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Topological states of matter in correlated electron systems

Qiang-Hua Wang

National Lab of Solid State Microstructures, Nanjing University, Nanjing 210093, China

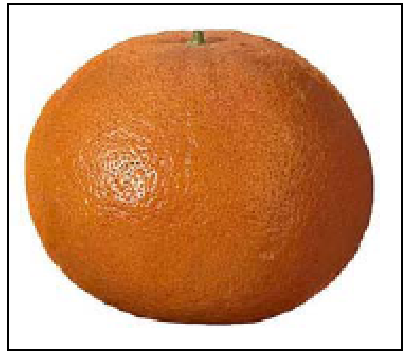
Collaborators： Dunghai Lee (Berkely), F Wang (MIT), F Yang (Beijing)
Students: Wan-Sheng Wang and Yuan-Yuan Xiang

Wang WS, et al, PRB 2012; Xiang YY, et al, PRB 2012

Outline

- Introduction and motivation
- T-breaking topological phases in doped Graphene and kagome lattices
- T-invariant topological superconductors
- Conclusions

Topology in daily life

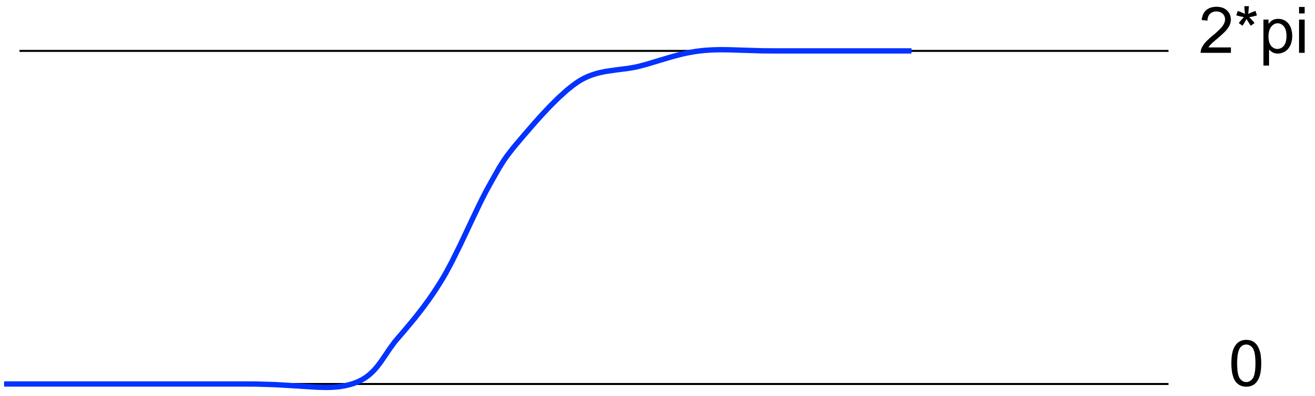


Zero handle

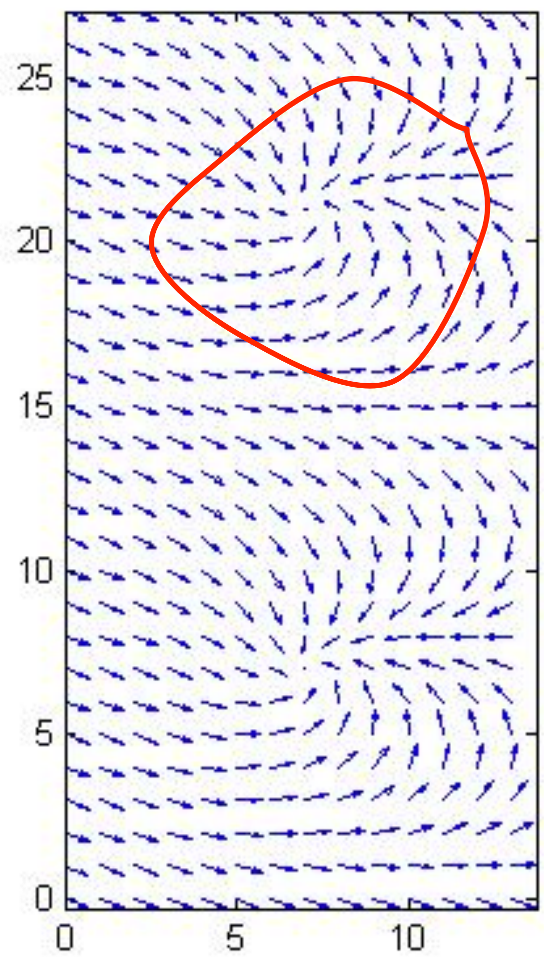
One handle

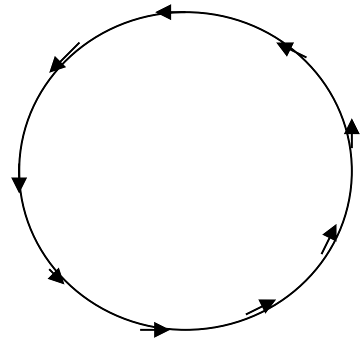
Topological states in 1D: kink and soliton

$$H = \int dx [K(\partial_x \phi)^2 + 1 - \cos\phi]$$



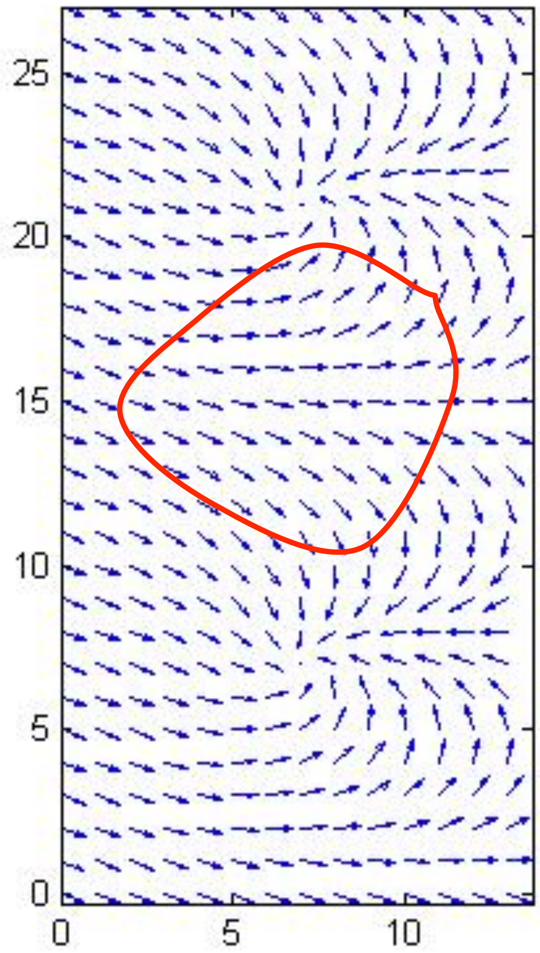
Topological states in 2D: vortex

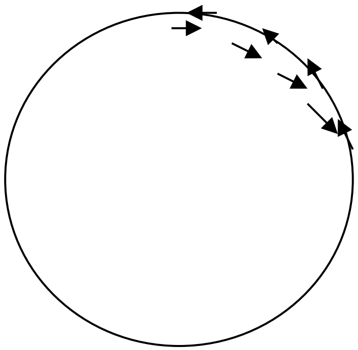




W=1

Topological states in 2D: vortex

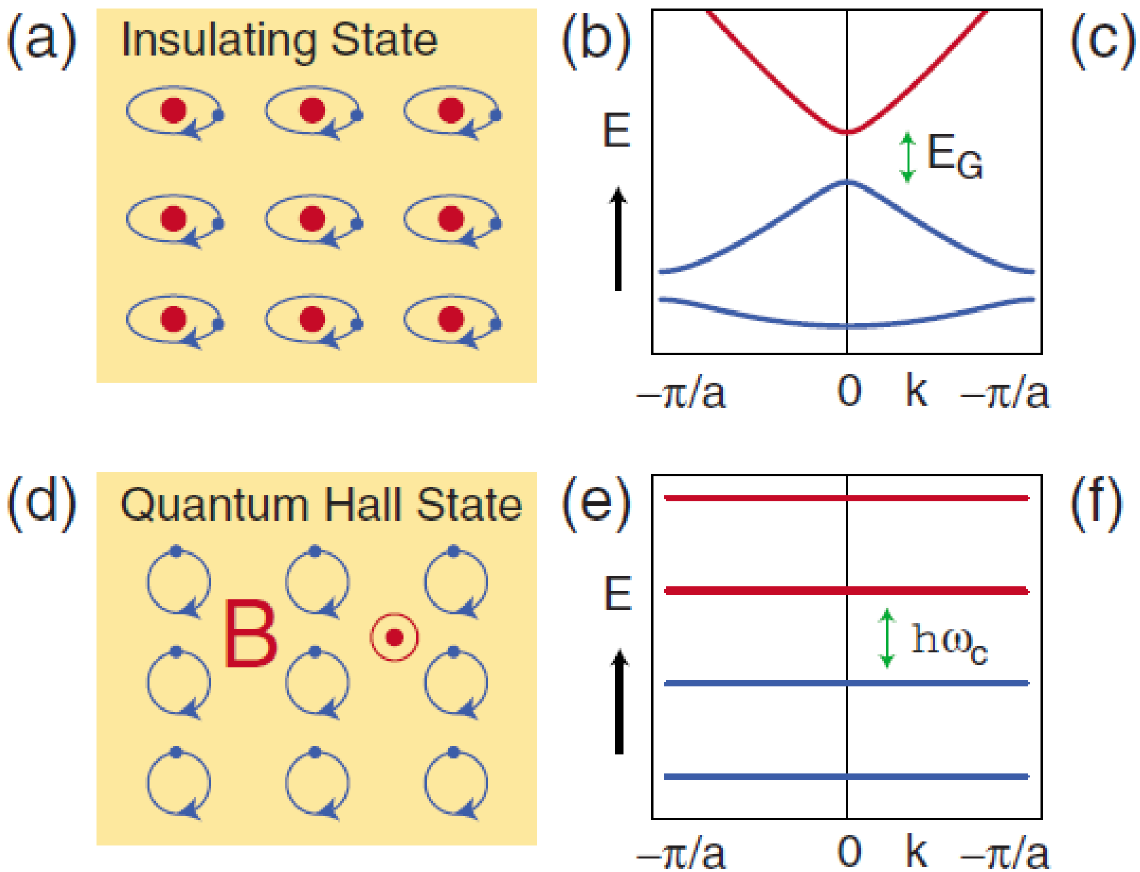




W=0

Hasan and Kane, RMP 2010

Topology and topological states of matter

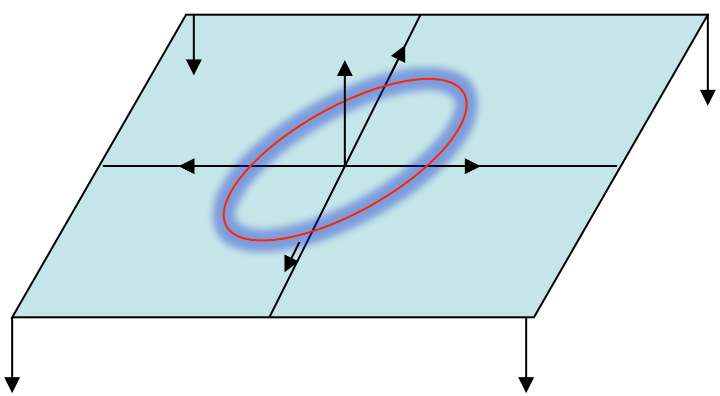


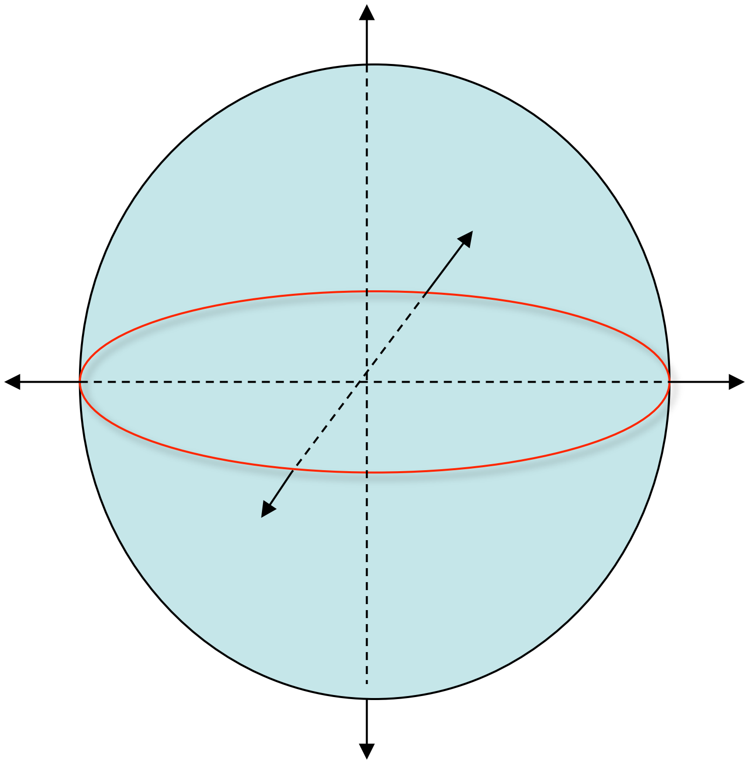
Topology and mapping k → H(k)

$$H = \sum_k \psi_k^+ [\vec{B}(k)\cdot\Gamma + \varepsilon_k I]\psi_k,$$

$$\{\Gamma_\alpha, \Gamma_\beta\} = 2\delta_{\alpha\beta}.$$

$$C_1 = \frac{1}{4\pi}\int dk_x \int dk_y \hat{\mathbf{d}}\cdot\frac{\partial\hat{\mathbf{d}}}{\partial k_x}\times\frac{\partial\hat{\mathbf{d}}}{\partial k_y}.$$





TKNN invariant, Chern number and edge states

Thouless, Kohmoto, Nightingale, and den Nijs 1982

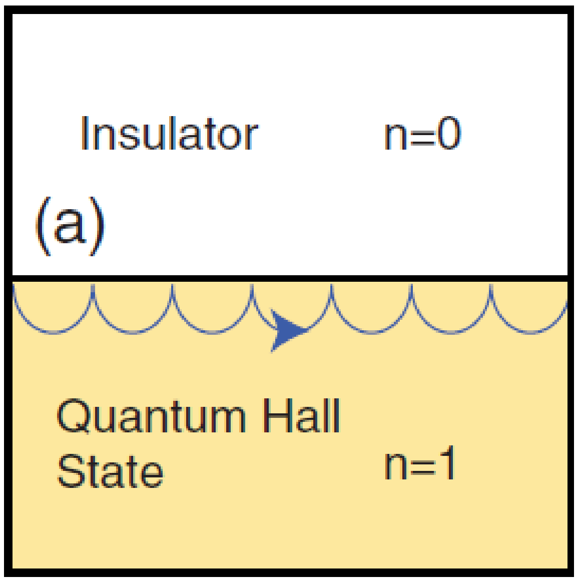
Topological number can not change smoothly.

Topologically distinct phases are connected by a) gap closing in the quantum phase transition point in parameter space, or b) gapless edge states in real space.

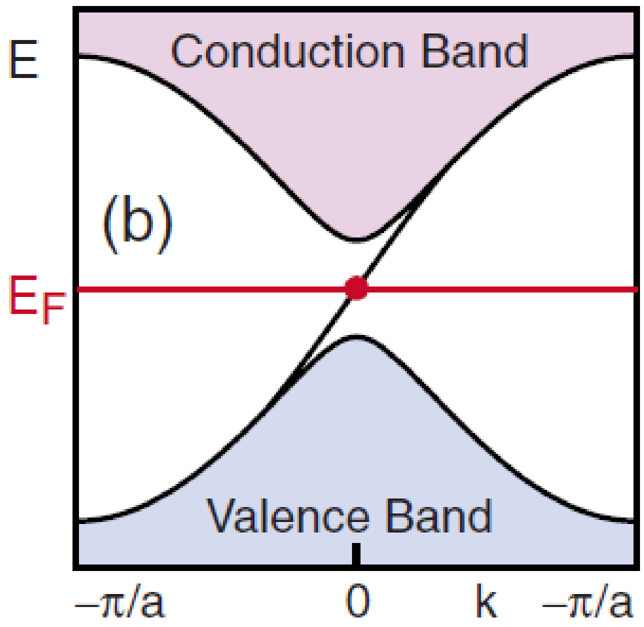
There is a 1 to 1 correspondence between the change of Z across the boundary and the number of edge states.

TKNN invariant, Chern number and edge states

Skippy cyclotron orbits



Edge state in the Haldane model



Spin polarized p+ip superconductor, Z=1

$$H = \frac{1}{2} \sum_{p} (c^{\dagger}_{\mathbf{p}}, c_{-\mathbf{p}}) \begin{pmatrix} \epsilon_{\mathbf{p}} & \Delta p_{+} \\ \Delta^{*} p_{-} & -\epsilon_{\mathbf{p}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{p}} \\ c^{\dagger}_{-\mathbf{p}} \end{pmatrix}$$

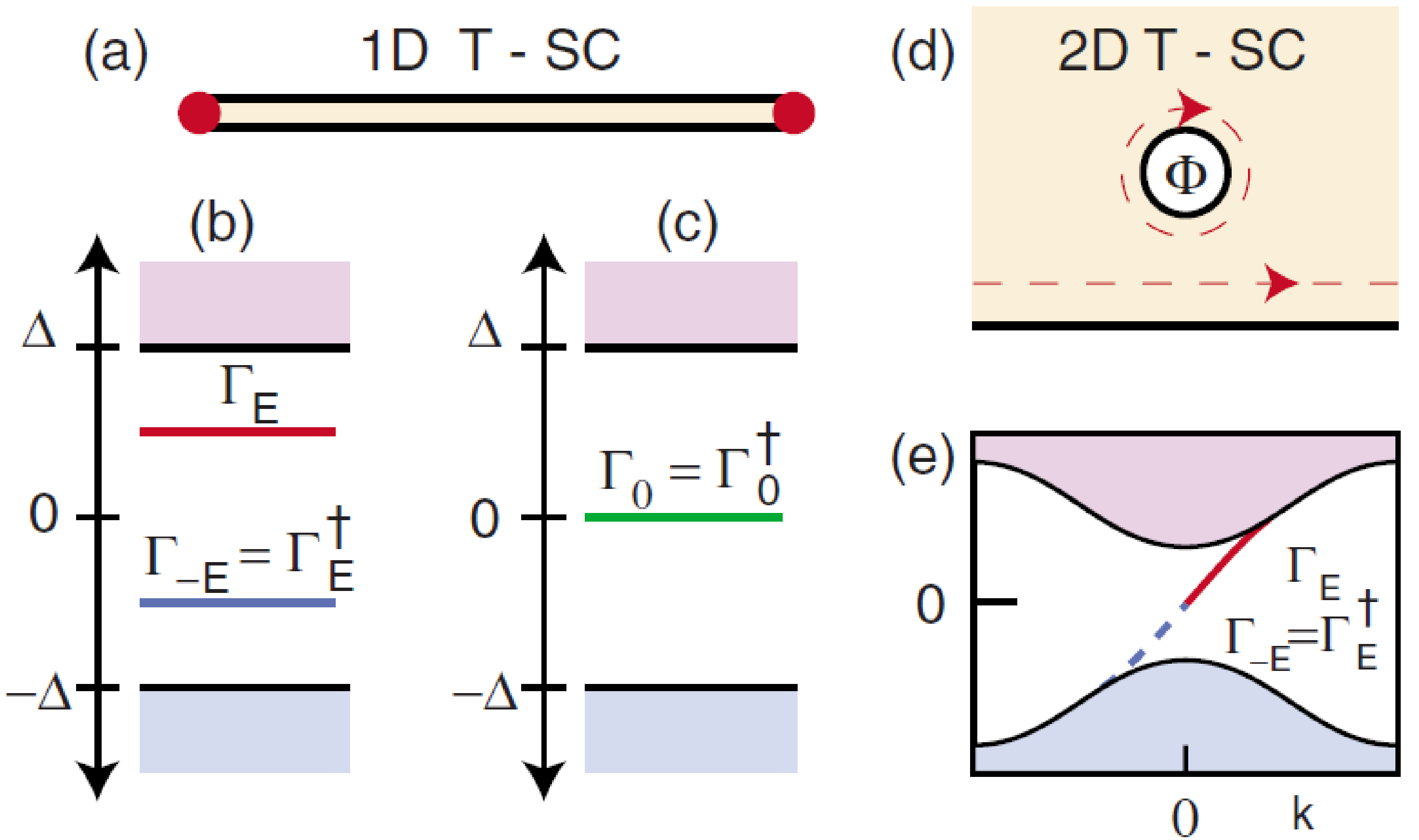
$$p_{\pm} = p_x \pm i p_y$$

$$\mathcal{H}_{\mathrm{BdG}}(\mathbf{k}) = [\mathcal{H}_0(\mathbf{k}) - \mu]\tau_z + \Delta_1(\mathbf{k})\tau_x + \Delta_2(\mathbf{k})\tau_y.$$

The effective field B(k) cover the Bloch sphere once.

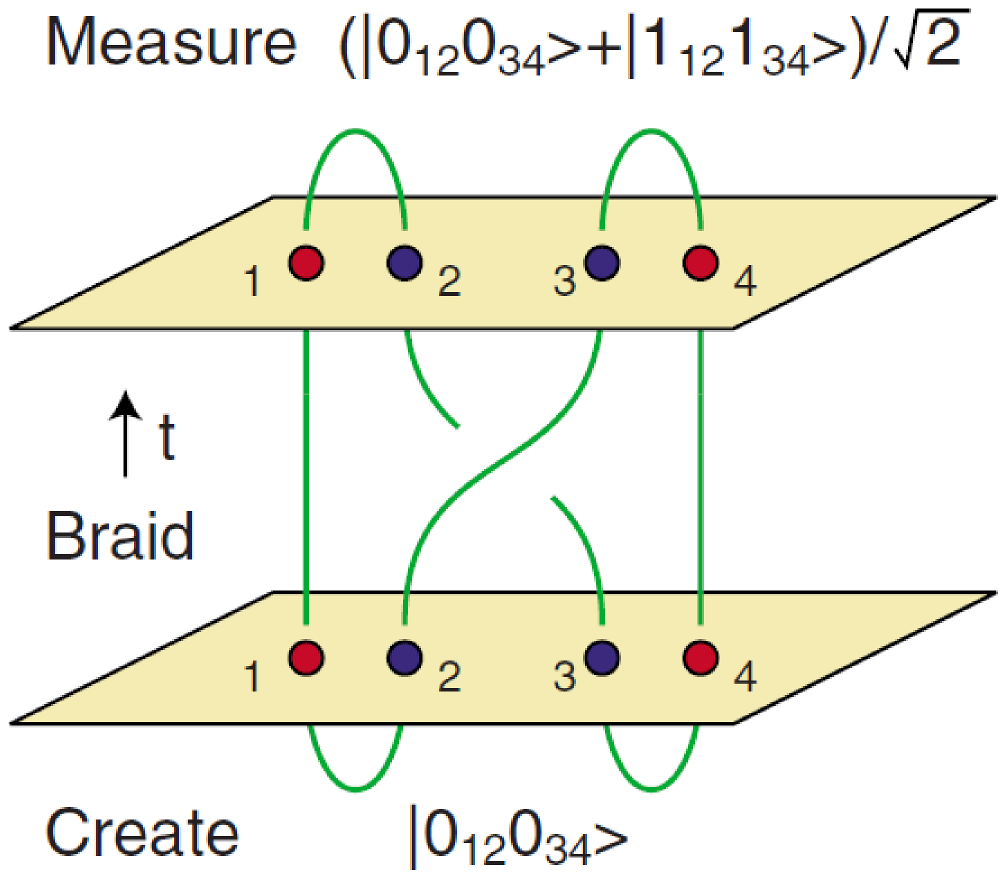
Read and Green 2000

Majorana fermions in 1d and 2d cases

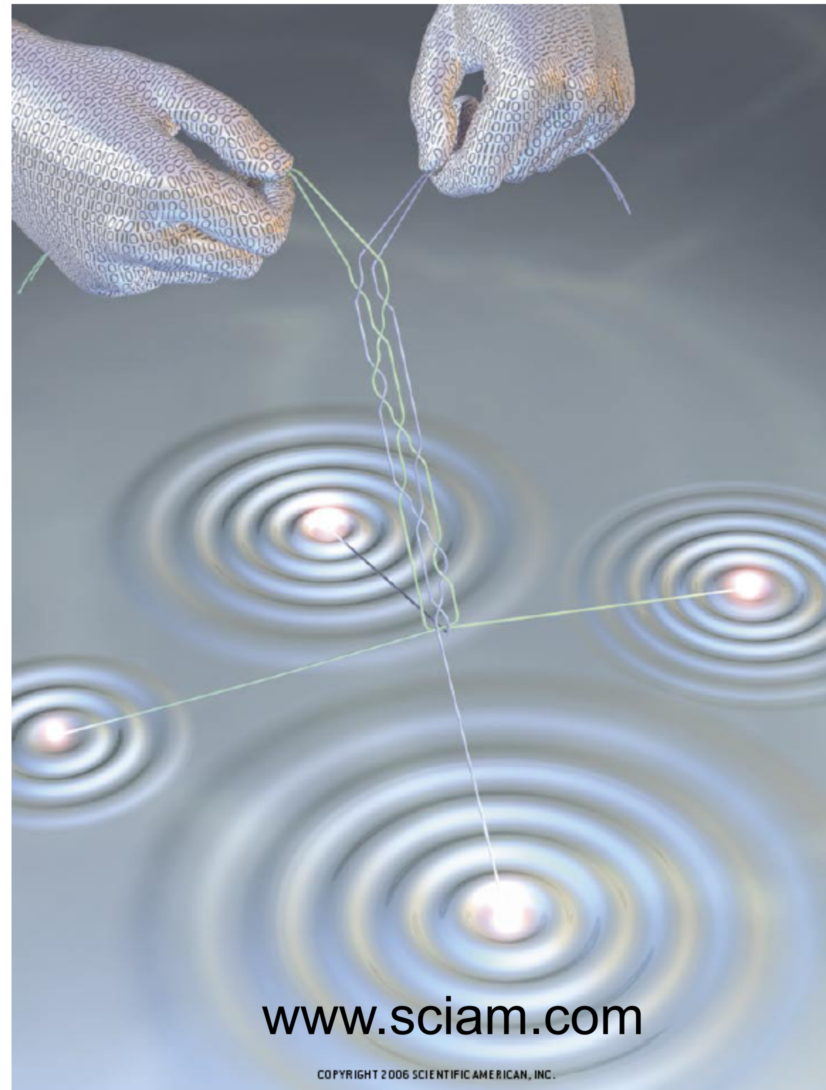


+E and –E forms a canonical fermion, not protected
E=0 comes in pair and sit on opposite edges, protected

Majorana fermion and non-Abelian statistics



Topological quantum computing



Spin singlet d+id superconductor

$$H = \sum_k \psi_k^+ \begin{pmatrix} H_0(k) - \mu & \Delta d_+(k) \\ \Delta^* d_-(k) & \mu - H_0(k) \end{pmatrix} \psi_k,$$

$$\psi_k^+ = (c_{k\uparrow}^+, c_{-k\downarrow})^T,$$

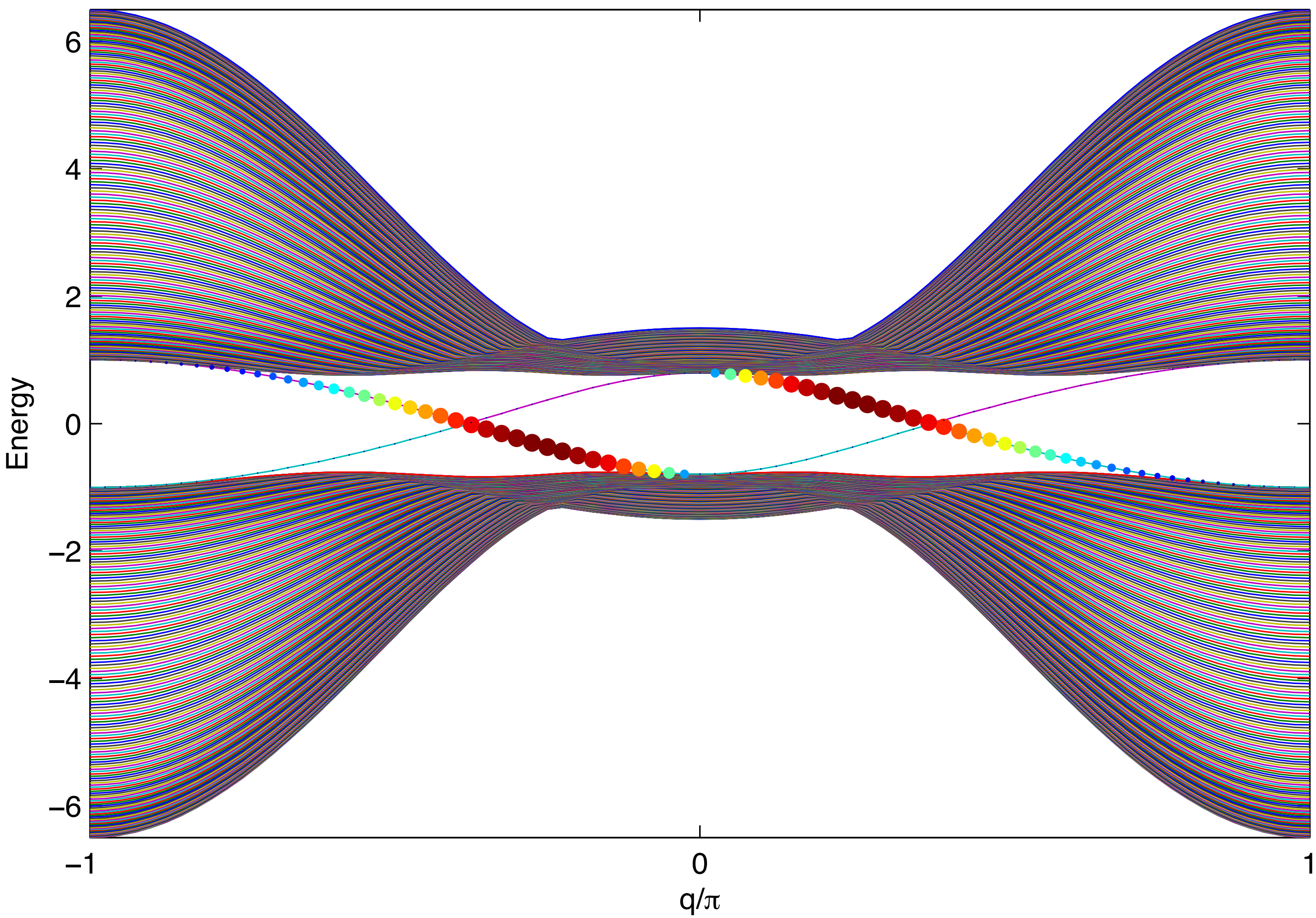
$$\psi_k = (c_{k\uparrow}, c_{-k\downarrow}^+)^T,$$

$$d_+(k) \sim \exp(2i\theta_k)$$

$$\mathcal{H}_{\mathrm{BdG}}(\mathbf{k}) = [\mathcal{H}_0(\mathbf{k}) - \mu]\tau_z + \Delta_1(\mathbf{k})\tau_x + \Delta_2(\mathbf{k})\tau_y.$$

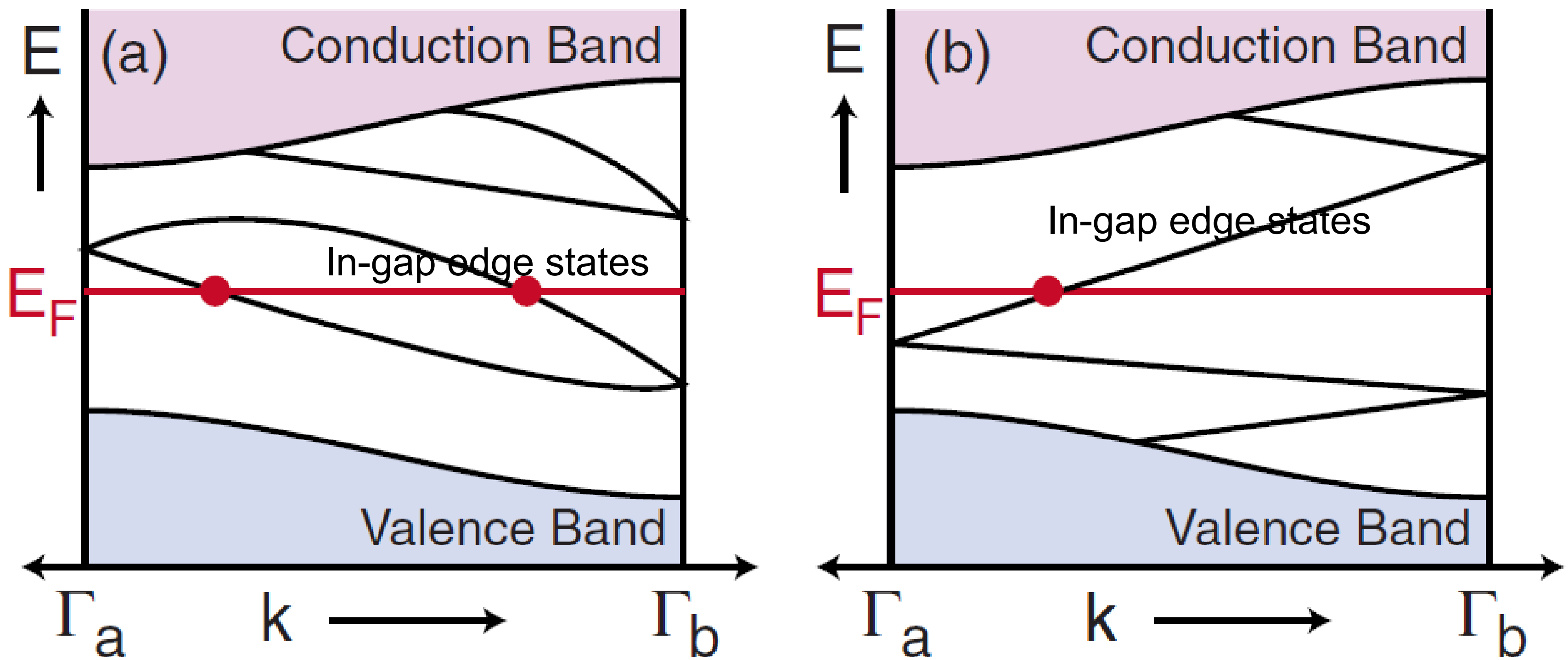
The effective field B(k) cover the Bloch sphere twice, thus Z=2

Edge states for d+id pairing (Z=2)



Z_2 number in T-invariant insulators

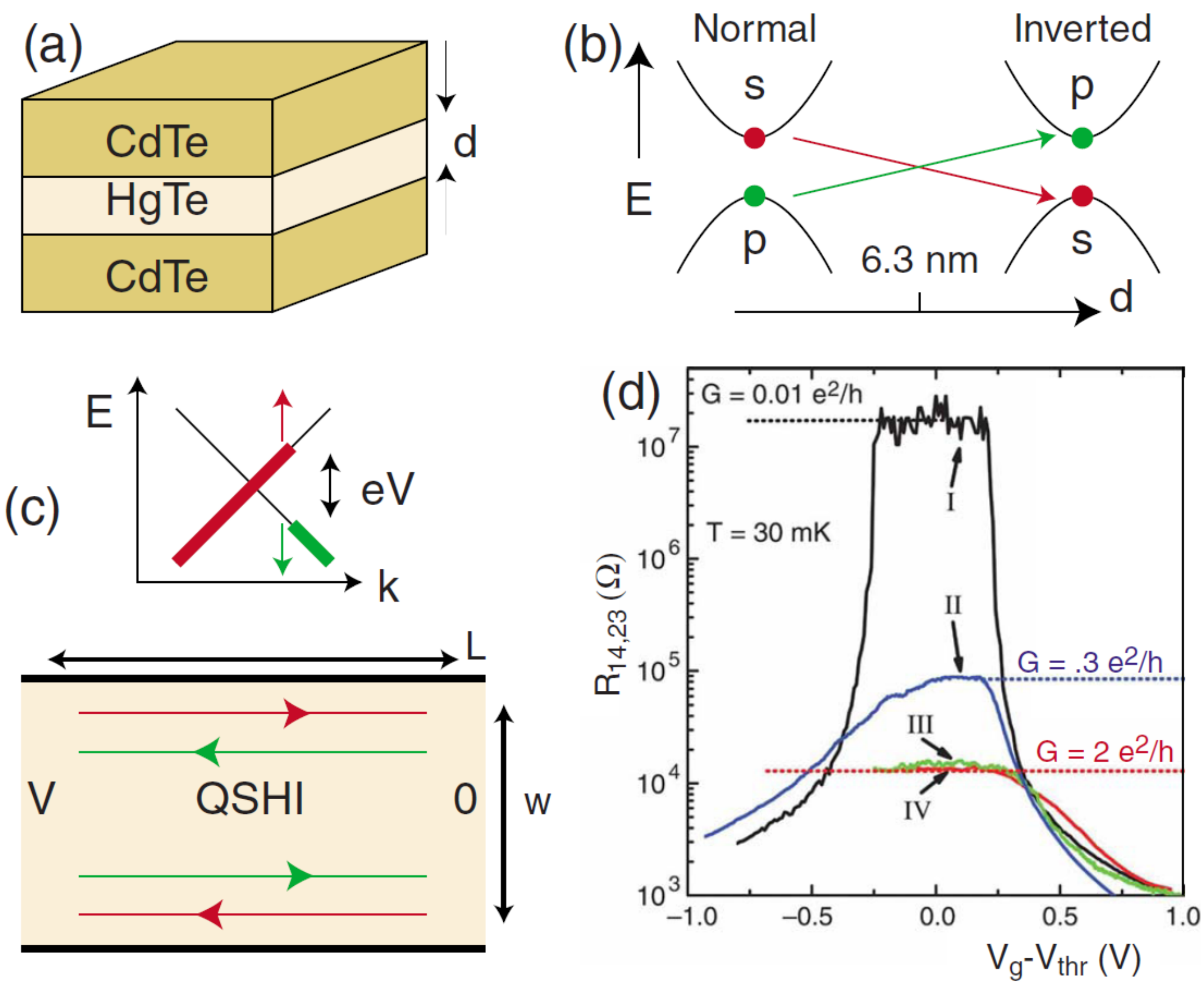
Kramers degeneracy on T-invariant momenta Γ



Enen crossings： Z_2=0

Odd crossings： Z_2=1

Quantum spin Hall system



Konig et al, 2007

T-invariant topological insulator /superconductor

$$H = \frac{1}{2}\sum_{p} \tilde{\Psi}^{\dagger} \begin{pmatrix} \epsilon_{\mathbf{p}} & \Delta p_{+} & 0 & 0 \\ \Delta^{*} p_{-} & -\epsilon_{\mathbf{p}} & 0 & 0 \\ 0 & 0 & \epsilon_{\mathbf{p}} & -\Delta^{*} p_{-} \\ 0 & 0 & -\Delta p_{+} & -\epsilon_{\mathbf{p}} \end{pmatrix} \tilde{\Psi}$$

$$p_{\pm} = i(p_x + i p_y)$$

For a topological insulator,

$$\psi_k^{+} = (a_{k\uparrow}^{+}, b_{k\uparrow}^{+}, a_{k\downarrow}^{+}, b_{k\downarrow}^{+})$$

$$\psi_k = (a_{k\uparrow}, b_{k\uparrow}, a_{k\downarrow}, b_{k\downarrow})^T \quad .$$

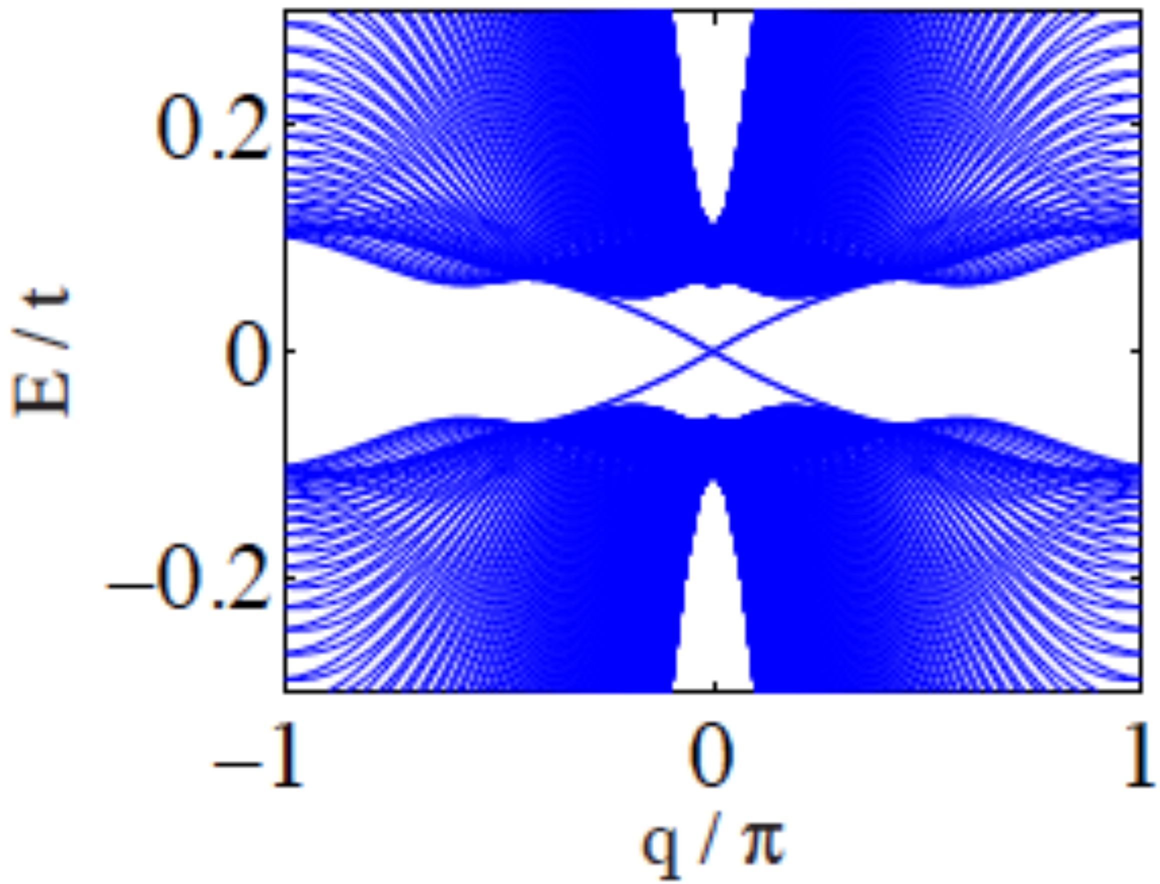
For a superconductor,

$$\psi_k^{+} = (a_{k\uparrow}^{+}, a_{-k\uparrow}, a_{k\downarrow}^{+}, a_{-k\downarrow})$$

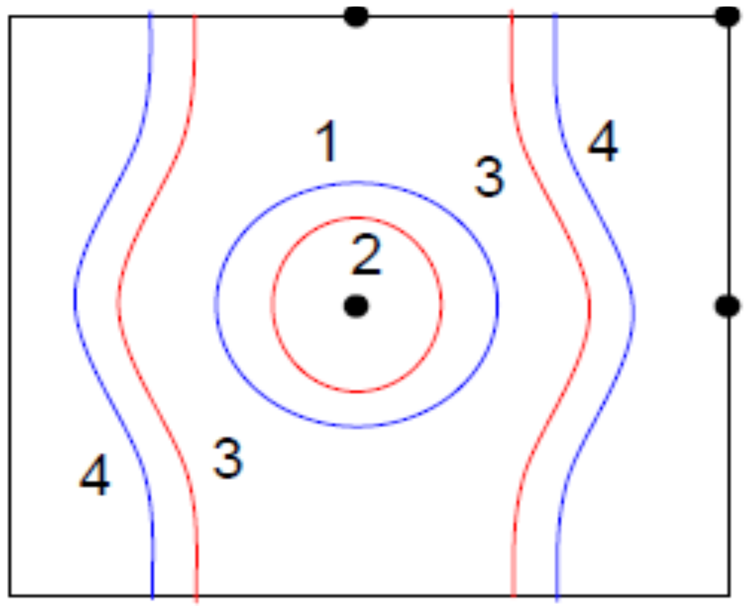
$$\psi_k = (a_{k\uparrow}, a_{-k\uparrow}^{+}, a_{k\downarrow}, a_{-k\downarrow}^{+})^T \quad .$$

Roy et al 2008; Schnyder et al 2008
Kitaev 2009; Qi et al 2009; Qi et al, RMP 2011

Edge state in a T-invariant topological superconductor

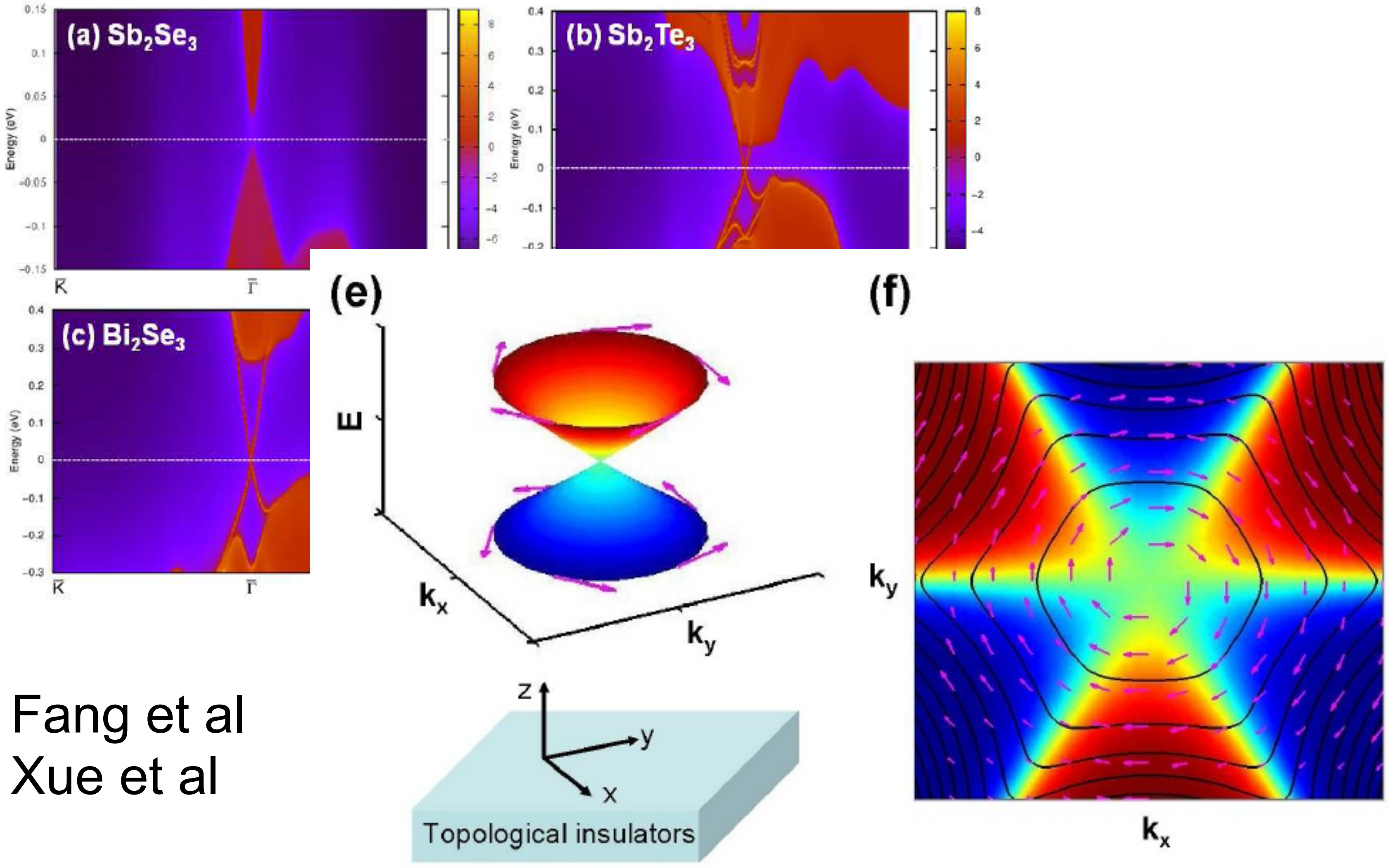


A convenient criterion for T-invariant topological superconductor



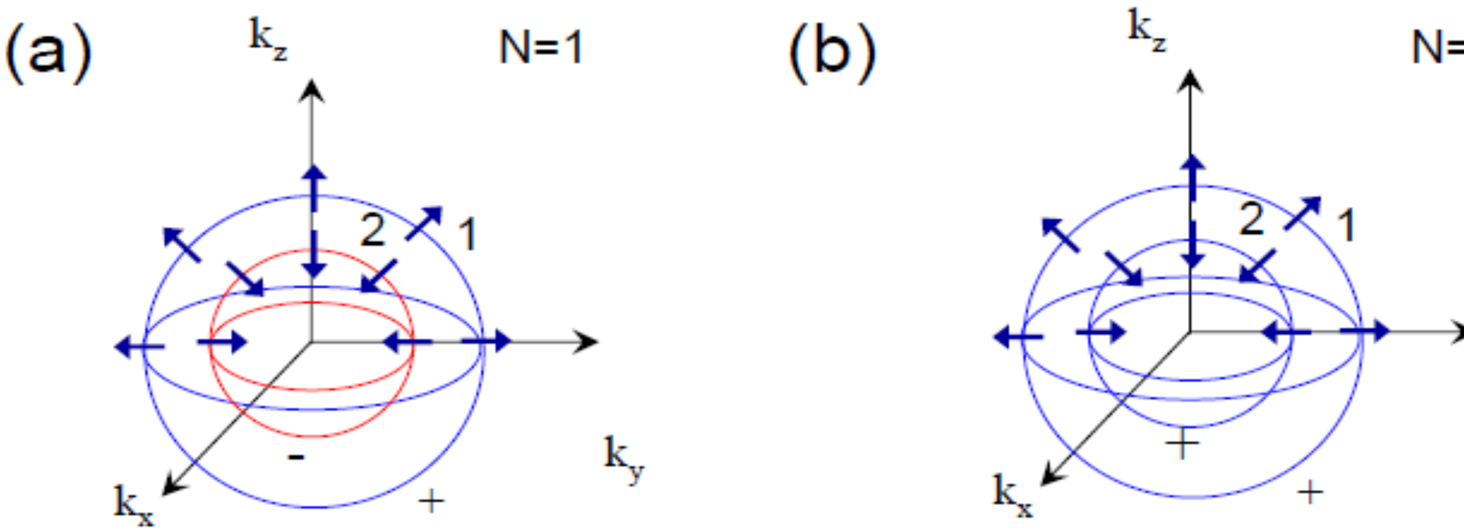
Roy et al 2008; Schnyder et al 2008
Kitaev 2009; Qi et al 2009; Qi et al, RMP 2011

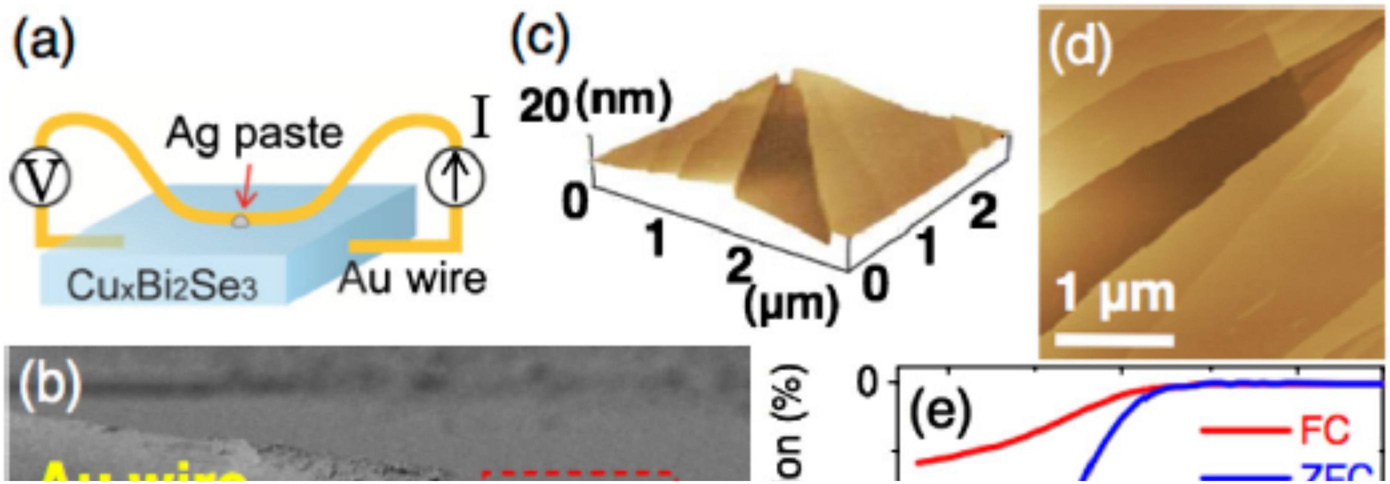
3d topological insulators

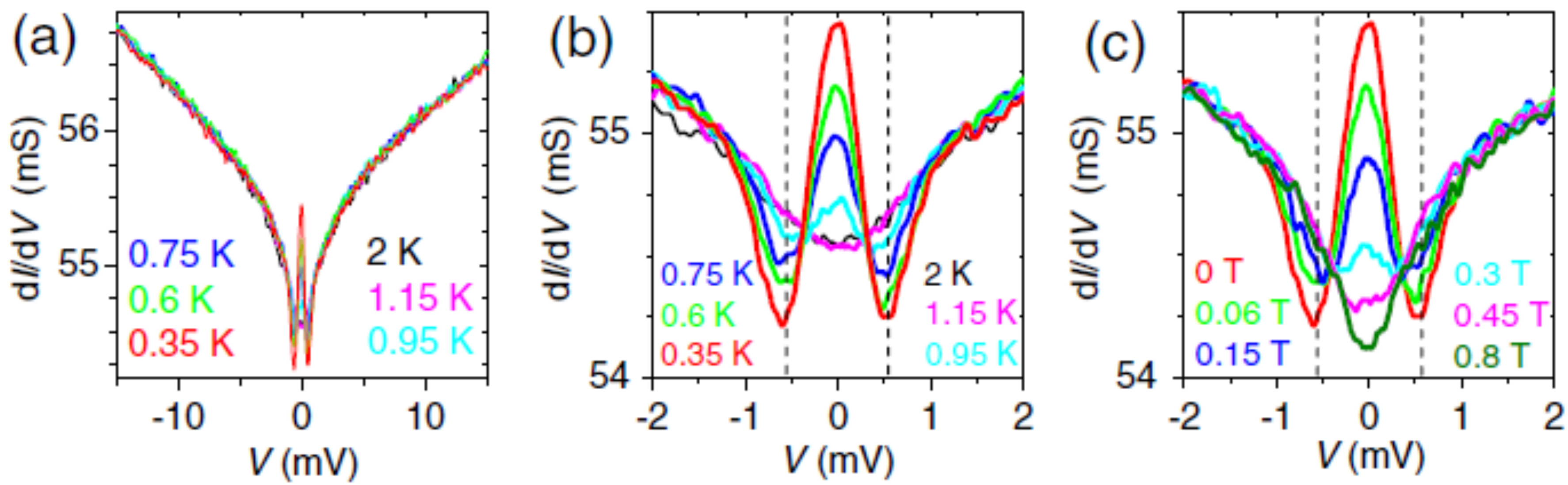


Fang et al
Xue et al

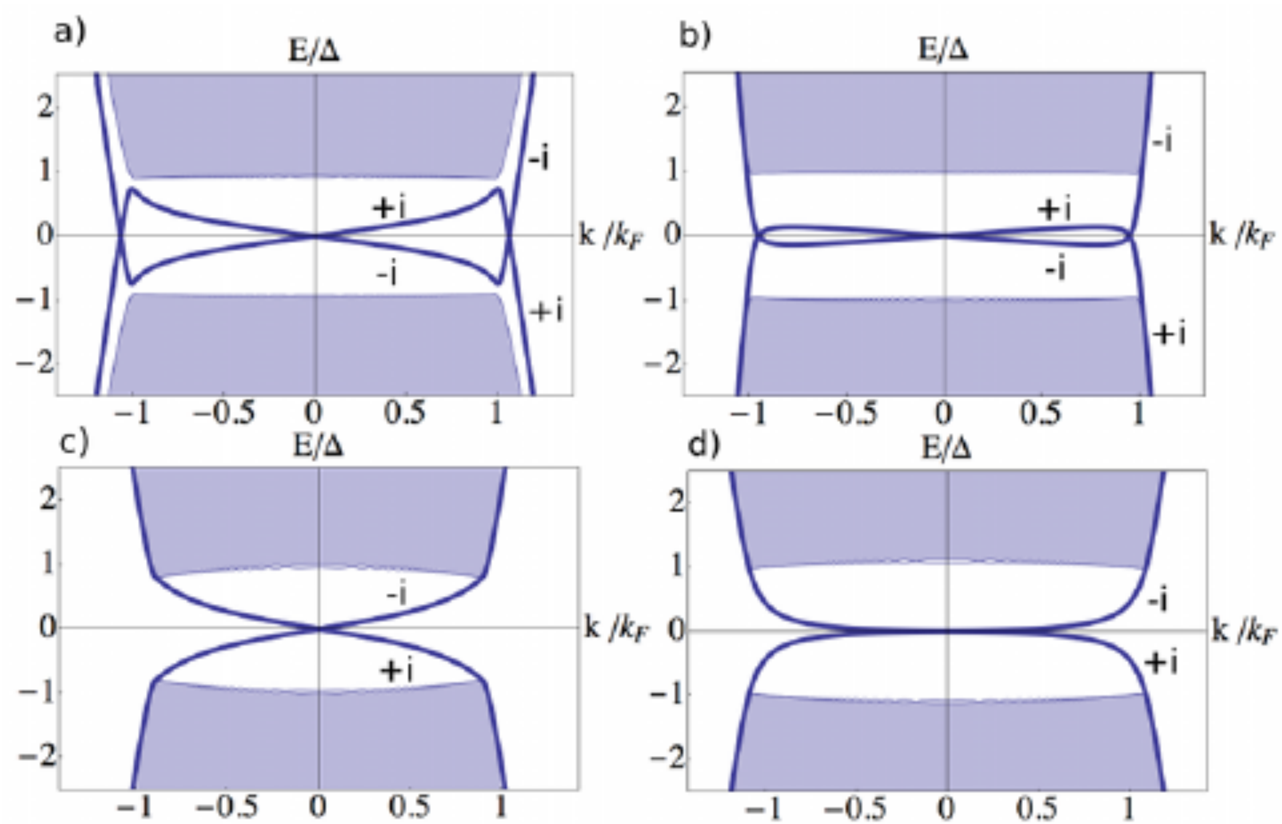
3d topological superconductors







Ando et al, PRL 2010



$$H_{\rm MF} = H + \sum_{<i\in 1, j\in 2>} \frac{\Delta}{2}(c^\dagger_{i\uparrow}c^\dagger_{j\downarrow} + c^\dagger_{i\downarrow}c^\dagger_{j\uparrow}) + h.c.$$

Fu and Berg, PRL 2010

Periodic table of topology

TABLE I. Periodic table of topological insulators and superconductors. The ten symmetry classes are labeled using the notation of Altland and Zirnbauer (1997) (AZ) and are specified by presence or absence of $\mathcal{T}$ symmetry Θ, particle-hole symmetry Ξ, and chiral symmetry $\Pi = \Xi\Theta$. ± 1 and 0 denote the presence and absence of symmetry, with ± 1 specifying the value of Θ^2 and Ξ^2. As a function of symmetry and space dimensionality d, the topological classifications ($\mathbb{Z}$, $\mathbb{Z}_2$, and 0) show a regular pattern that repeats when $d \to d+8$.

	Symmetry			d							
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	−1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	−1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	−1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

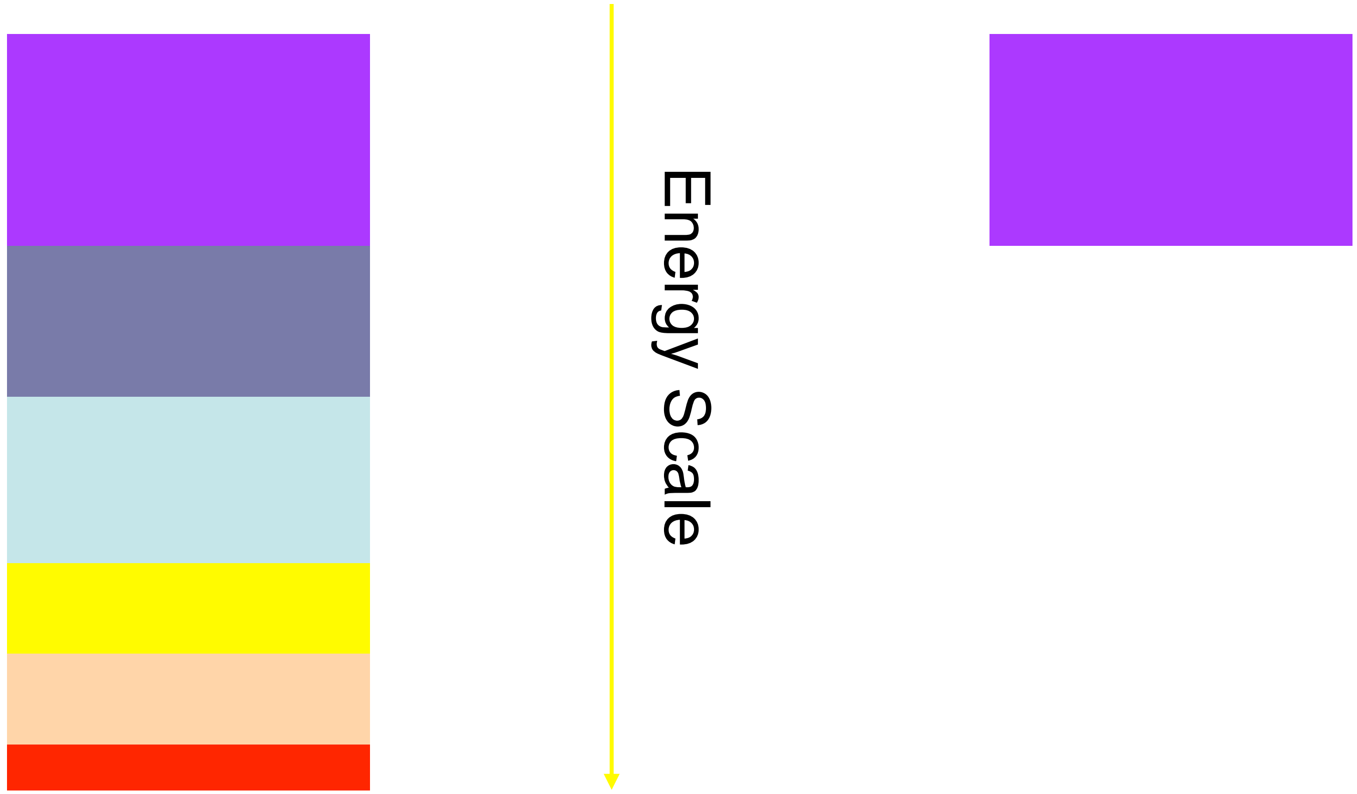
Challenges in the search of (intrinsic) topological superconductors

- Energy scale of topological insulators: eV
- Finding a BdG hamiltonian finishes only a half of the job
- Proximity effect generated topological superconductor depends solely on the edge (surface) states of topological insulator (many literatures in this direction)
- Intrinsic topological superconductor relies on the system itself, such as Sr2RuO4 and He-III B-phase.
- In repulsive systems, the energy scale involved in superconducting transition: 1 ~ 40meV. Energy hierarchy requires RG treatment.

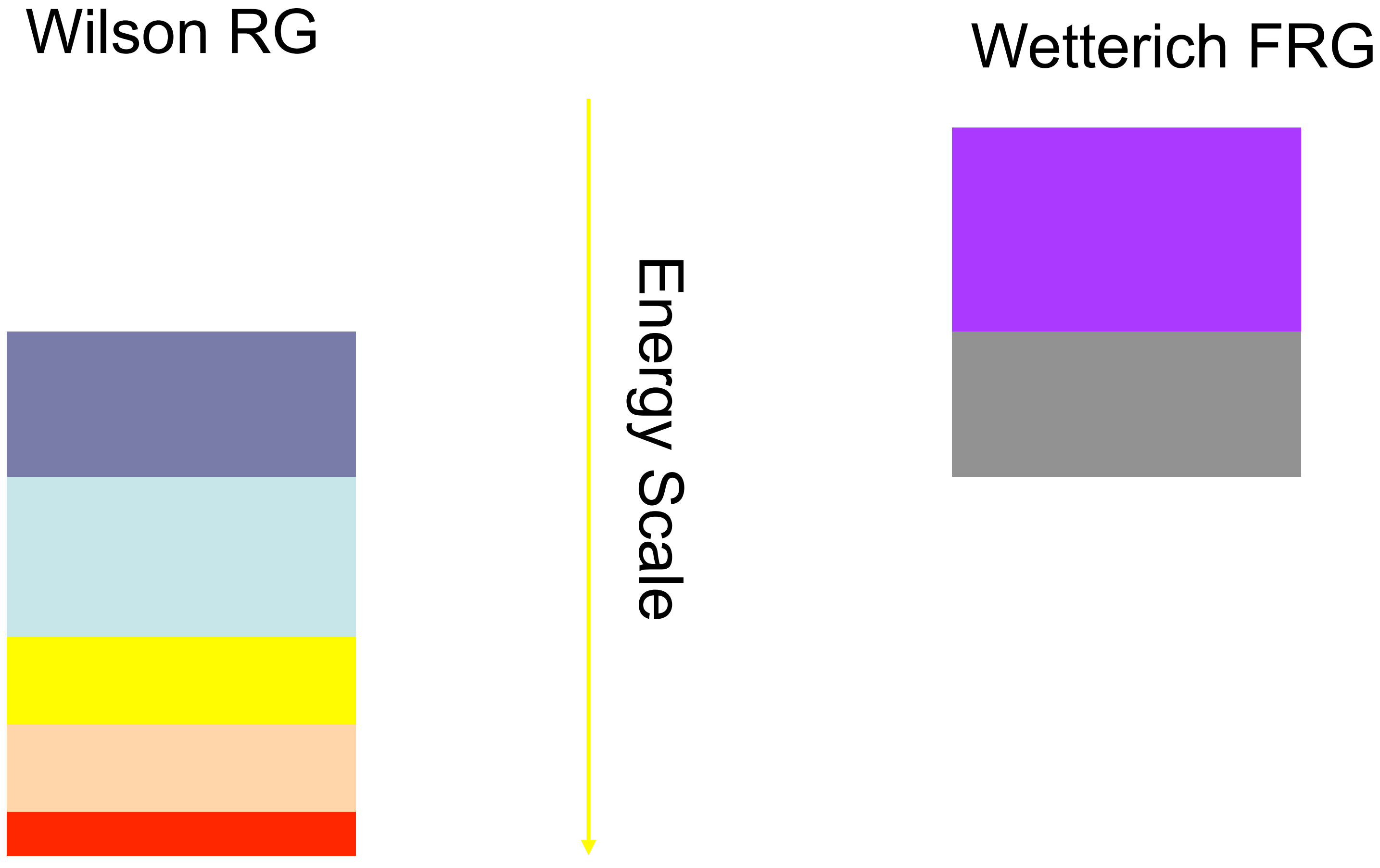
Ideas of RG and FRG

Wilson RG

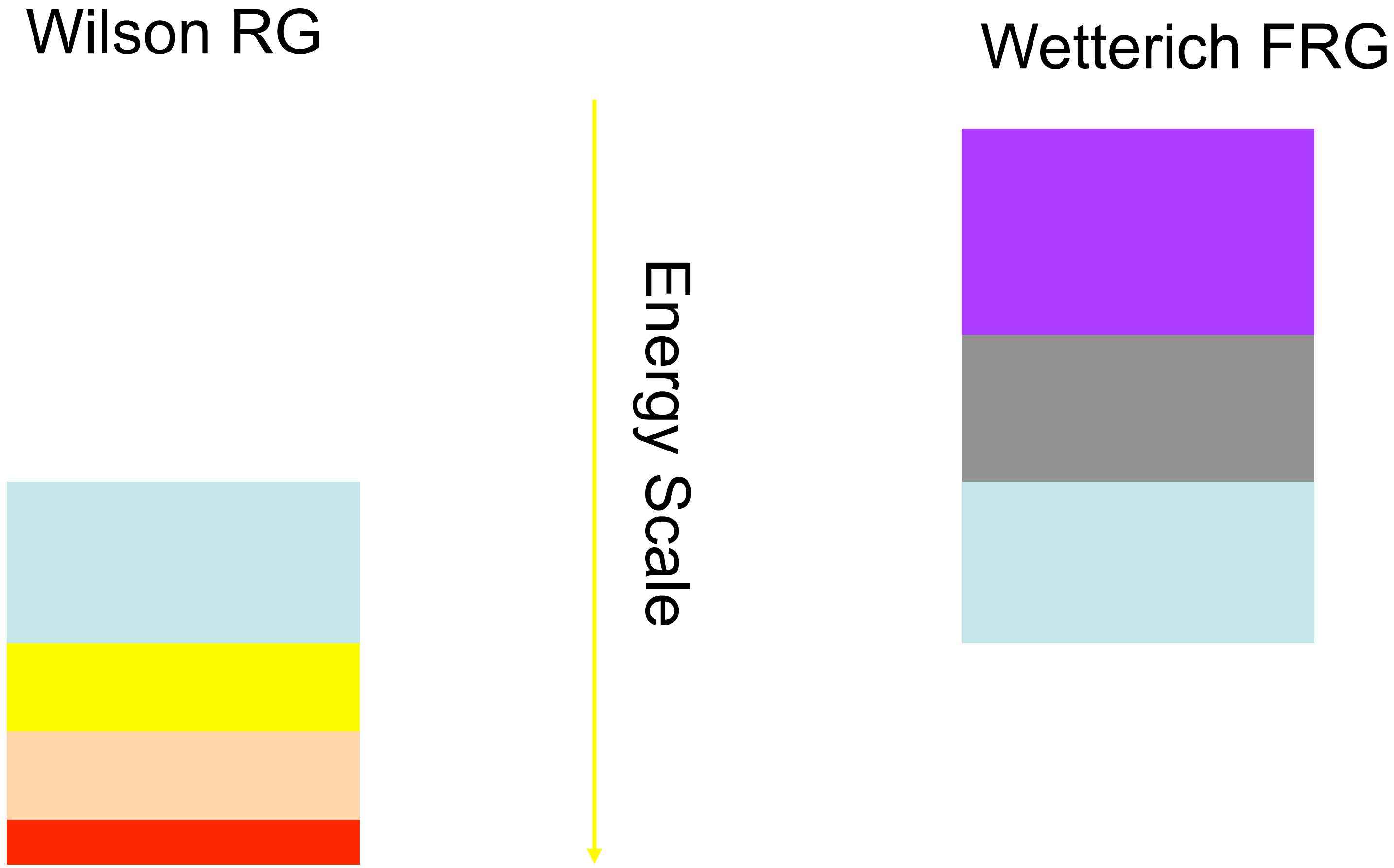
Wetterich FRG



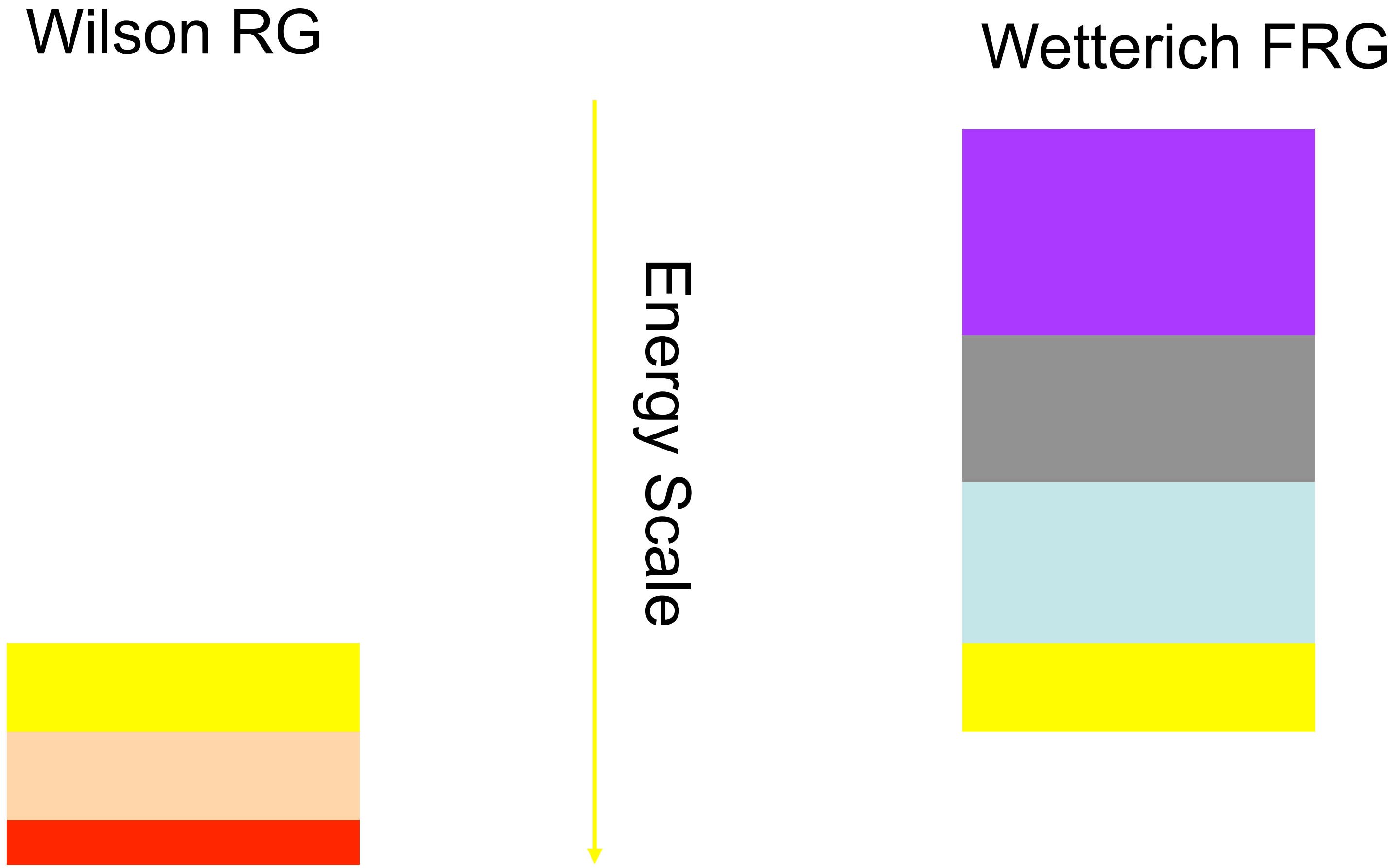
Ideas of RG and FRG



Ideas of RG and FRG



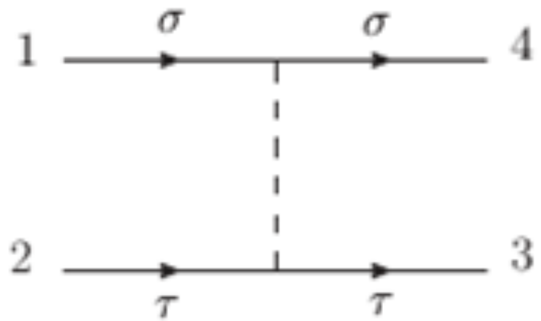
Ideas of RG and FRG



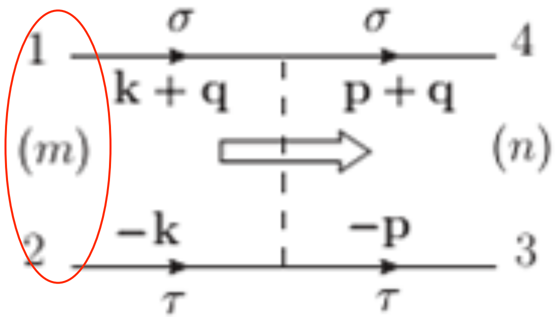
Singular-mode FRG

Cf: Husemann and Salmhofer

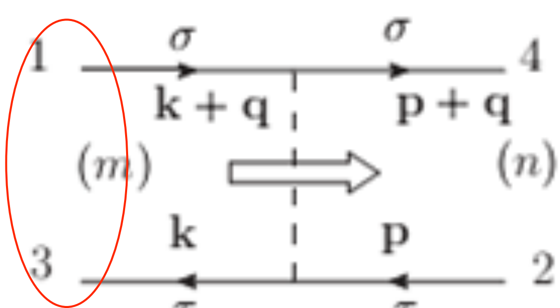
$c_1^\dagger c_2^\dagger(-\Gamma_{1234})c_3 c_4.$



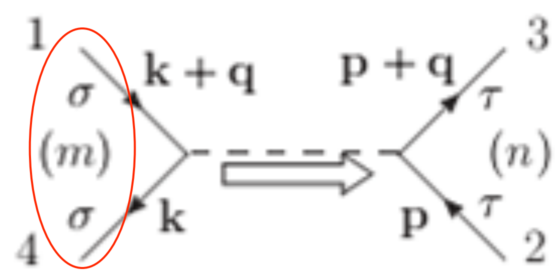
P



C

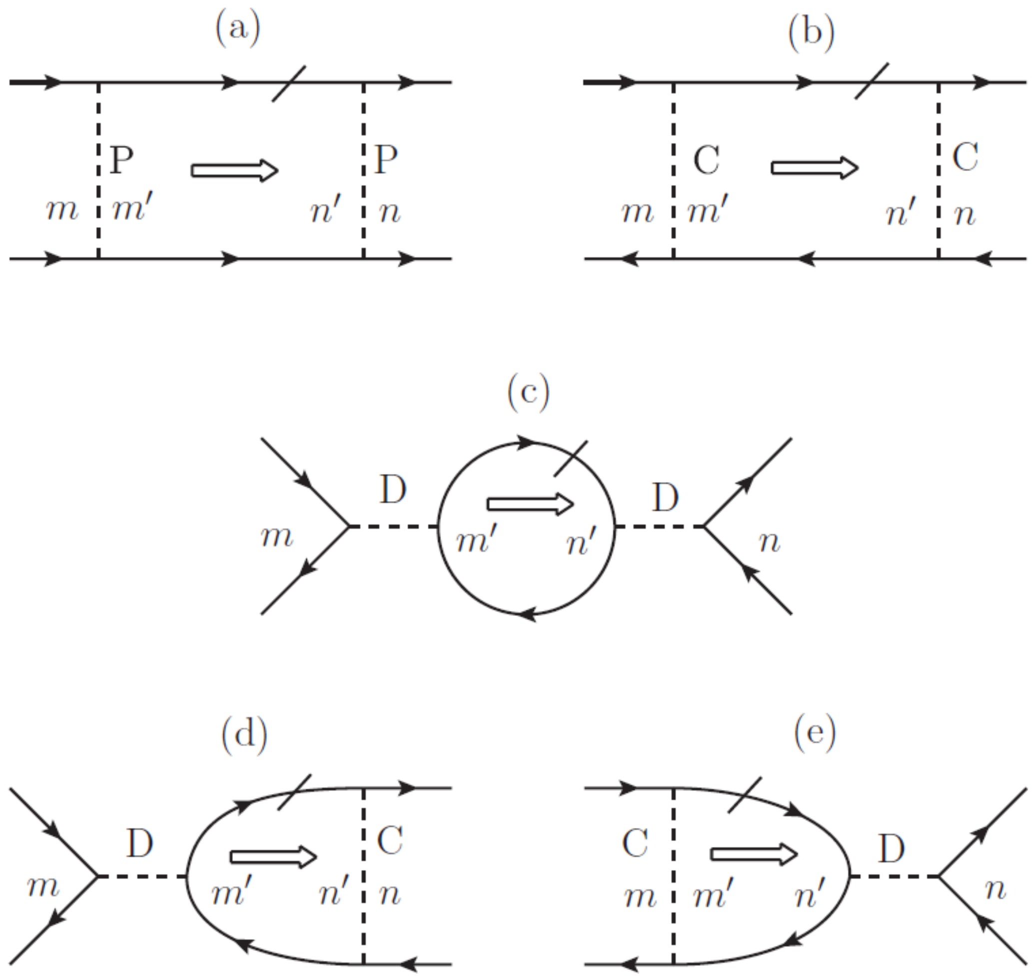


D



$\{f_m\}:$ orthonormal form factors

FRG flow



FRG flow

$$\partial P/\partial\Lambda = P\chi'_{pp}P,$$

$$\partial C/\partial\Lambda = C\chi'_{ph}C,$$

$$\partial D/\partial\Lambda = (C-D)\chi'_{ph}D + D\chi'_{ph}(C-D),$$

FRG flow

$$dP/d\Lambda = \partial P/\partial\Lambda + \hat{P}(\partial C/\partial\Lambda + \partial D/\partial\Lambda),$$
$$dC/d\Lambda = \partial C/\partial\Lambda + \hat{C}(\partial P/\partial\Lambda + \partial D/\partial\Lambda),$$
$$dD/d\Lambda = \partial D/\partial\Lambda + \hat{D}(\partial P/\partial\Lambda + \partial C/\partial\Lambda),$$

A simple view of mode-mode coupling

$$\pm c^{+}_{i\sigma} c_{i\sigma} c^{+}_{j\tau} c_{j\tau} \quad \Leftrightarrow \quad \pm c^{+}_{i\sigma} c^{+}_{j\tau} c_{j\tau} c_{i\sigma}$$

$$S_i \cdot S_j \quad \Leftrightarrow \quad -\frac{1}{2}(c^{+}_{i\uparrow} c^{+}_{j\downarrow} - c^{+}_{i\downarrow} c^{+}_{j\uparrow})(c_{j\downarrow} c_{i\uparrow} - c_{j\uparrow} c_{i\downarrow}) + \ldots \quad \Rightarrow \quad \uparrow\downarrow - \downarrow\uparrow$$

Singlet pair

$$-S_i \cdot S_j \quad \Leftrightarrow \quad -\frac{1}{4} c^{+}_{i\uparrow} c^{+}_{j\uparrow} c_{j\uparrow} c_{i\uparrow} + \ldots \quad \Rightarrow \quad \uparrow\uparrow, \quad \downarrow\downarrow, \quad \uparrow\downarrow + \downarrow\uparrow$$

Triplet pair

+ more general bond-type density wave interactions

Instabilities

- Q=0 p-p susceptibility always logarithmically divergent → universal Cooper instability wrt infinitesimal attraction

- p-h susecptibility usually finite (unless in case of perfect nesting or van Hove singularity) →Stoner instability wrt finite interaction

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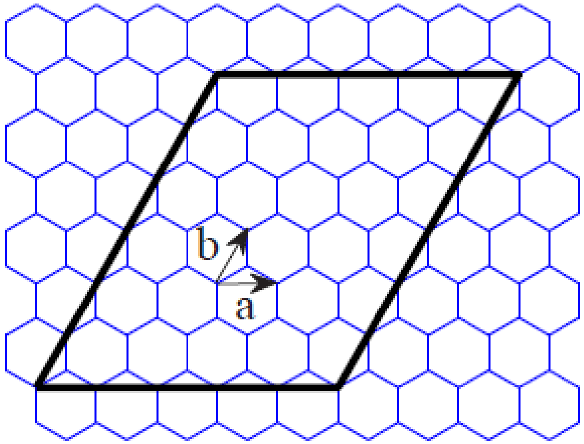
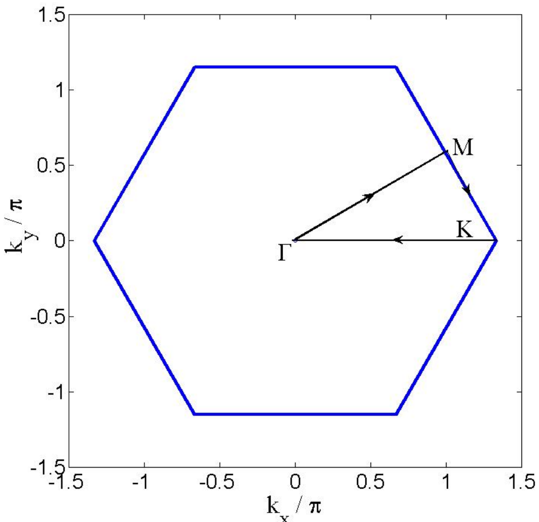
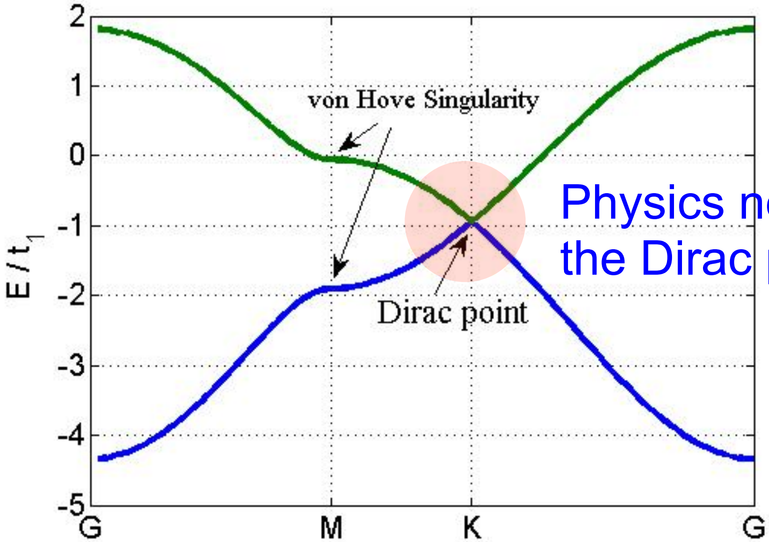
- T-breaking topological phases in doped Graphene and kagome lattices
- T-invariant topological superconductors
- Conclusions

Band structure of graphene

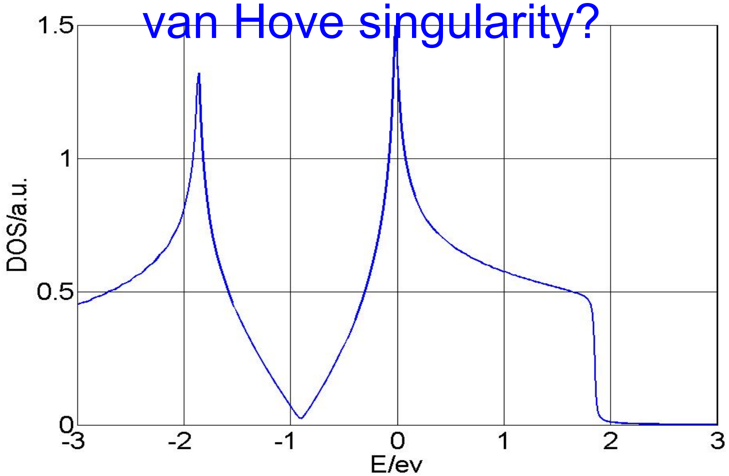






Physics near the Dirac point

Physics near van Hove singularity?



The band structure of graphene with
t1=2.8ev, t2=0.1ev , t3=0.07ev at ¼ doping

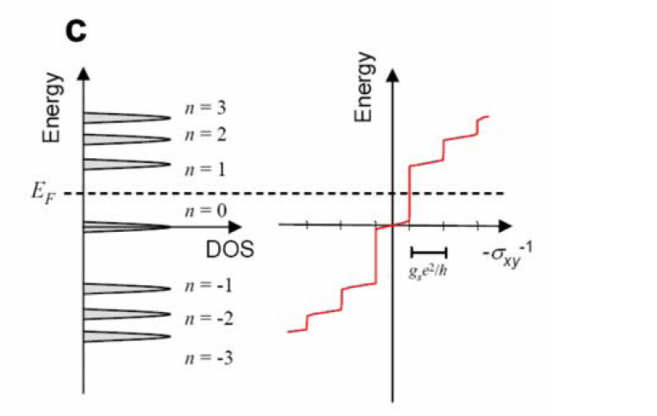
Relativistic quantum mechanics near the Dirac point

Semenoff, PRL53,2449,1984

$$-iv_F\boldsymbol{\sigma}\cdot\nabla\psi(\mathbf{r}) = E\psi(\mathbf{r}).$$

$$\psi_{\pm,\mathbf{K}}(\mathbf{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta_{\mathbf{k}}/2} \\ \pm e^{i\theta_{\mathbf{k}}/2}\end{pmatrix} \qquad \psi_{\pm,\mathbf{K}'}(\mathbf{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta_{\mathbf{k}}/2} \\ \pm e^{-i\theta_{\mathbf{k}}/2}\end{pmatrix}$$

$$\sigma_{xy} = \frac{I}{V_H} = \frac{c}{V_H}\frac{\delta E}{\delta\Phi} = \pm 2(2N+1)\frac{e^2}{h},$$

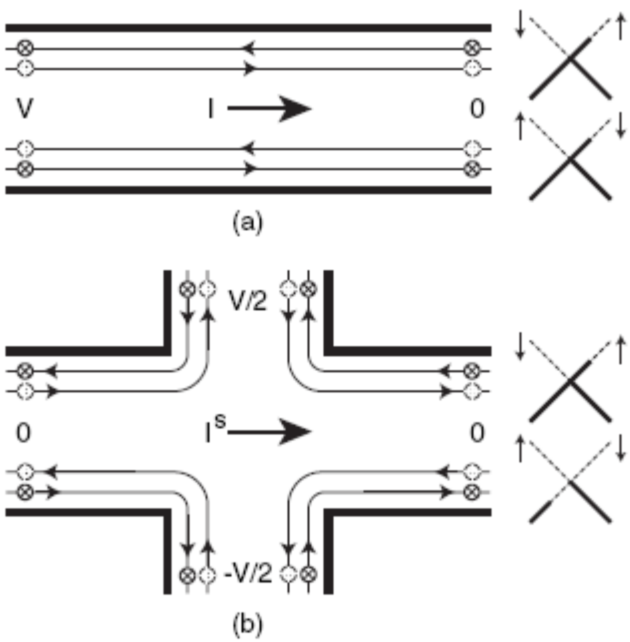


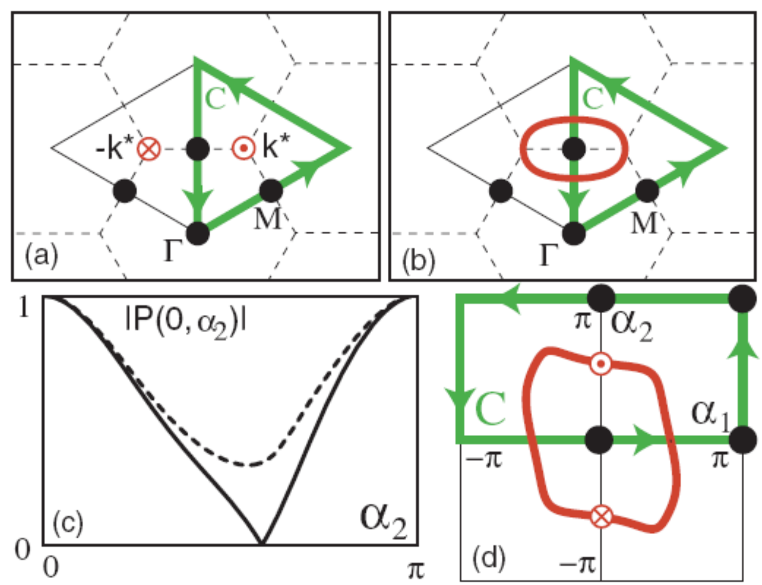
Zhang et al., Nature 438,201(2005)

Spin quantum Hall effect?

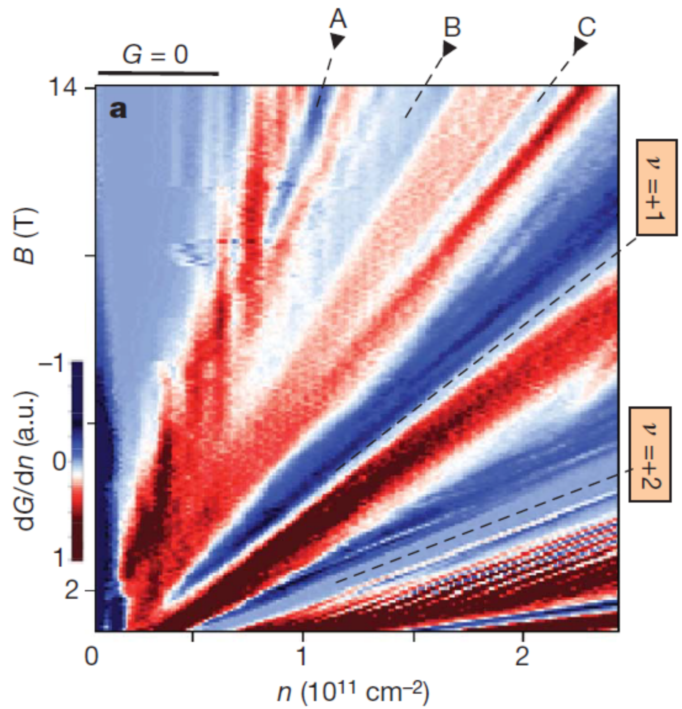
C. L. Kane and E.J. Mele , QSHE in Graphene , PRL95,226801(2005)

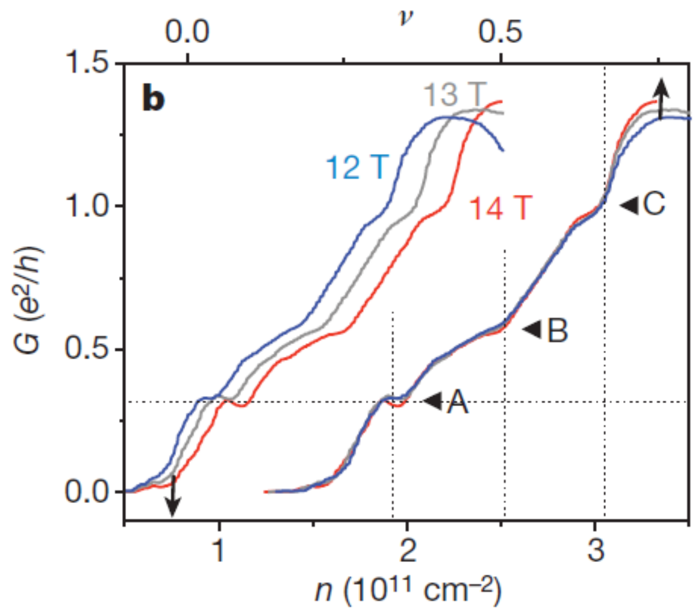
C. L. Kane and E. J. Mele, Z2 topological Order and the QSHE, PRL95, 146802 (2005)

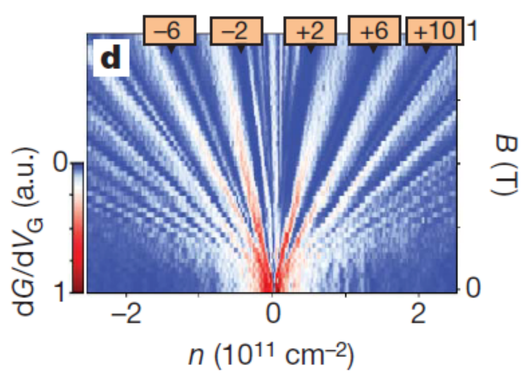




Correlations revealed by fractional QH







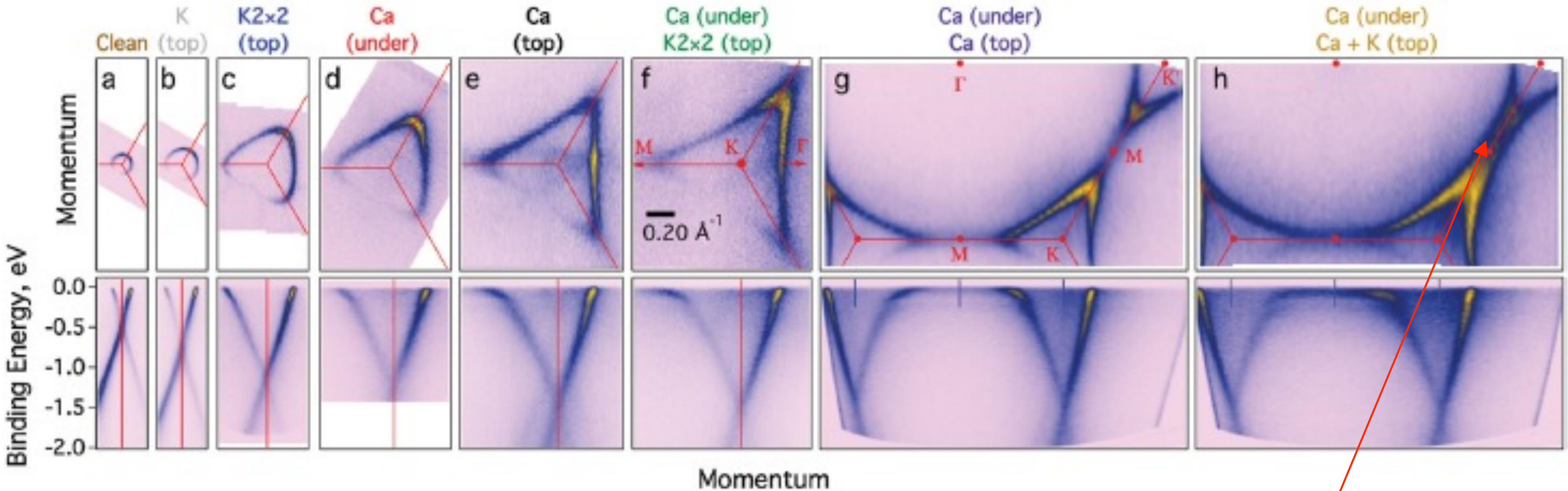
	ν	G (e^2/h)
A	0.30 ± 0.02	0.32 ± 0.02
B	0.46 ± 0.02	0.54 ± 0.02
C	0.68 ± 0.05	0.94 ± 0.02

X.Du et al., Nature 462,192; Bolotin et al., Nature 462,196

Doping graphene…

Eli Rotenberg

PRL **104**, 136803 (2010) PHYSICAL REVIEW LETTERS week ending 2 APRIL 2010



Extended van Hove singularity

What's so special of graphene

- van Hove singularity and correlation effect

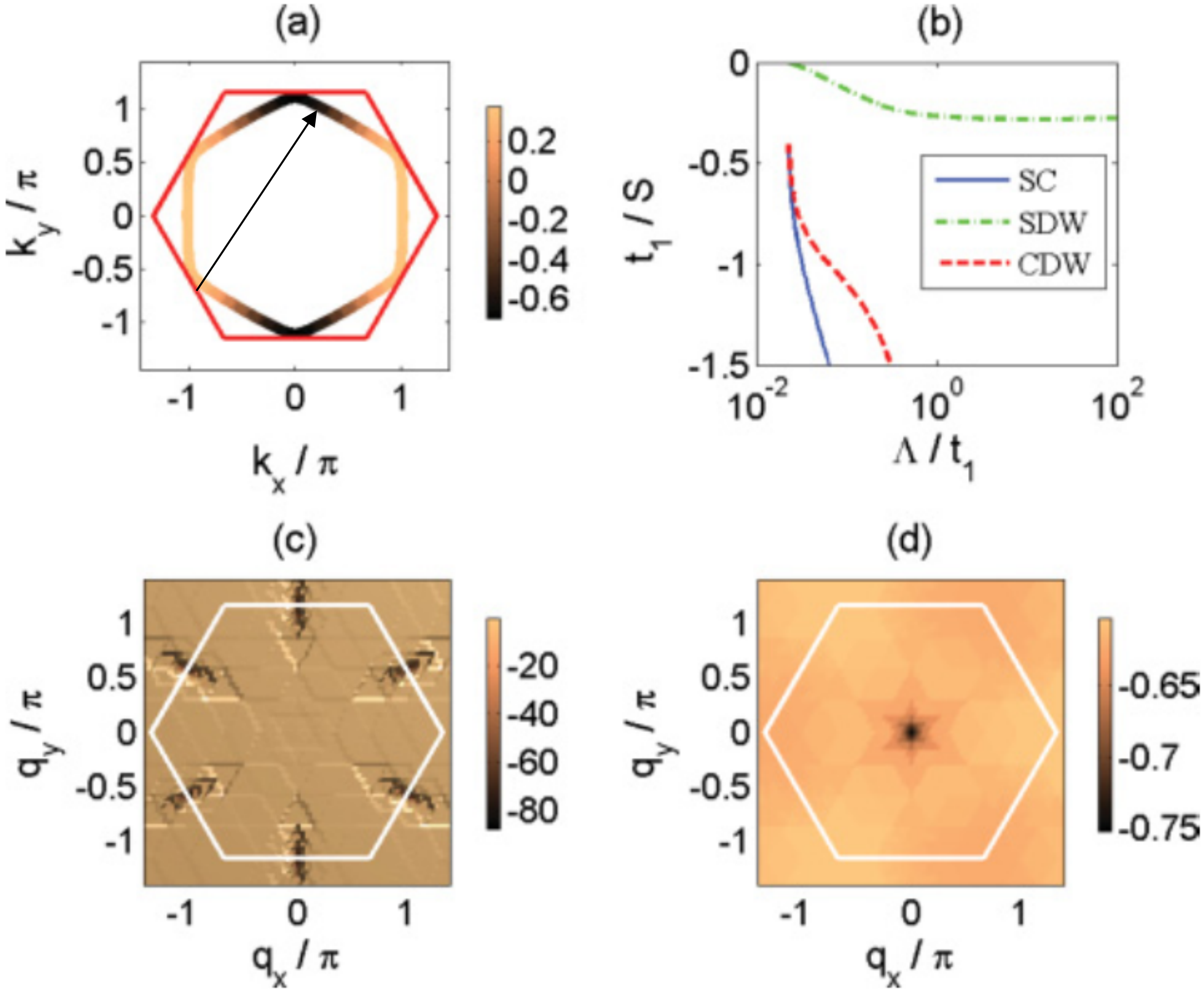
- Under C_{6v} point group， $(x^2-y^2$,xy) and (x,y) are doublets. Candidates for the gap function.

- T-breaking mixing of degenerate pairing gaps very likely, leading to a full gap

- Possible pairing symmetries: s, d+id, p+ip, f

x=1/4, U=3.6t, V=0

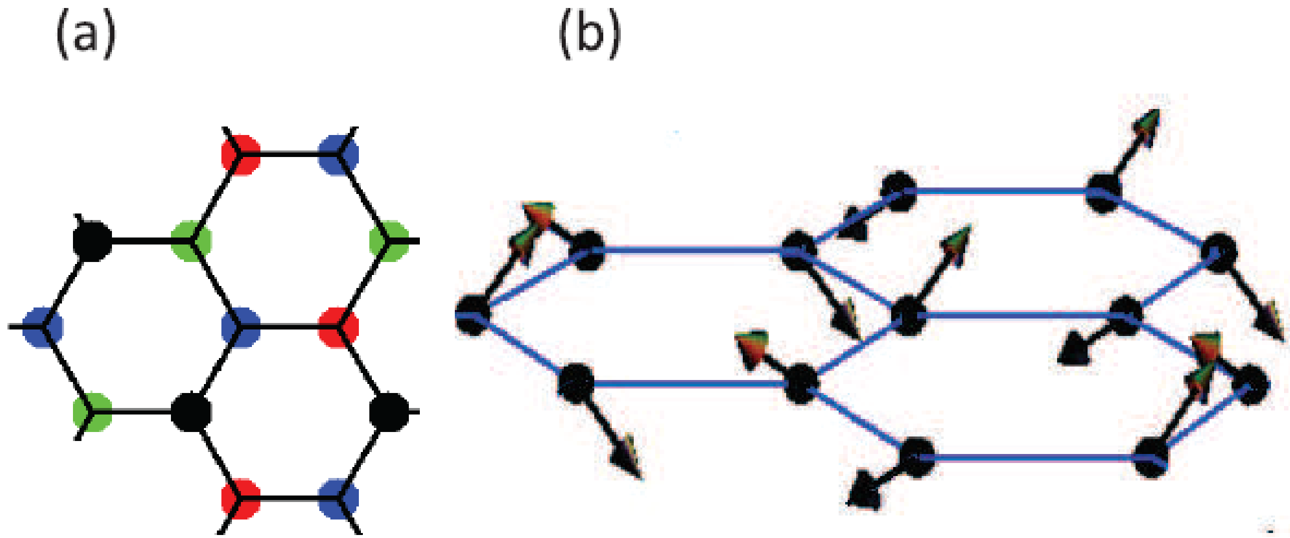
Van Hove singularity and perfect nesting

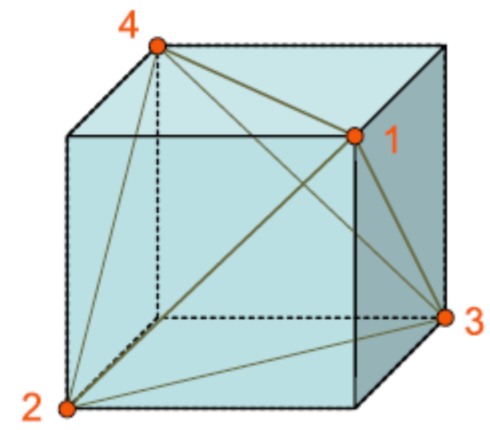


V_sdw

V_sc

Chiral SDW

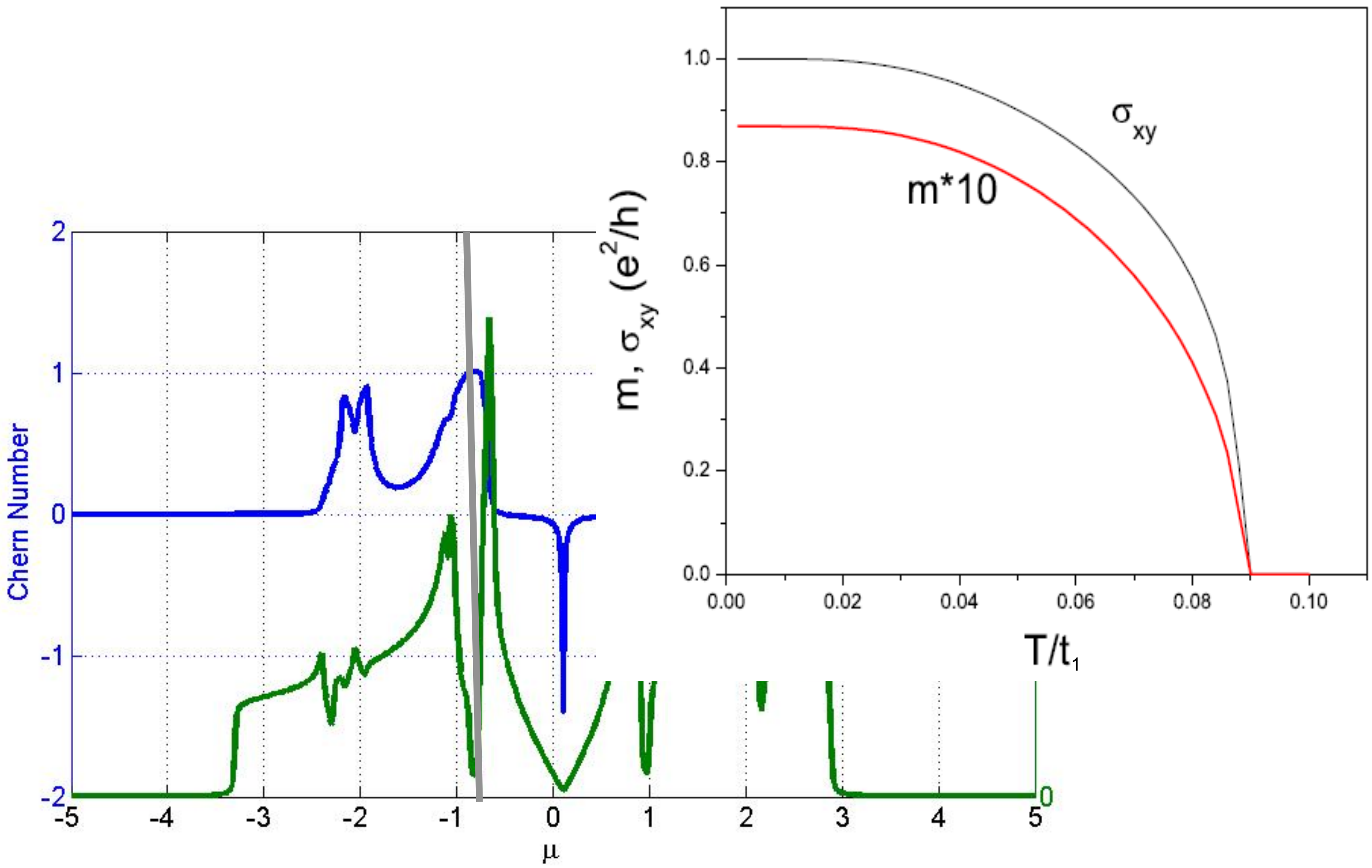




LiTao, arxiv 1103.2420, honeycomb lattice

Martin and Batista, PRL101,156402, triangle lattice

Chern number and quantized anomalous Hall conductivity



Non-perturbative quantum Monte Carlo

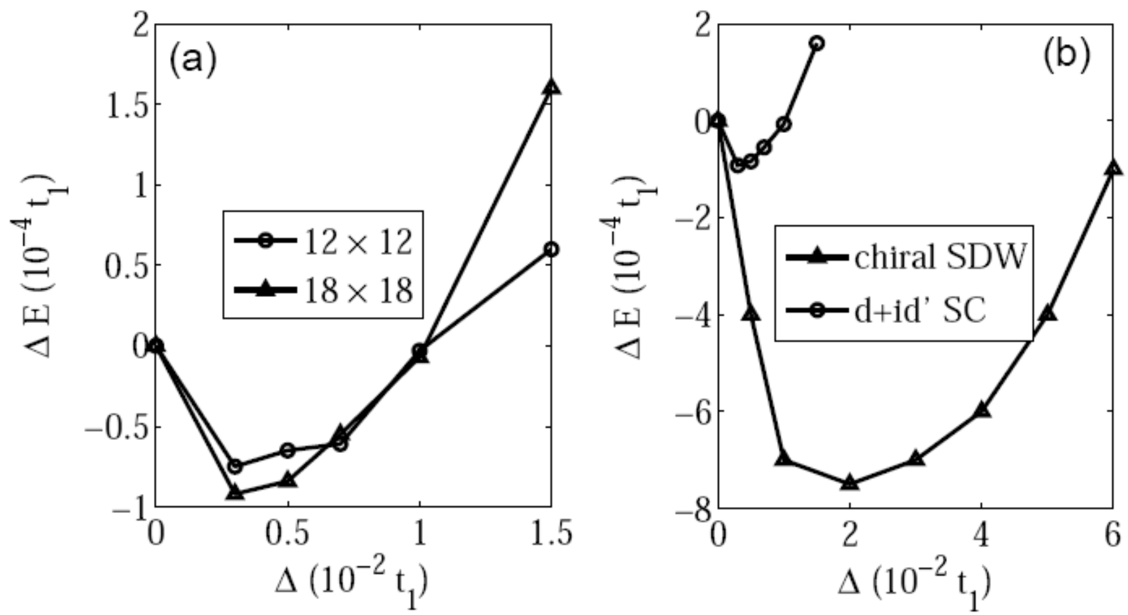
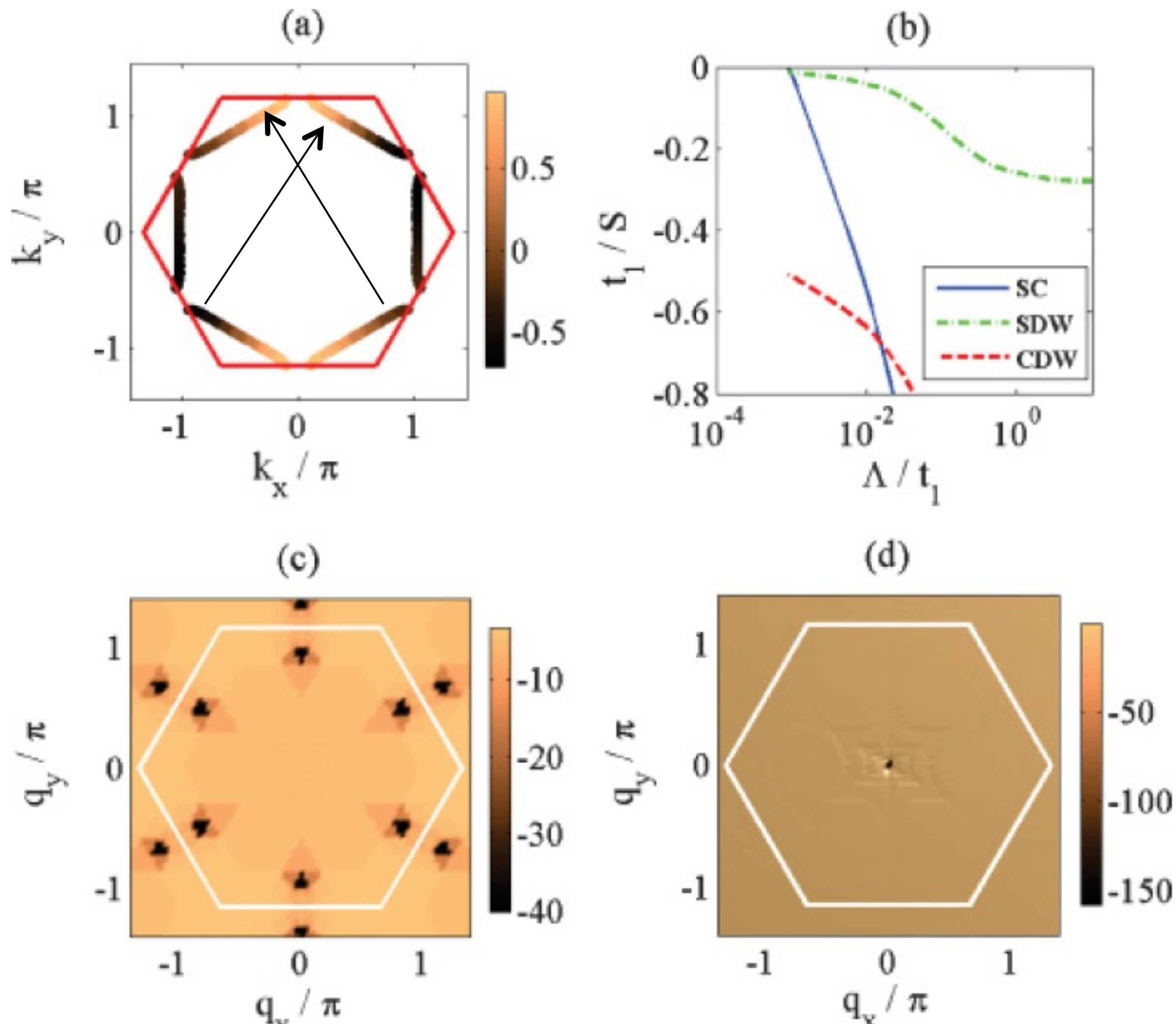


FIG. 3: Variational Monte Carlo results for $\delta = 1/4$. (a) The energy gain per site due to $d+id'$ SC order on 12×12 (circles) and 18×18 (triangles) lattices, showing negligible finite-size effect.(b) The energy gain per site due to $d+id'$ SC (circles) and chiral SDW (triangles) order parameters on an 18×18 lattice.

x=0.211, U=3.6t, V=0

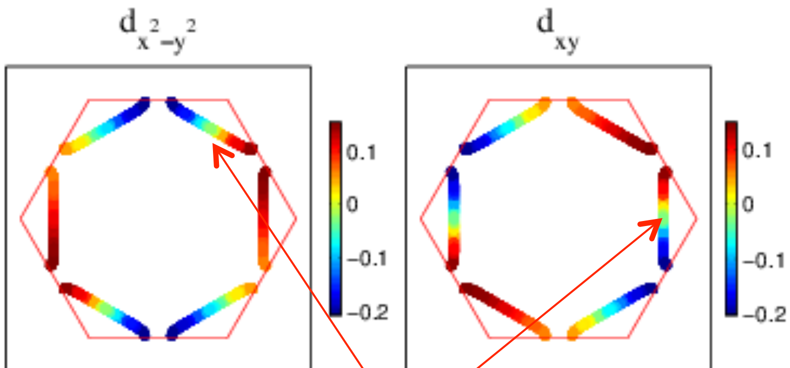


V_sdw

V_sc

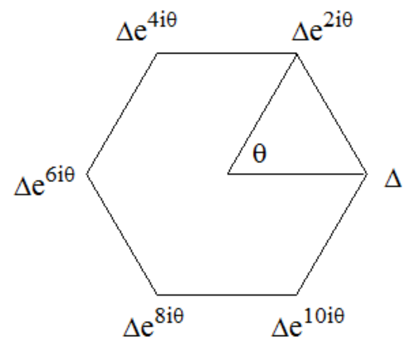
T-breaking d+id

Two degenerate d-wave pairing:



Nodal gap

d+id

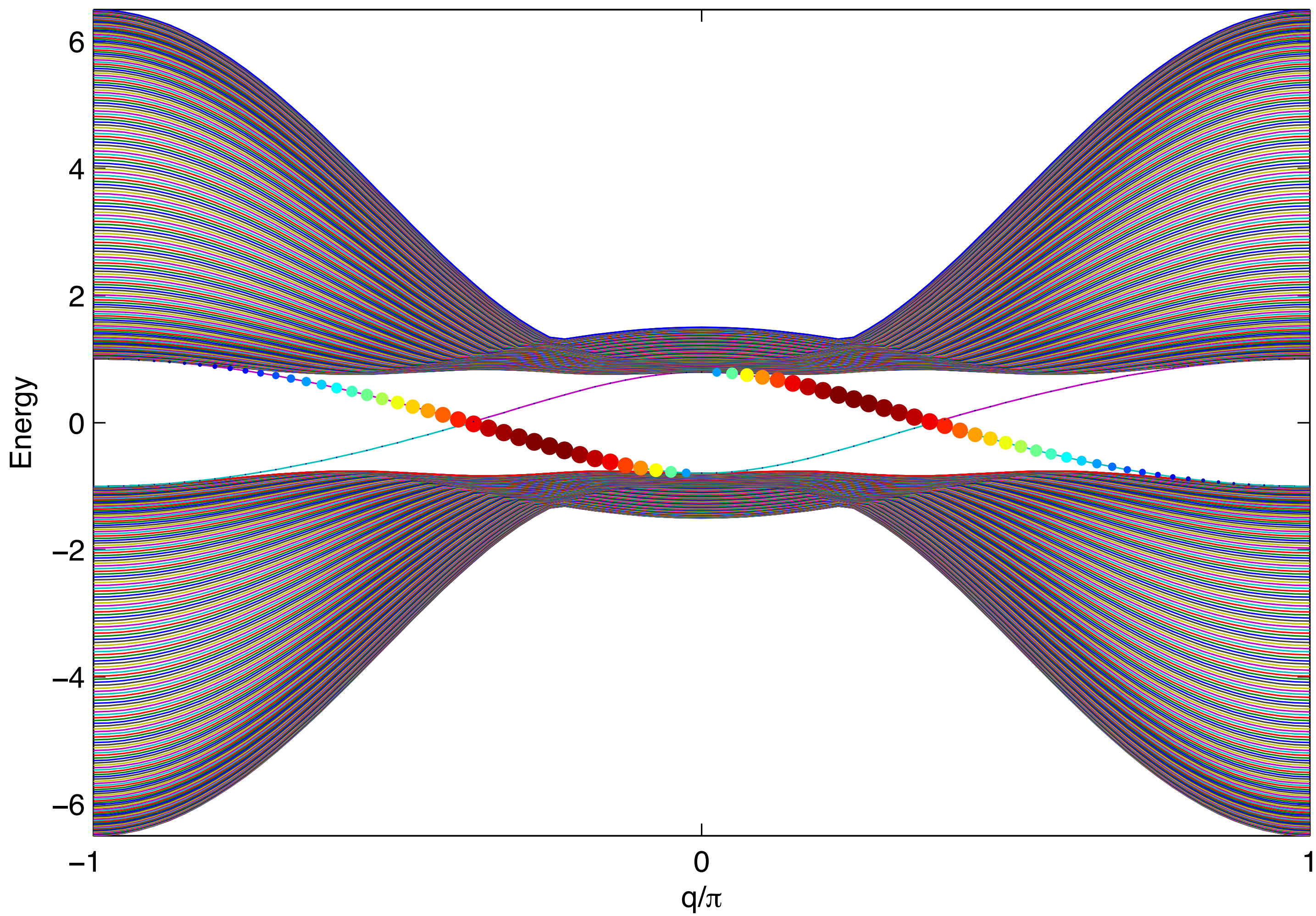


Full gap

MF or GL theory predict that the d+id pairing is energetically more favorable

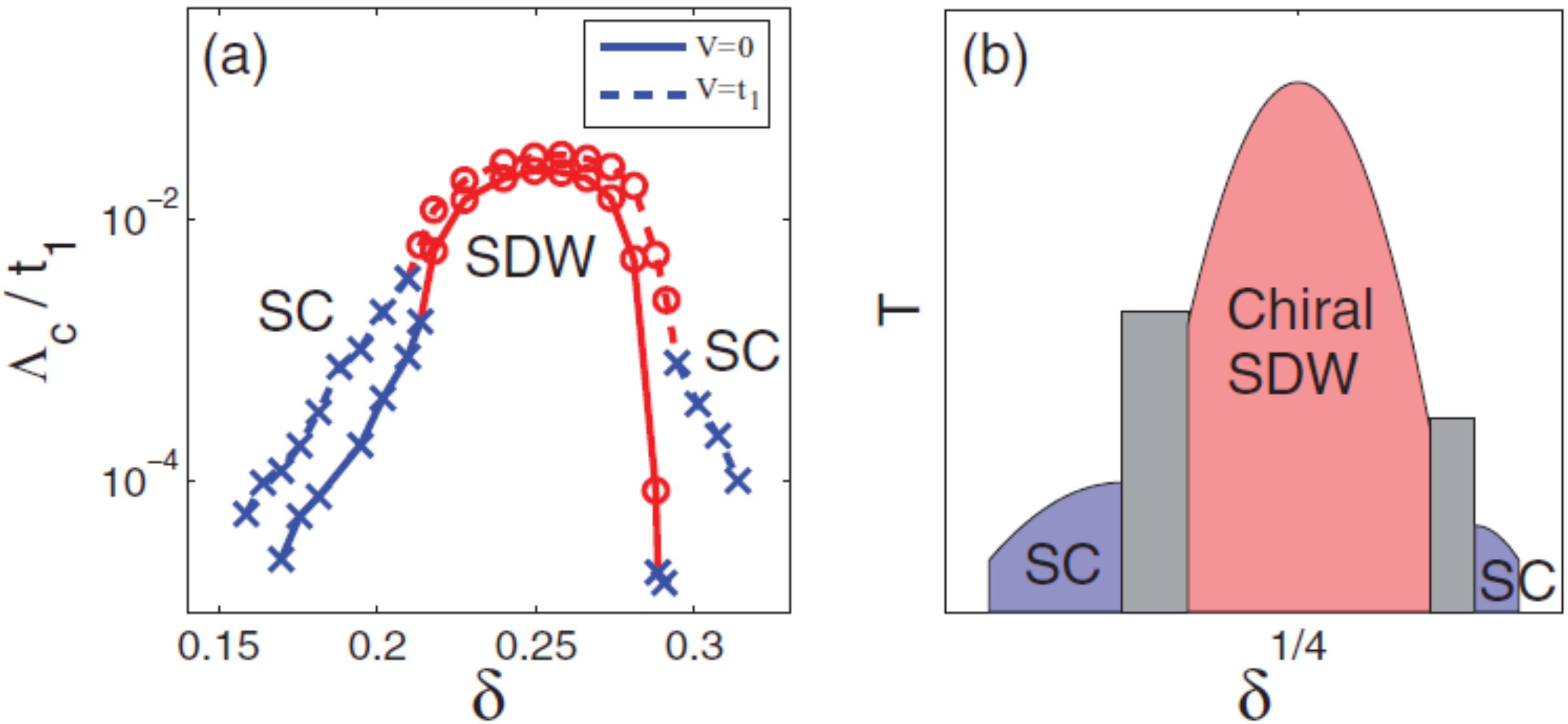
R Nandkishore et al, Nature Physics 8, 158 (2012)

Edge states for d+id pairing (Z=2)



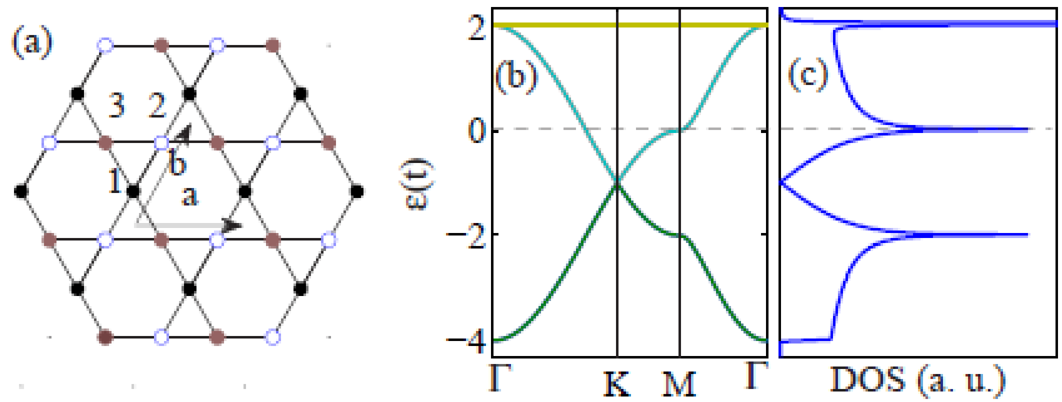
Cf: Keisel et al, arxiv 1109.2953

Phase diagram

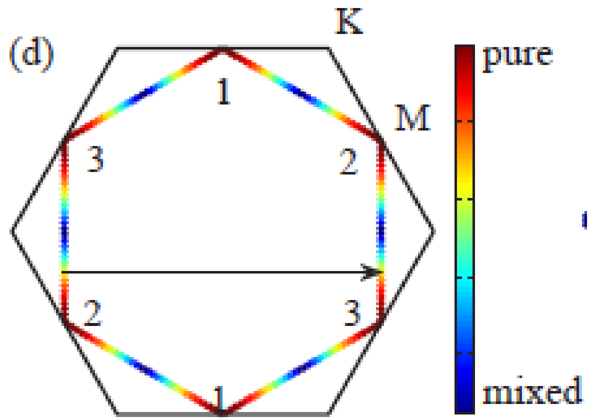


Both chiral SDW and chiral d+id are topological.

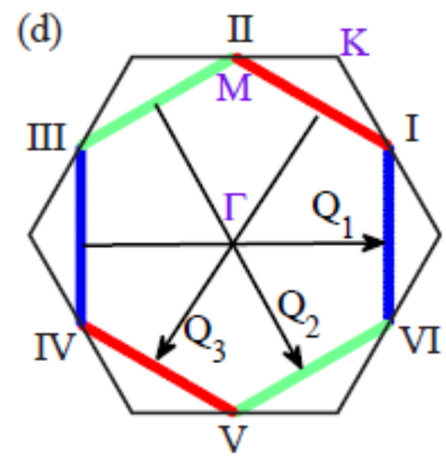
Kagome lattice



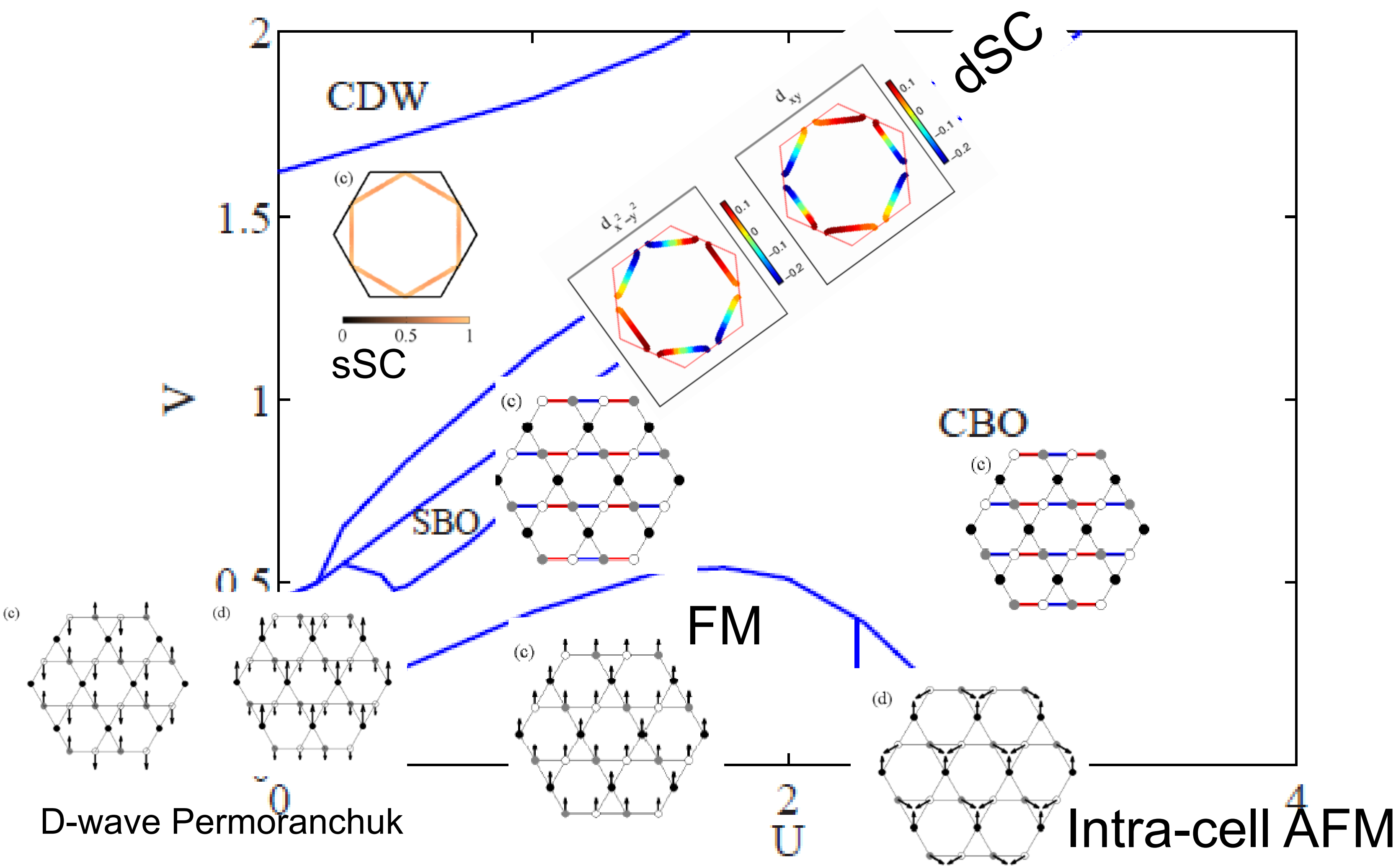
Upper van Hove filling



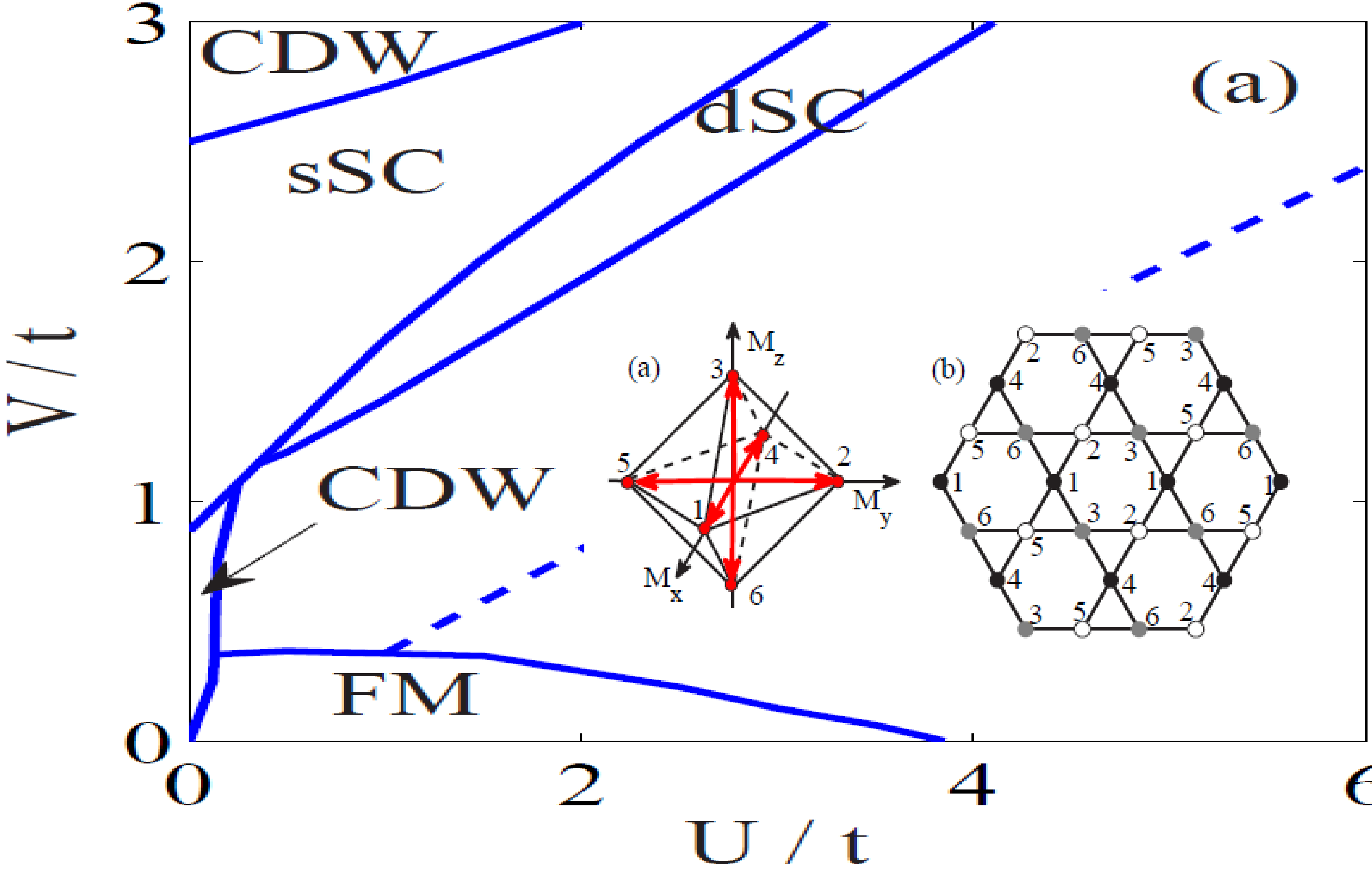
Lower van Hove filling



Upper van Hove filling



Lower van Hove filling



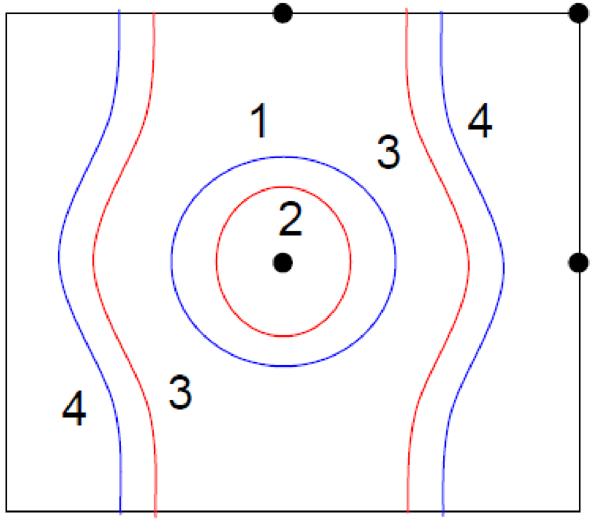
- Topological states of matter and challenges of the search of intrinsic topological superconductors
- T-breaking topological phases in doped Graphene
- T-invariant topological superconductors, a road map
- Conclusions

Gap function of a T-invariant superconductor

$$B^\dagger = \sum_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}} \Delta_{\mathbf{k}} \Psi^{\dagger T}_{-\mathbf{k}}$$

$$\Delta(\mathbf{k}) = [\phi(\mathbf{k})\sigma_0 + \vec{d}(\mathbf{k}) \cdot \vec{\sigma}] i\sigma_2,$$

singlet triplet



Qi et al, Kitaev et al:

1) Even number of spin-split pockets, each encircles an odd number of T-invariant momenta.

2) Number of pockets with + and - signs: Even = Odd + Odd

Gap-function of a T-invariant superconductor

$$\gamma_k = (-\sin k_y, \sin k_x, 0)$$

$$H_0 = \Psi_k^+(\varepsilon_k\sigma_0 + \lambda\gamma_k \cdot \sigma)\Psi_k \quad \rightarrow \quad \psi_k^+(\varepsilon_k \pm \lambda |\gamma_k|)\psi_k,$$

$$\left|-k,a\right\rangle = i\sigma_2 K\left|k,a\right\rangle$$

$$If \quad d_k \sim \gamma_k, \quad T-in\mathrm{var}iant, \quad and$$

$$H_P = \Psi_k^+(\phi_k\sigma_0 + d_k \cdot \sigma)i\sigma_2(\Psi_{-k}^+)^T$$

$$\rightarrow -\psi_{k,a}^+(\quad \phi_k \pm |d_k| \quad)(\psi_{-k,b}^+)^T\delta_{a,b}$$

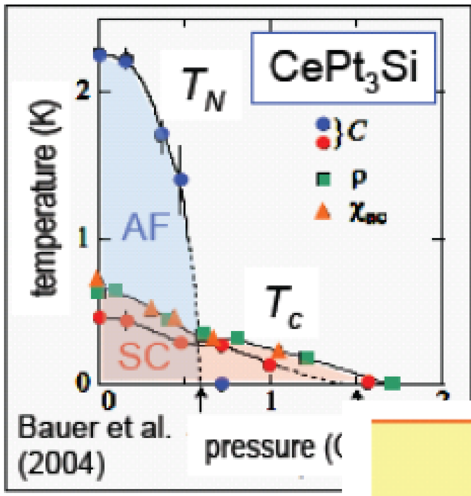
$$i(p_x + ip_y)\downarrow\downarrow + i(p_x - ip_y)\uparrow\uparrow$$

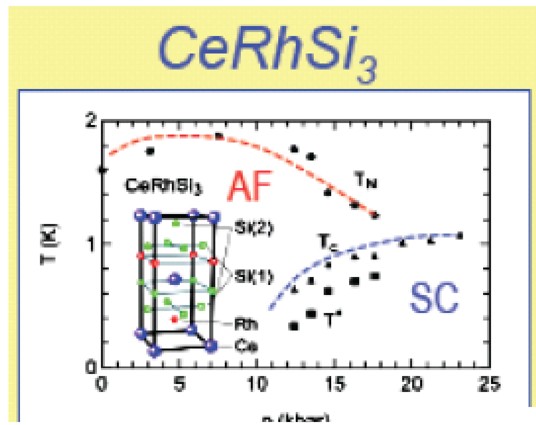
Our road map

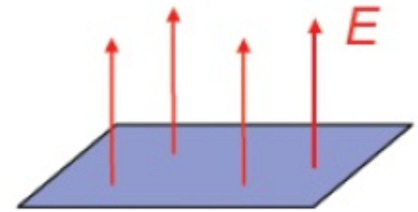
- Seek ferromagnetic spin fluctuations to favor triplet pairing
- Seek point group with odd parity degenerate irreducible representation (such as C_{4v} and C_{6v})
- Seek a system with 2(2n+1) spin-split pockets
- Rashba coupling causes degenerate triplets to recombine into a T-invariant gap, plus small induced singlet component.

Non-centrosymmetric systems

Ce-based heavy Fermion SC

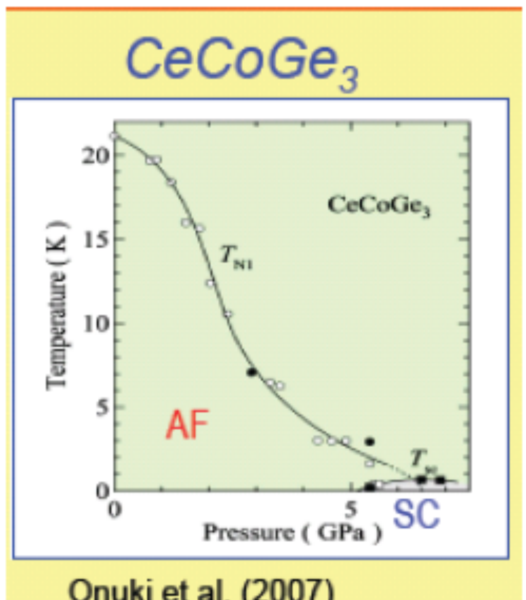


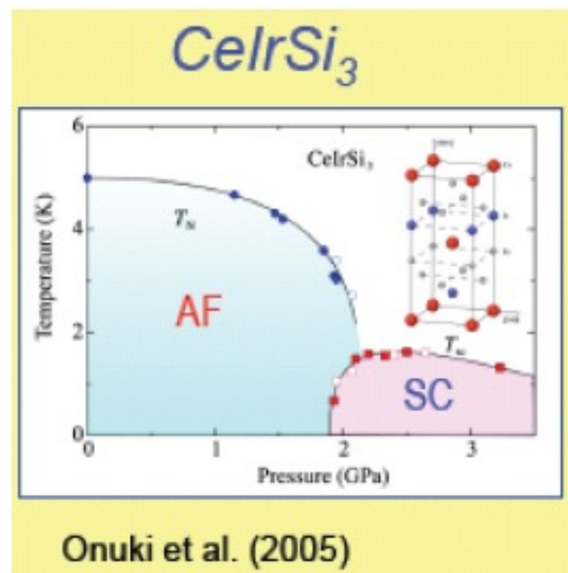




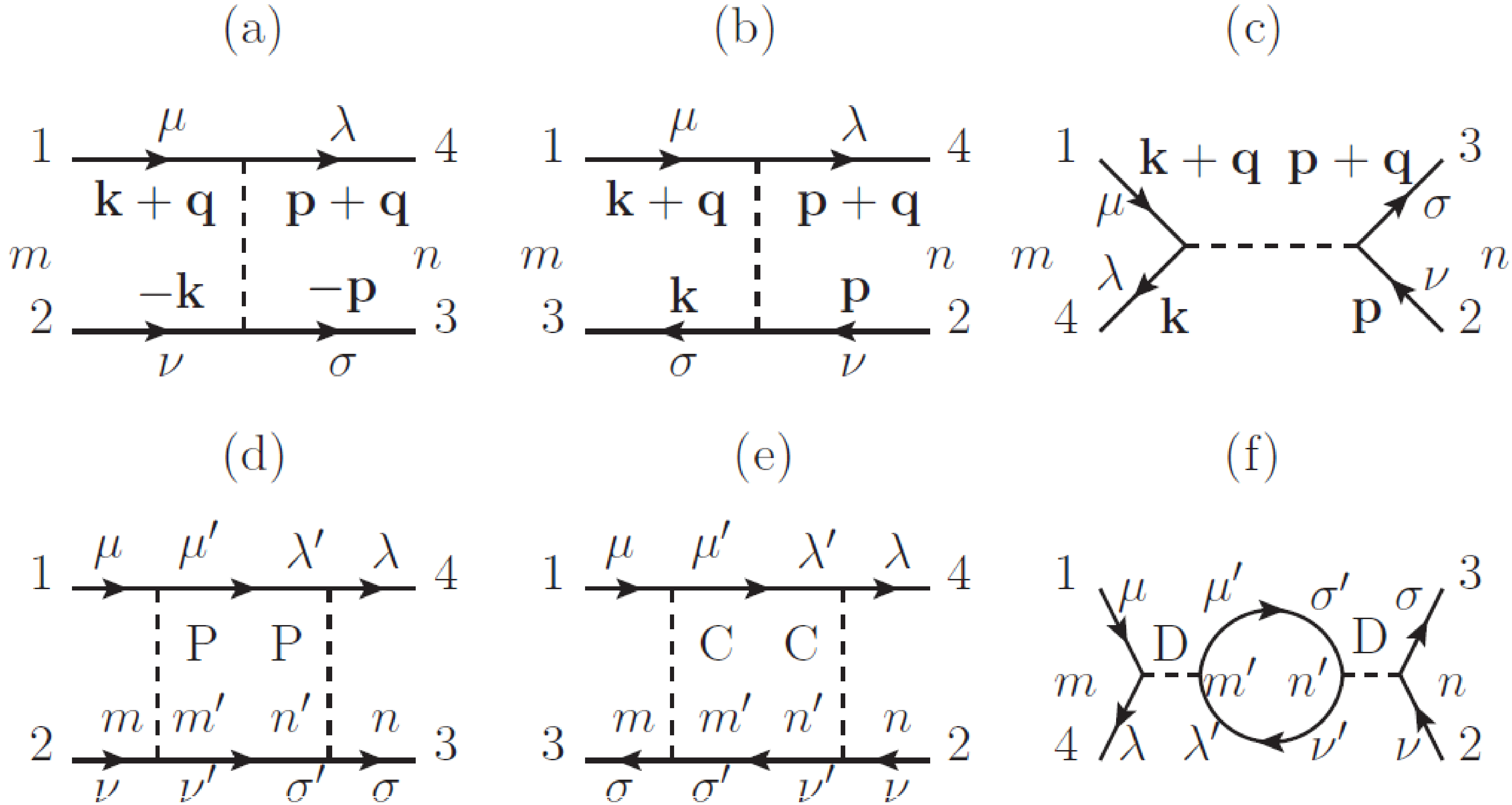
Rashba type
spin-orbit coupling

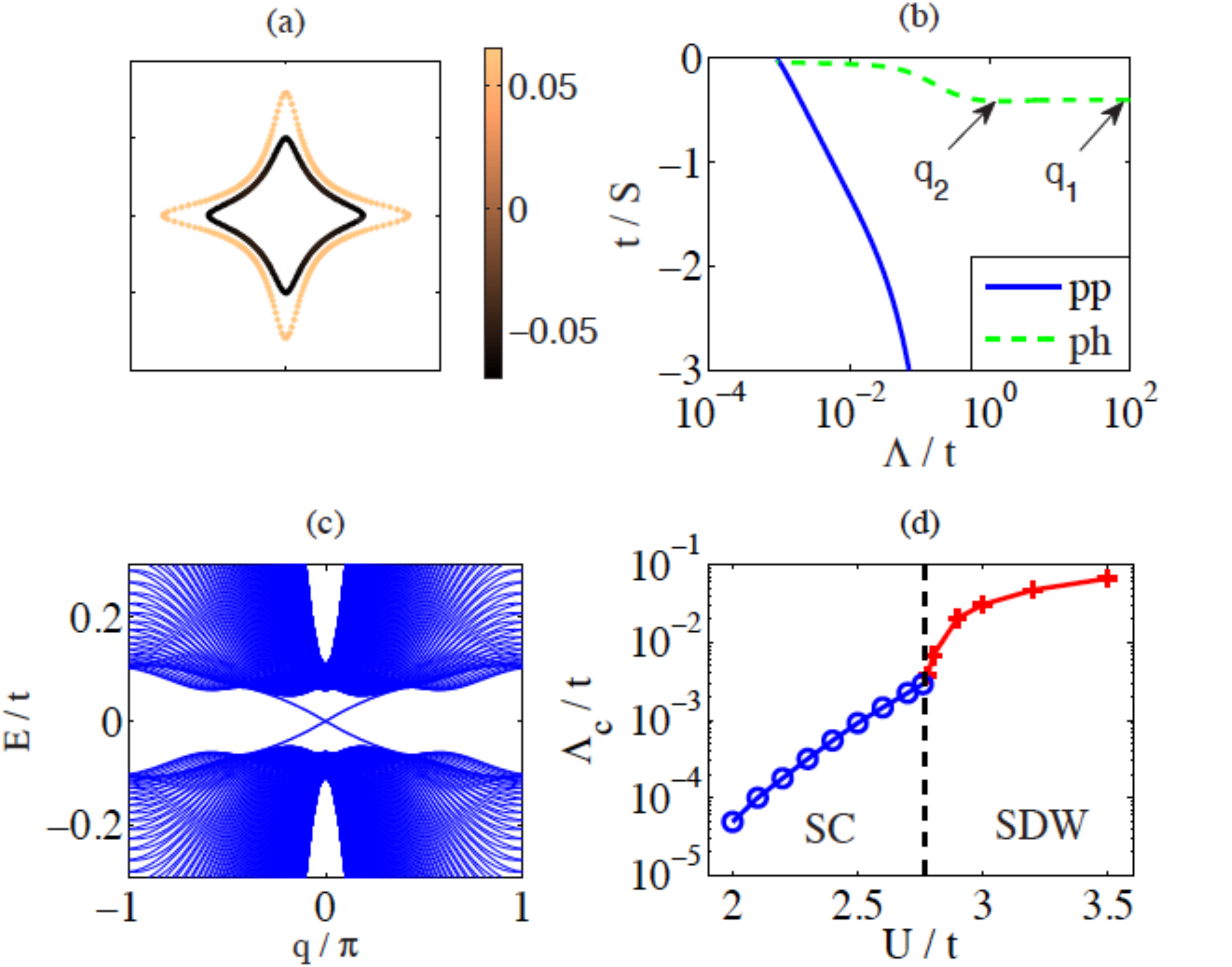
Samokhin et al, Frigeri et al





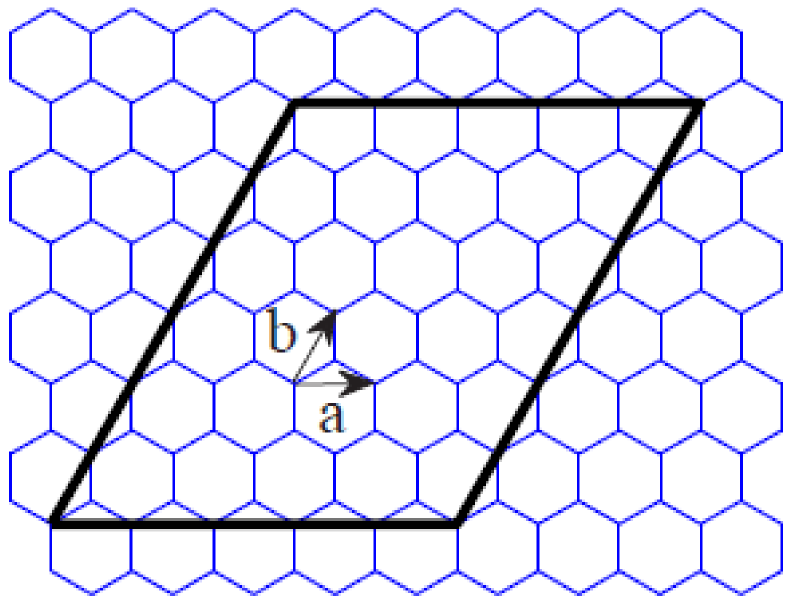
Spin-resolved fully anti-symmetrized SM-FRG



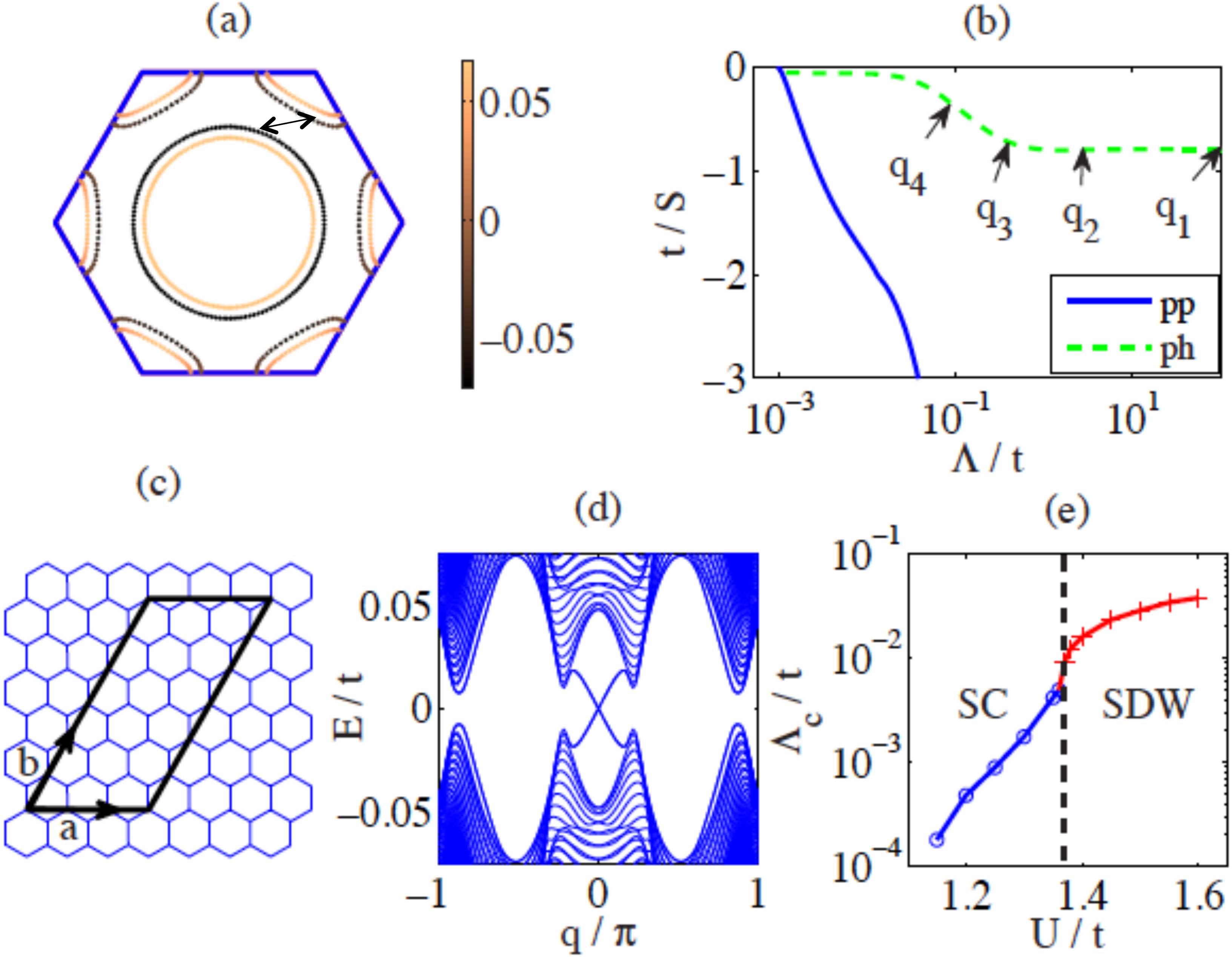


(a)
0.05
0
−0.05
(b)
0
−1
−2
−3
t / S
q2
q1
pp
ph
10^{-4}
10^{-2}
10^{0}
10^{2}
Λ / t
(c)
0.2
0
−0.2
E / t
−1
0
1
q / π
(d)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Λ_c / t
SC
SDW
2
2.5
3
3.5
U / t

Topological pairing triggered by small-q inter-pocket scattering



$$H_0 = -\sum_{i\delta} \Psi_i^\dagger t_\delta \Psi_{i+\delta} - i\lambda \sum_{i\delta_{nn}} \Psi_i^\dagger (\hat{z} \times \vec{\delta}_{nn} \cdot \vec{\sigma}) \Psi_{i+\delta_{nn}} - \mu \sum_i \Psi_i^\dagger \Psi_i. \tag{3}$$


(a)
0.05
0
−0.05
(b)
0
−1
−2
−3
t / S
q4
q3
q2
q1
pp
ph
10^−3
10^−1
10^1
Λ / t
(c)
b
a
(d)
0.05
0
−0.05
E / t
−1
0
1
q / π
(e)
10^−1
10^−2
10^−3
10^−4
Λc / t
SC
SDW
1.2
1.4
1.6
U / t

Possible candidates with ferromagnetic spin fluctuations

Li_2Pd_3B, Li_2Pt_3B, URhGe, $HoMo_6Se_8$, $ErRh_4B_4$,

Aoki and Flouquet, JPSJ 81, 011003 (2012).

Iron under high pressure

Shimizu et al, NATURE 412, 316 (2001).

$LaAlO_3/SrTiO_3$

Reyren, etal, Science 317, 1196 (2007).

- Motivation
- T-breaking topological phases in doped Graphene
- T-invariant topological superconductors, a road map
- Conclusions

Conclusions

- Graphene near ¼ doping is either a Chern insulator or a chiral d+id superconductor.
- Ferromagnetic instability is the key to T-invariant topological insulator, plus Rashba coupling and 2(2n+1) spin split fermi pockets (encircling T -invarint momenta).
- T-invariant topo-SC can be triggered by 1) proximity to van Hove singularity and 2) by small-q inter-pocket scattering

Thank you for your attention