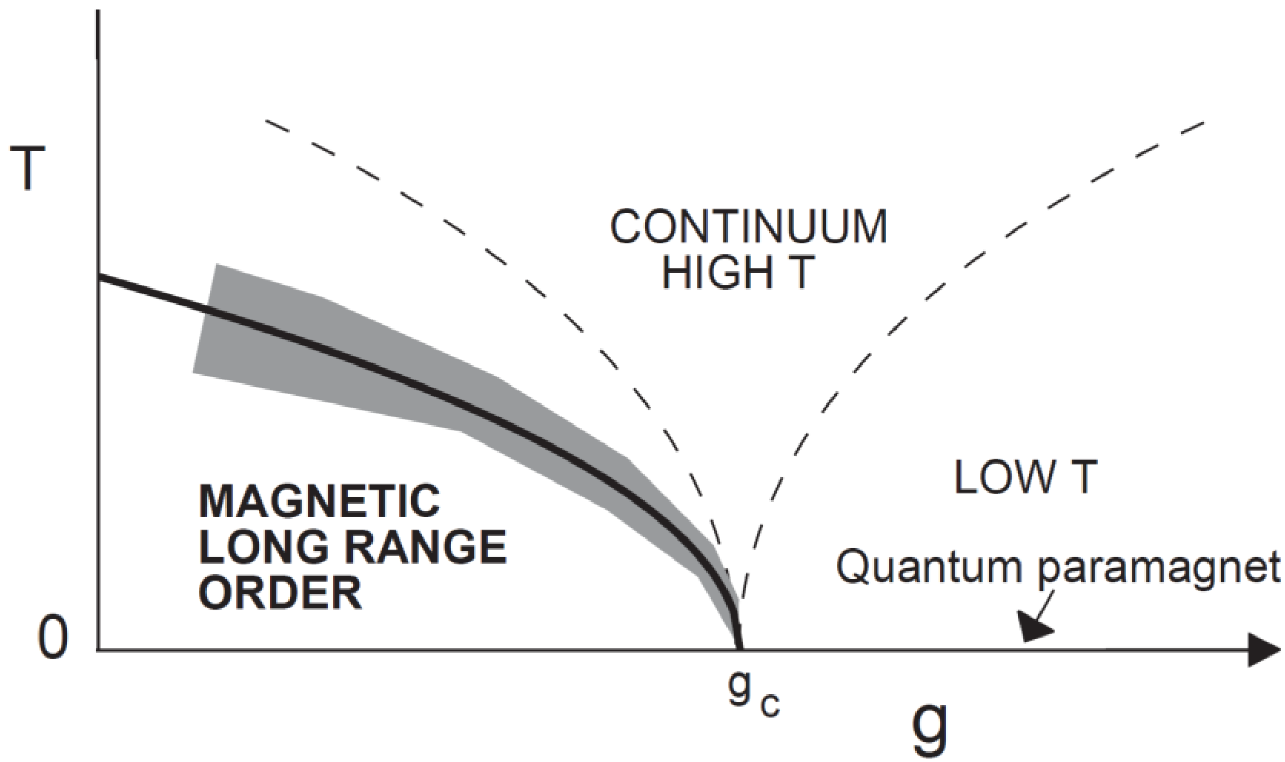
3. Line-nodal critical theory

$$\mathcal{S}_{line} = \int_{k,\omega} k_f \left(\sqrt{\omega^2 + v_\perp^2 k_\perp^2 + v_z^2 k_z^2} + r\right)|\phi(k,\omega)|^2$$

z=1 + hyperscaling violation

Phase diagram

Usual phase diagrams



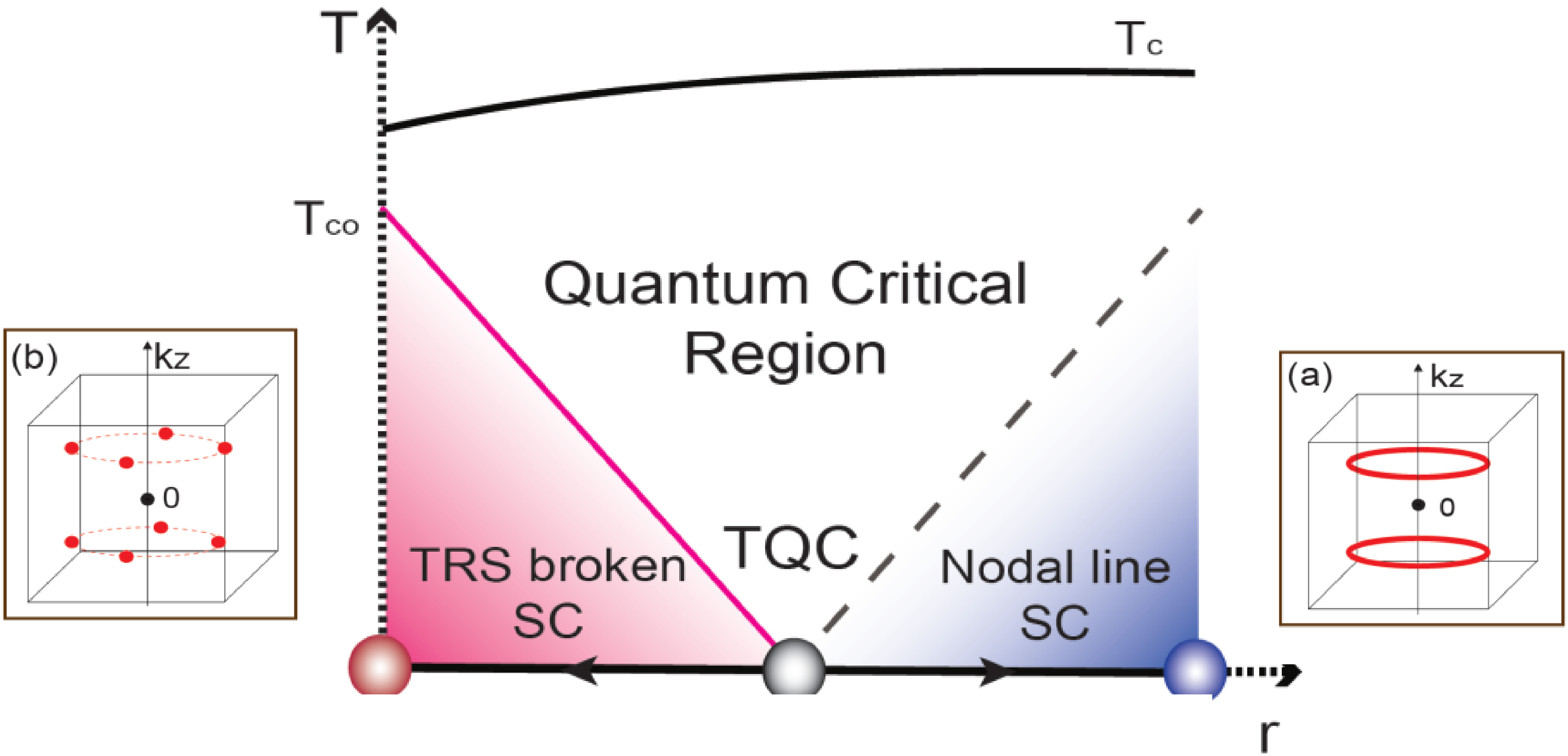
S. Sachdev, Quantum Phase Transitions

Basically, $T^2 \sim |g-g_c|$

$$T_c \sim (g_c - g)^{1/2}$$

$$\langle \phi \rangle \sim (g - g_c)^{1/2}$$

Phase diagram



Significantly larger quantum critical region due to fermion excitation

$$T_c \sim (g_c - g) \qquad\qquad \langle\phi\rangle \sim (g - g_c)$$

The linear temperature phase boundary!

Take-home Message II

Topological phase transitions are interesting.

Interplay between symmetry and topology!

Phases with Quantum Anomalies

Continuous Symmetry $\rightarrow$ Conservation law (Noether's thm)

$$\partial_\mu J^\mu = 0 \quad \frac{dQ}{dt} = 0$$

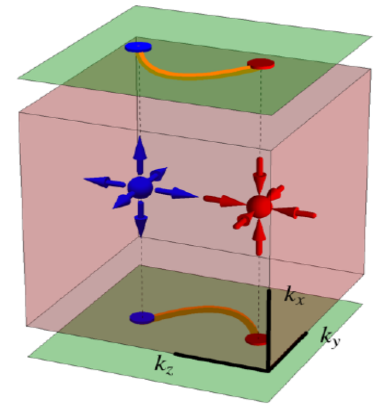
Anomalous Symmetry $\rightarrow$ Conservation law is "spoiled"

$$\partial_\mu J^\mu = \mathcal{A} \neq 0$$

Ex) $\partial_\mu J_5^\mu = \frac{1}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}$

Phases with Quantum Anomalies

Example : Weyl / Dirac semi-metal



$$\mathcal{L} = i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R$$

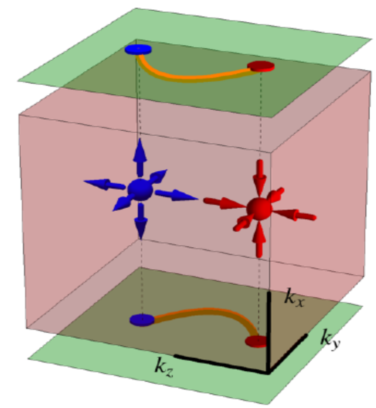
$$J^\mu = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R$$
$$J_5^\mu = \bar{\psi}_L\gamma^\mu\psi_L - \bar{\psi}_R\gamma^\mu\psi_R$$

$$\partial_\mu J^\mu = 0$$
$$\partial_\mu J_5^\mu = \frac{1}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}$$

WSM / DSM :
almost non-interacting, very stable!
Yet, non-local transport!

Phases with Quantum Anomalies

Example : Weyl / Dirac semi-metal



$$\mathcal{L} = i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R$$

$$J^\mu = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R$$
$$J_5^\mu = \bar{\psi}_L\gamma^\mu\psi_L - \bar{\psi}_R\gamma^\mu\psi_R$$

$$\partial_\mu J^\mu = 0$$
$$\partial_\mu J_5^\mu = \frac{1}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}$$

WSM / DSM :
almost non-interacting, very stable!
Yet, non-local transport!

More interests in anomalies!

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The "parity" anomaly on an unorientable manifold

Edward Witten
School of Natural Sciences, Institute for Advanced Study, Einstein Drive, Princeton, New Jersey 08540, USA
(Received 24 June 2016; revised manuscript received 5 September 2016; published 28 November 2016)

‘t Hooft anomaly matching

Non-perturbative nature : tool for strong coupling physics.

‘t Hooft anomaly matching
- Anomalies at UV fixed point and IR fixed point should be matched.
- Local deformation of theories do not change anomaly. (topological)

Roughly speaking, anomaly is *conserved.*

Implication of continuous symmetry anomaly
- Existence of the **massless** degrees of freedom *(Coleman and Grossman 1982)*

$$\Gamma_{\mu\nu\lambda}(q_1,q_2,q_3)\delta^{(4)}(q_1+q_2+q_3) = \int \prod_i d^4x_i e^{iq_ix_i} T < 0|J_\mu(x_1)J_\nu(x_2)J_\lambda(x_3)|0>, \qquad q_3^\lambda \Gamma_{\mu\nu\lambda}(q_1,q_2,q_3) = \mathcal{A}\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta .$$

‘t Hooft anomaly matching

Non-perturbative nature : tool for strong coupling physics.

‘t Hooft anomaly matching

- Anomalies at UV fixed point and IR fixed point should be matched.
- Local deformation of theories do not change anomaly. (topological)

Roughly speaking, anomaly is *conserved.*

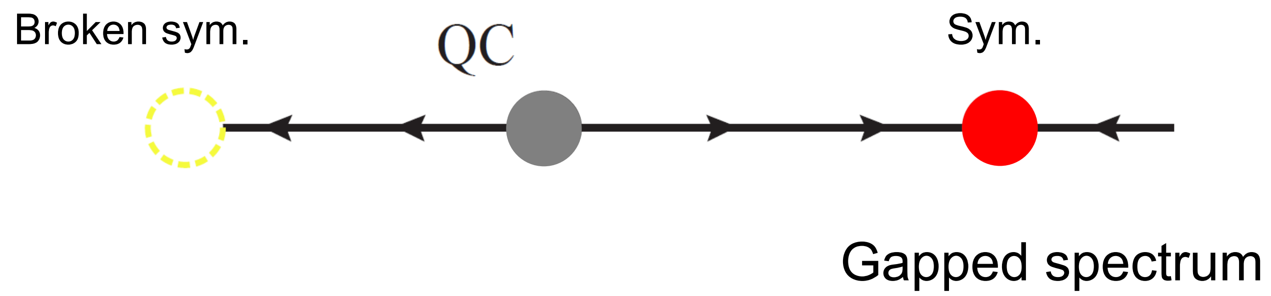
Implication of continuous symmetry anomaly

- Existence of the **massless** degrees of freedom *(Coleman and Grossman 1982)*

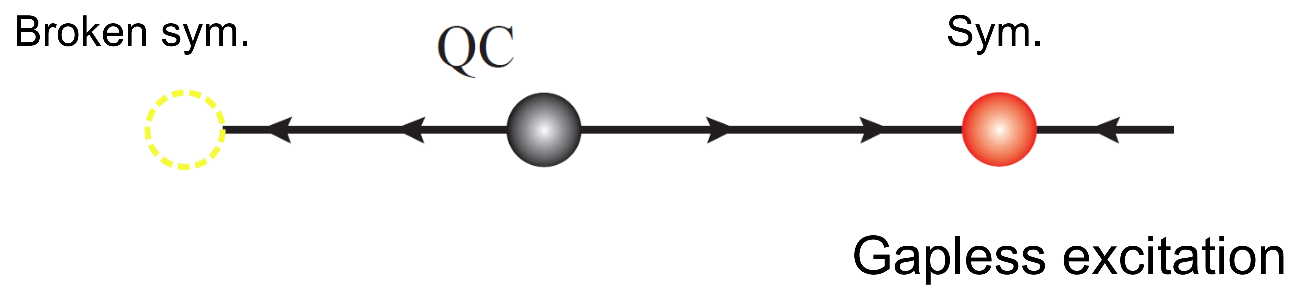
1. Take a system with DSM
2. Introduce strong correlation
3. Phase diagrams??

Phases with Quantum Anomalies

Conventional LGW paradigm (without anomaly),



Exotic criticality (with anomaly)



Phases with Quantum Anomalies

Ex) 1d spin-chain

Hasting-Oshikawa-Lieb-Schultz-Mattias theorem

Spin ½ systems with a translation symmetry : always gapless!

$$H = \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$$

The spin system is described by

$$\mathcal{S} = \int d^2x \frac{1}{2g^2} (\partial \vec{\phi})^2 + i\mathcal{S}_{WZW}$$

$$\mathcal{S}_{WZW} = c_2 \int_{X_3} \epsilon_{i_1 i_2 i_3 i_4} \epsilon^{\mu_1 \mu_2 \mu_3} \phi^{i_1} \partial_{\mu_1} \phi^{i_2} \partial_{\mu_2} \phi^{i_3} \partial_{\mu_3} \phi^{i_4}$$

O(4) non-linear sigma model with Wess-Zumino-Witten model

Phases with Quantum Anomalies

Ex) 1d spin-chain

$$\mathcal{S} = \int d^2x \frac{1}{2g^2} (\partial \vec{\phi})^2 + i\mathcal{S}_{WZW} \qquad \vec{\phi} = (n_x, n_y, n_z, \phi_{VBS})$$

Competing order physics between Neel and valence-bond-solid

Phases with Quantum Anomalies

Ex) 1d spin-chain

$$\mathcal{S} = \int d^2x \frac{1}{2g^2}(\partial\vec{\phi})^2 + i\mathcal{S}_{WZW} \qquad \vec{\phi} = (n_x, n_y, n_z, \phi_{VBS})$$

Competing order physics between Neel and valence-bond-solid

The gapless excitation is protected by quantum anomalies!

SCIENTIFIC REPORTS

OPEN **Competing Orders and Anomalies**

Eun-Gook Moon[1,2]

3d spin systems can have similar quantum anomalies!

Phases with Quantum Anomalies

Ex) 1d spin-chain

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Competing order physics between Neel and valence-bond-solid

The gapless excitation is protected by quantum anomalies!

SCIENTIFIC REPORTS

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Eun-Gook Moon[1,2]

3d spin systems can have similar quantum anomalies!

Future Questions :

1) 3d version of HOLSM?
2) Relation with quantum spin liquids?
3) Experimental signals?
4) …

Take-home Message III

Quantum anomalies are interesting.

Gapless excitations are guaranteed!

Recipe???

1. Strong correlation physics (ex: d & f orbitals)

2. Tune parameters around QPT (pressure, doping,…)

3. Measure / calculate physical quantities (resistivity, susceptibility,…)

4. Find unusual behaviors (ex: NFL, top, anomaly)

Summary

Exotic quantum criticalities signal novel physics.

Non trivial symmetric ground states may realize exotic quantum criticalities.

Non-Fermi liquids, topological phases, and quantum anomalies are specific examples.

Collaboration between theory and experiment is necessary!

Thank you for your attention!

Appendix A: Massless excitation with anomalies

It is well understood that massless excitation is guaranteed by continuous symmetry anomalies.[47–49] The presence of continuous group's anomalies enforces singularities of analytical structures of currents correlation functions. To be self-contained, we introduce the proof with slight modification following the notation in Coleman and Grossman.[49]

In $4D$, the anomalous Ward identity is in three currents correlation function,

$$\Gamma_{\mu\nu\lambda}(q_1, q_2, q_3)\delta^{(4)}(q_1 + q_2 + q_3) = \\ \int \prod_i d^4x_i e^{iq_ix_i} T < 0|J_\mu(x_1)J_\nu(x_2)J_\lambda(x_3)|0 >,$$

and the current conservation gives

$$q_3^\lambda \Gamma_{\mu\nu\lambda}(q_1, q_2, q_3) = \mathcal{A}\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta. \tag{A1}$$

All non-abelian Lie algebra indices are absorbed into the anomaly coefficient $\mathcal{A}$.

The correlation function is symmetric under simlutaneous permutations of (q_1, q_2, q_3) and (μ, ν, λ). Now let us investigate analytic structure of the correlation function. Due to permutation and covariance, the structure must be in the form

$$\Gamma_{\mu\nu\lambda} = F(q_i^2)\left[\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta q_{3\lambda} + \epsilon_{\nu\lambda\alpha\beta}q_2^\alpha q_3^\beta q_{1\mu} + \epsilon_{\mu\nu\alpha\beta}q_3^\alpha q_1^\beta q_{2\nu}\right]$$

We omit possible tensors which cannot contribute to the anomalies. Note that the momentums are off-shell, so one can access all available regions and we focus on the region

$$q_1^2 = q_2^2 = q_3^2 = -Q^2.$$

The correlation function contracted with $q_{3\lambda}$ gives

$$q_3^\lambda \Gamma_{\mu\nu\lambda}(q_1, q_2, q_3) = -F(Q^2) Q^2 \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta. \tag{A2}$$

Then, the anomaly equation (A.1) gives

$$F(Q^2) = -\frac{\mathcal{A}}{Q^2}.$$

The pole structure at zero mass nicely show the presence of massless excitation (also see[48] for dispersion analysis). The singularity even further enforces that UV and IR information needs to be matched.

In the paper by Coleman and Grossman, they add more conditions such as non-singularties from vertex corrections, and they conclude the helicity of massless degrees of freedom is $\pm\frac{1}{2}$, which indicates the symmetric phase is massless fermions as in our minimal model. The authors argue that the assumptions are not that strong, so it would be very interesting the conditions are proved / disproved in future research.

The above discussion only relies on the anomaly properties and nothing more, thus it is applied to everywhere in phase diagrams. But, it is only applied to anomalies of continuous symmetries since the current conservation plays a crucial role. For the discrete gauge group, which is especially important in SPT physics, the presence of anomalies does not guarantee massless excitation.[8–11]

We note that in $2D$, the minimal symmetry for spin 1/2 chains to be massless is $SU(2)\times Z_2$ corresponding $SU(2)\times Z_2$[16] which is smaller than $SO(4) \sim SU(2)\times SU(2)$, and it is manifest some subgroups of the continuous group is enough on lattice systems, and it would be interesting to find criteria to determine the subgroups in higher dimensions.