

# From entanglement to quantum cryptography

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# Outline

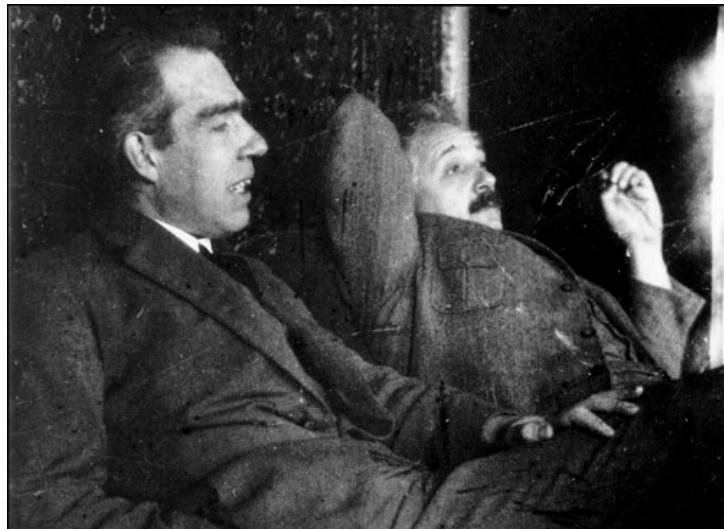
- Quantum entanglement
  - EPR pair
- Quantum cryptography
  - Cryptography
  - Why Quantum Cryptography
  - Why is it secure?
- \*MDI QKD
- Some developments in quantum crypto



# Quantum Entanglement

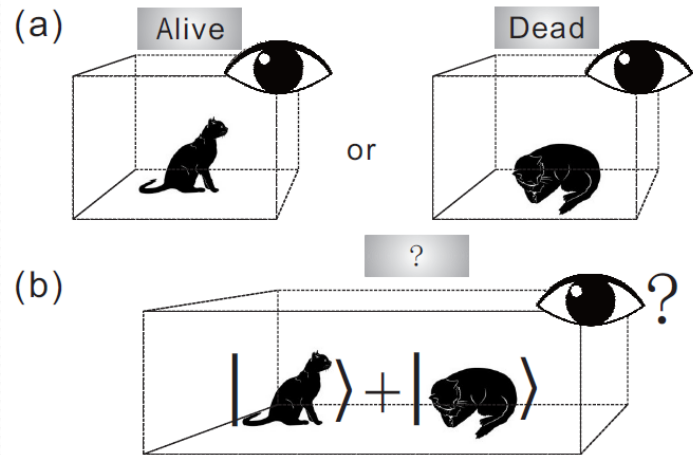
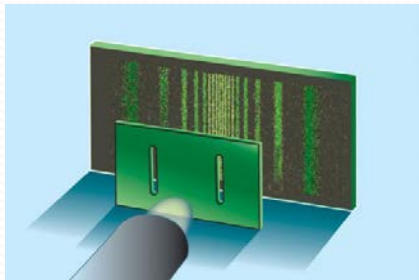
# Quantum intrinsic randomness

- Einstein, “God does not play dice!”
- Bohr, “Albert, stop telling God what to do!”



# What is Coherence?

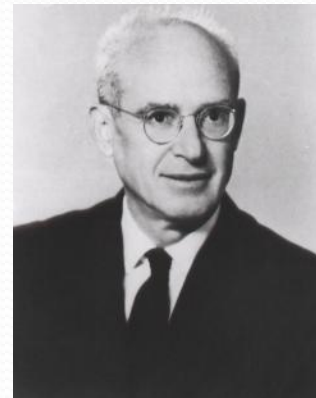
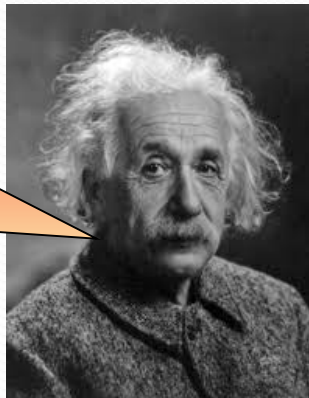
- Coherent interference
- Schrodinger's cat



# Einstein-Podolsky-Rosen Paradox

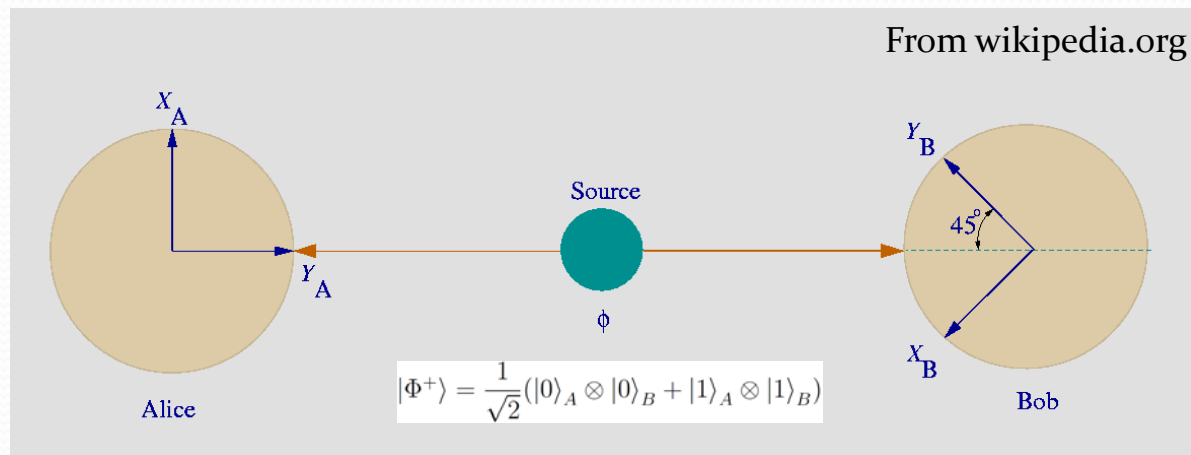
- Is Quantum Mechanics complete?
- Local hidden variable
- Entanglement
  - A pair of particles: measure on one particle would instantaneously affect the state of the other

Spooky  
action at a  
distance



# Bell's inequality

- Quantum mechanics vs. local hidden variable



$$\begin{aligned}
 S &= -\langle x_a x_b \rangle + \langle x_a y_b \rangle + \langle y_a x_b \rangle + \langle y_a y_b \rangle \\
 &= \langle (-x_a + y_a) x_b \rangle + \langle (x_a + y_a) y_b \rangle \\
 &\leq | -x_a + y_a | + | x_a + y_a | \leq 2
 \end{aligned}$$

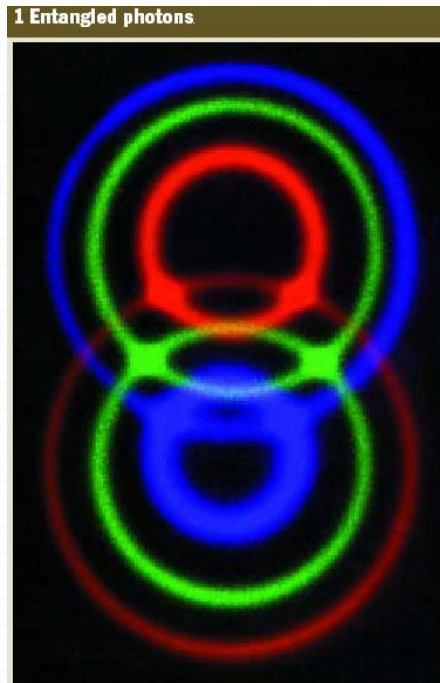
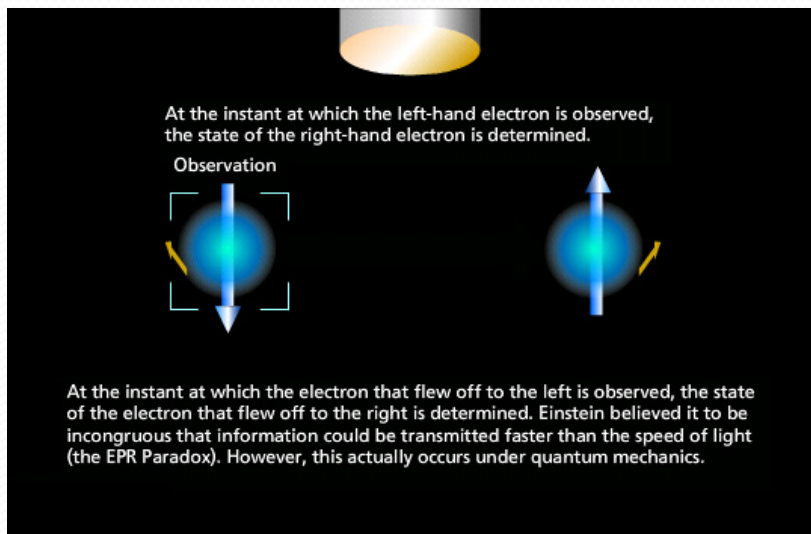
$$\begin{aligned}
 \langle x_a x_b \rangle &= -\frac{1}{\sqrt{2}} \\
 \langle x_a y_b \rangle &= \langle y_a x_b \rangle = \langle y_a y_b \rangle = \frac{1}{\sqrt{2}} \\
 S &= 2\sqrt{2}
 \end{aligned}$$





# EPR pairs in reality

- Various physical systems



- Alain Aspect '81: Bell's inequality experiment demo

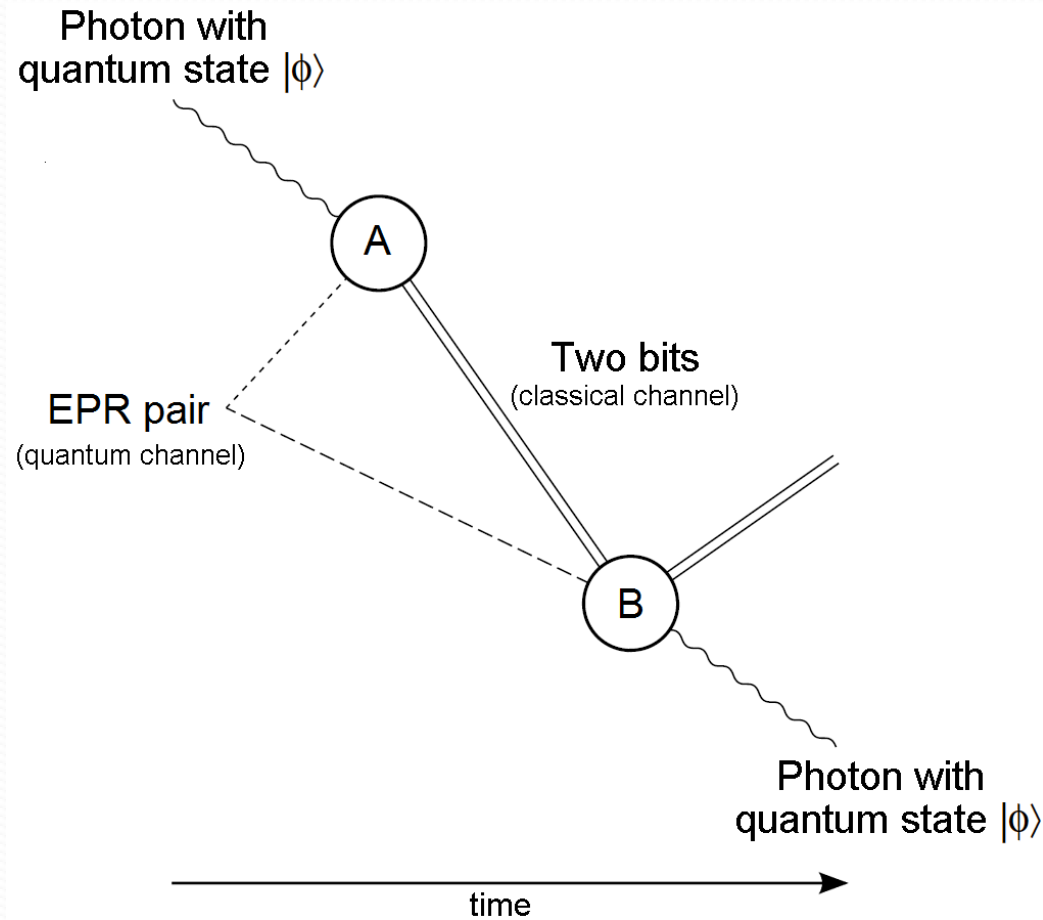
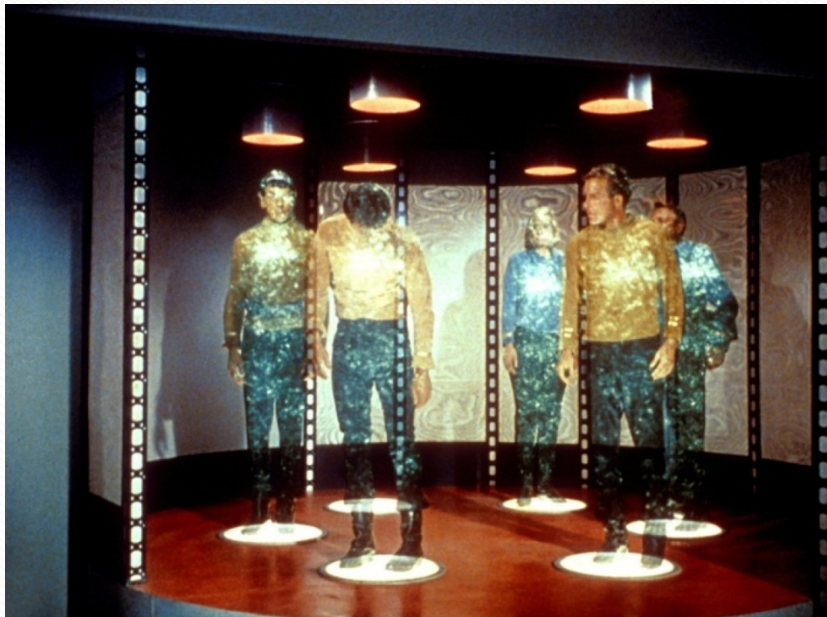


# Entanglement

- EPR pair
  - $|00\rangle + |11\rangle = |++\rangle + |--\rangle$
- Non-locality --- Entanglement
  - Local hidden variable: been ruled out by Bell's inequality test
  - Non-local correlation: why quantum mechanics is “weird”
- Natural source for secure key
  - Strong correlation
  - Randomness in nature
  - Cannot be eavesdropped --- no local hidden variable
- EPR pair means perfect key

# Quantum Teleportation

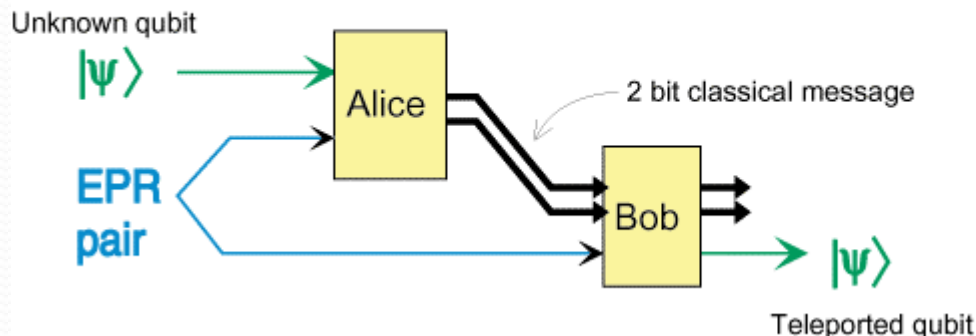
- Sci-Fi
  - Instant transportation
  - Teleport materials



# Teleportation

- How to send a quantum state  $a|0\rangle + b|1\rangle$  faithfully?
  - Classically, the values of  $a$  and  $b$  are continuous, so it requires a large amount of information transmission
  - Heisenberg's uncertainty principle
- What if Alice and Bob have a pre-shared entangled state?
  - Only needs to transfer 2 classical bits

Quantum Teleportation uses 2 classical bits to send 1 qubit





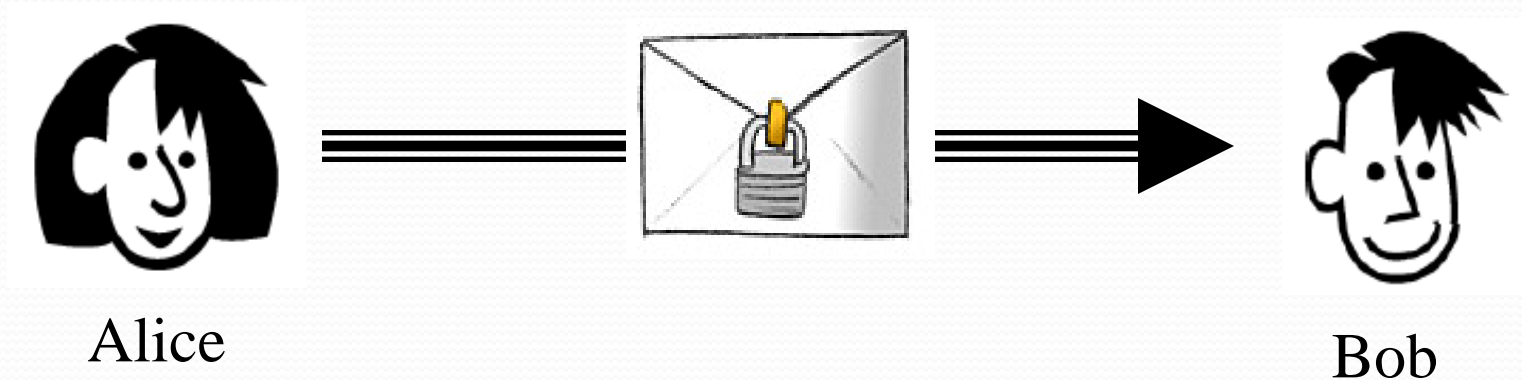
# Quantum Cryptography

# Information security



# Cryptography

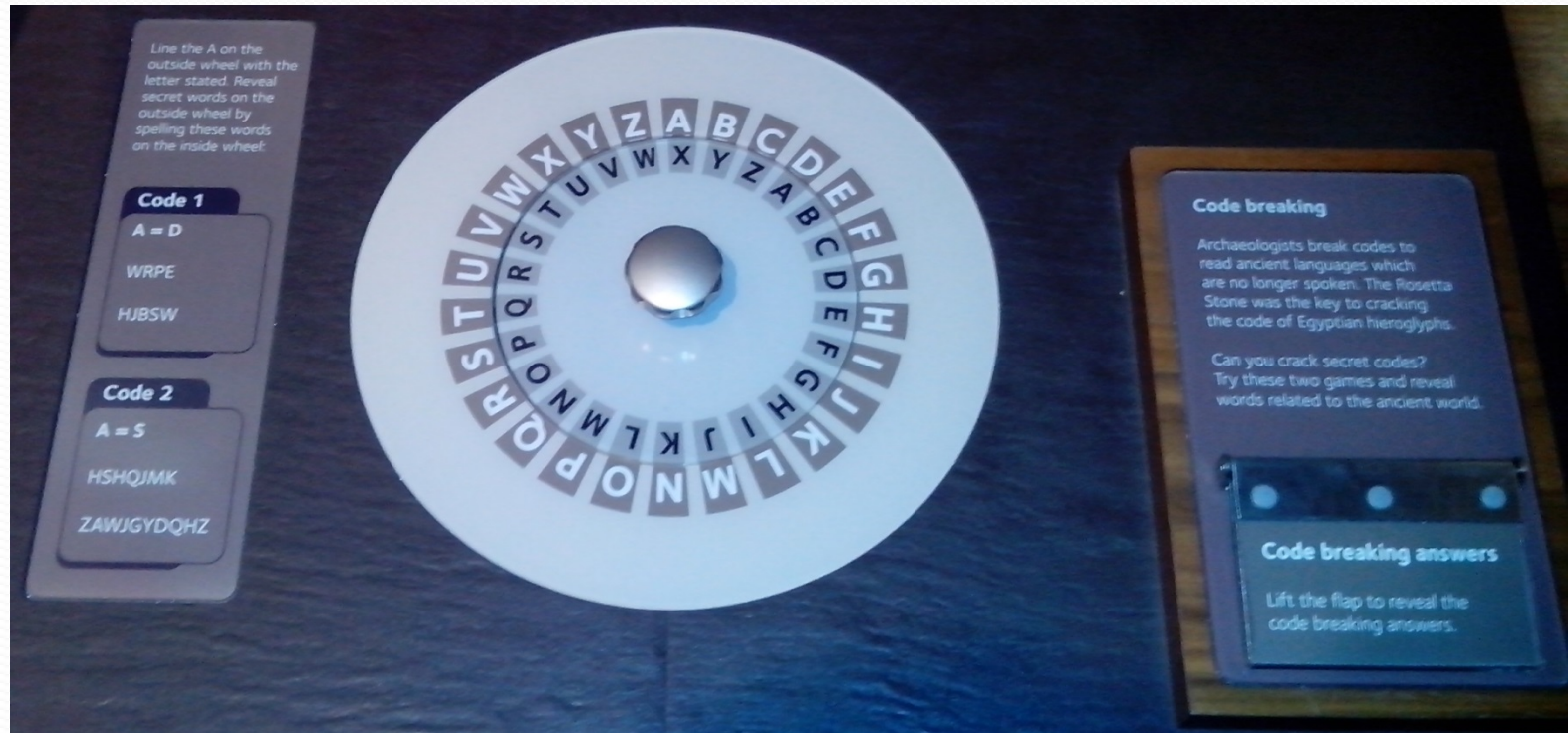
- Cryptography
  - The science of encoding and decoding secret messages.
- Cryptology
  - Public key cryptosystem (such as RSA, Diffie-Hellman)
- Cryptanalysis
  - Hacking





# Cryptography in history

- 2000 BC
- War





# Objective

- Confidentiality
- Integrity
- Authenticity
- Nonrepudiation
- Access Control

# Cryptographic Methods

- Symmetric
  - Same key for encryption and decryption
  - Key distribution problem
  - E.g., DES, AES, IDEA, Blowfish, **OTP**, ...
- Asymmetric
  - Mathematically related key pairs for encryption and decryption
  - Public and private keys
  - E.g., Diffie-Hellman, RSA, ...
- Hybrid
  - Combines strengths of both methods
  - Asymmetric distributes symmetric key, also known as a session key
  - Symmetric provides bulk encryption
  - Example: SSL negotiates a hybrid method

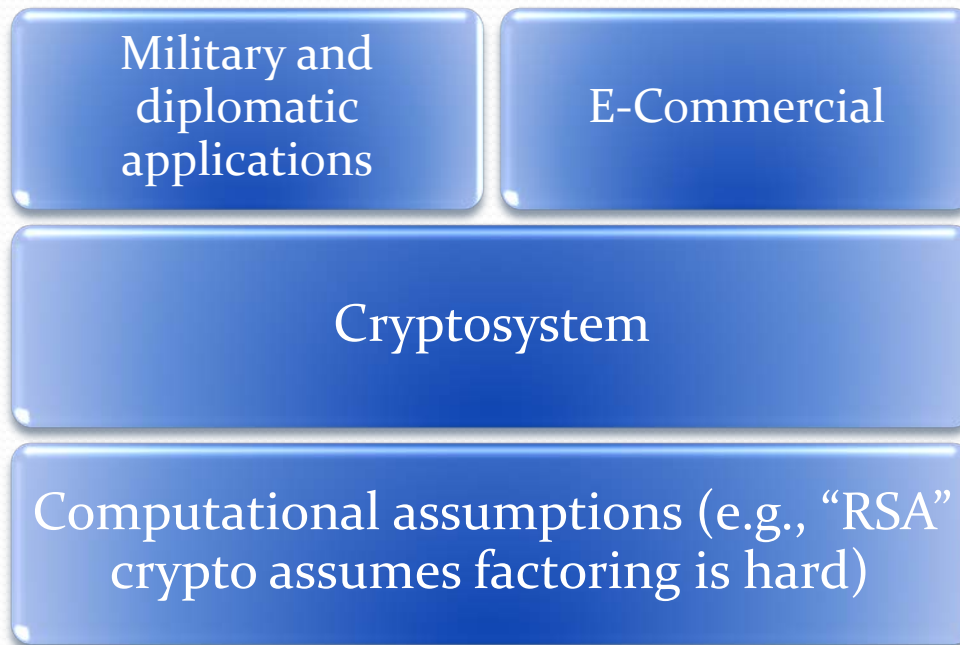
# Cryptanalysis

- The study of methods to break cryptosystems
- Often targeted at obtaining a key
- Attacks may be passive or active
- Kerckhoff's Principle
  - The only secrecy involved with a cryptosystem should be the key
- Cryptosystem Strength
  - How hard is it to determine the secret associated with the system?
- Brute force
  - Trying all key values in the keyspace
- Dictionary Attack
  - Find plaintext based on common words
- Factoring Attacks
  - Find keys through prime factorization
  - **Quantum computer**



# Conventional Cryptography

- Application framework



# What is wrong with conventional cryptography?

- Unanticipated Advances in Hardware and Algorithms.
- Quantum Code-breaking
- “Cryptographers do not sleep well.”
- Cryptography is for the paranoid.
- In history, **every** advance in code-making has been defeated by advances in code-breaking with disastrous consequences to users.

# Unanticipated Advances in Hardware and Algorithms

- Code-breaking of Enigma led to fore-runners of computers!

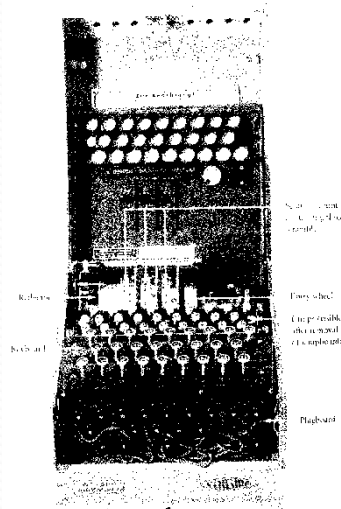


Figure 49 A German Enigma machine in use. The rotor wheels are visible.

German Enigma Machine  
10 million billion possible combinations!  
Looked unbreakable.

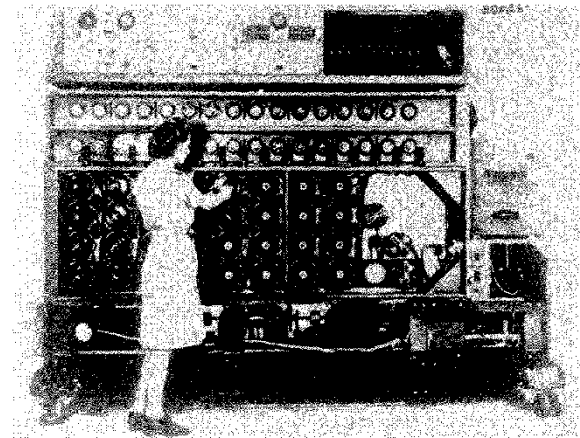


Figure 50 A Bletchley Park bombe in action.

Allied code-breaking machine “bombe”.  
Enigma Broken!

# Quantum Code-breaking

- Shor's algorithm: Factoring is easy with a quantum computer!
- Quantum computing can **efficiently** break:
  - RSA
  - Discrete logarithm problem: Diffie-Hellman key exchange
  - Elliptic-curve cryptographic systems
- “If a quantum computer is ever built, much of conventional Cryptography will fall apart!” (Brassard)



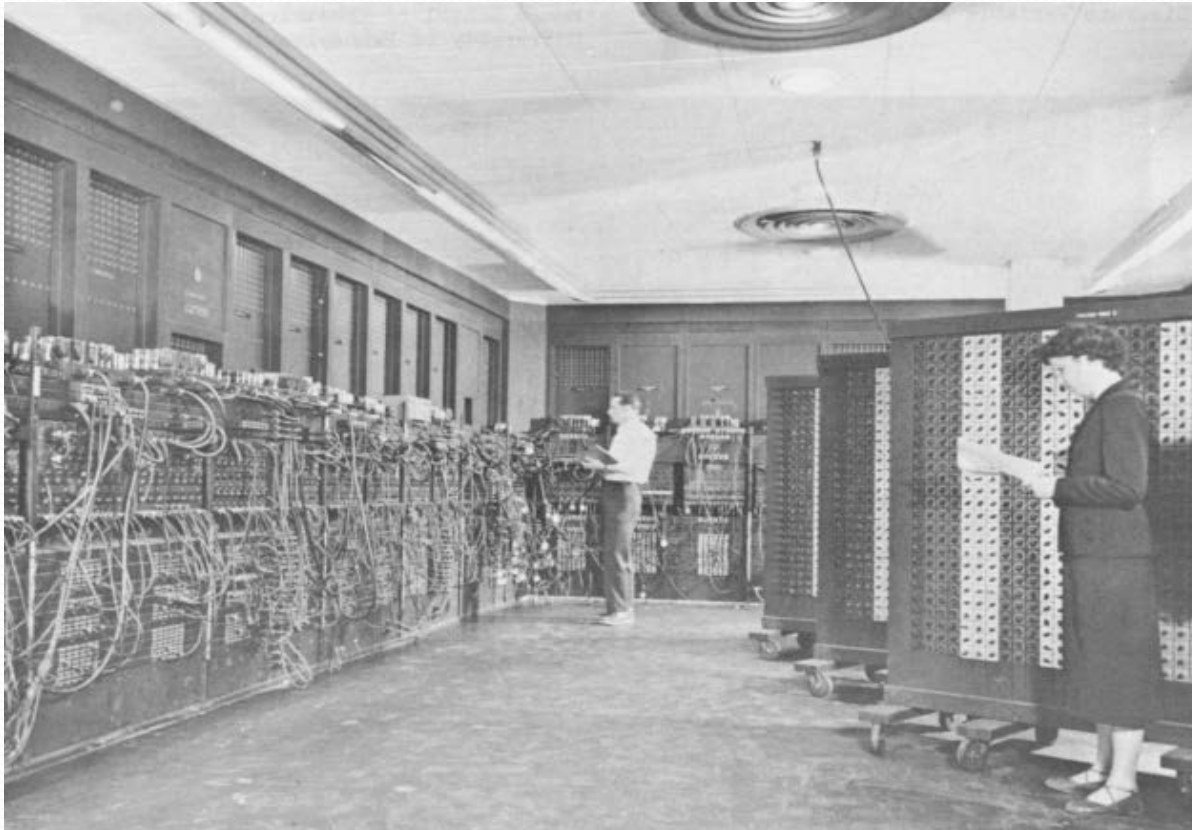


# Forward security?

- Trade secrets and government secrets are kept as secrets for decades (say 70 years).
- A Big Problem RIGHT NOW:
  - If adversary can factor in 2086, she can then decrypt all traffic sent in 2016. (She can save all communications in 2016.)
- Was there any computer 70 years ago?
- What will a computer look like 70 years from now?
- It is ridiculous to think that we know the future 70 years from now.

# ENIAC 1946

- The world's first electronic digital computer



# Cell phone 2016

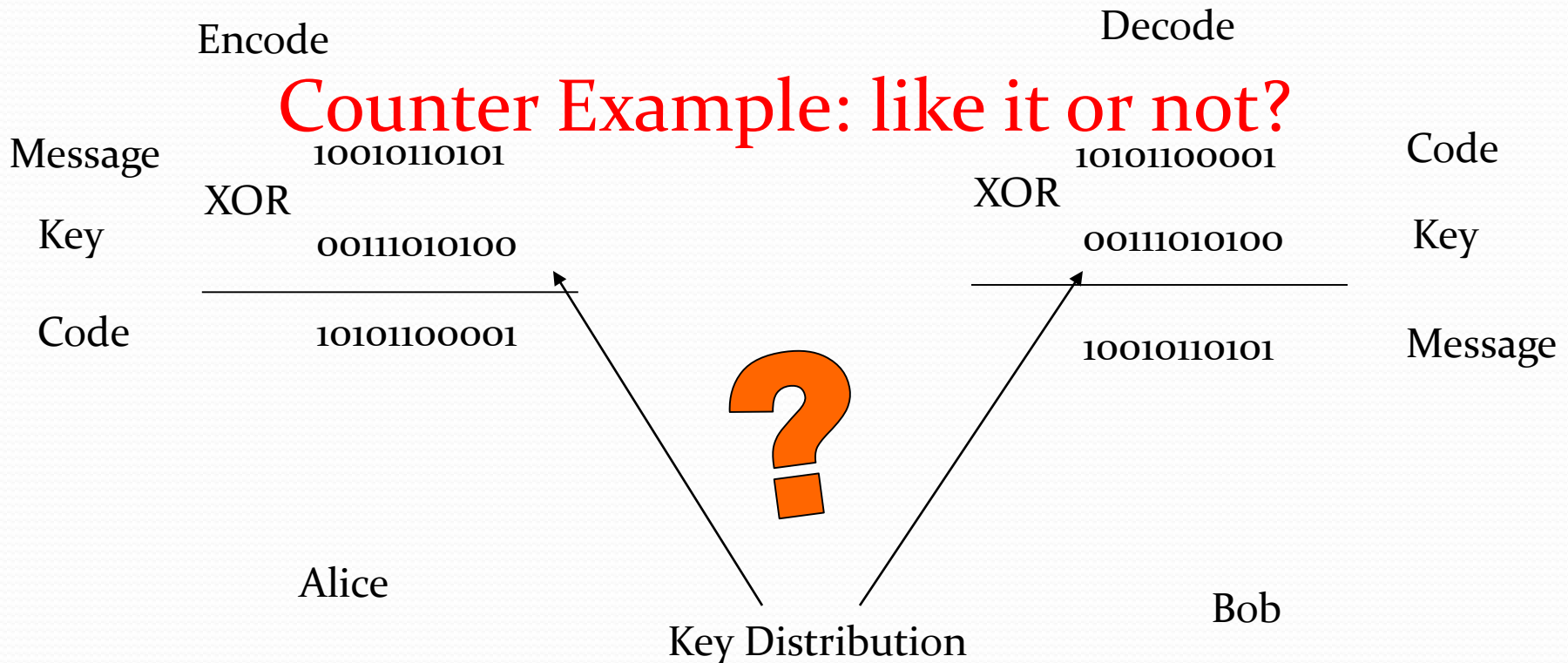


# Computer in 2086



# Private key cryptosystem

- One-time pad (OTP): proven secure by Shannon
- Alice and Bob share two identical keys secretly.

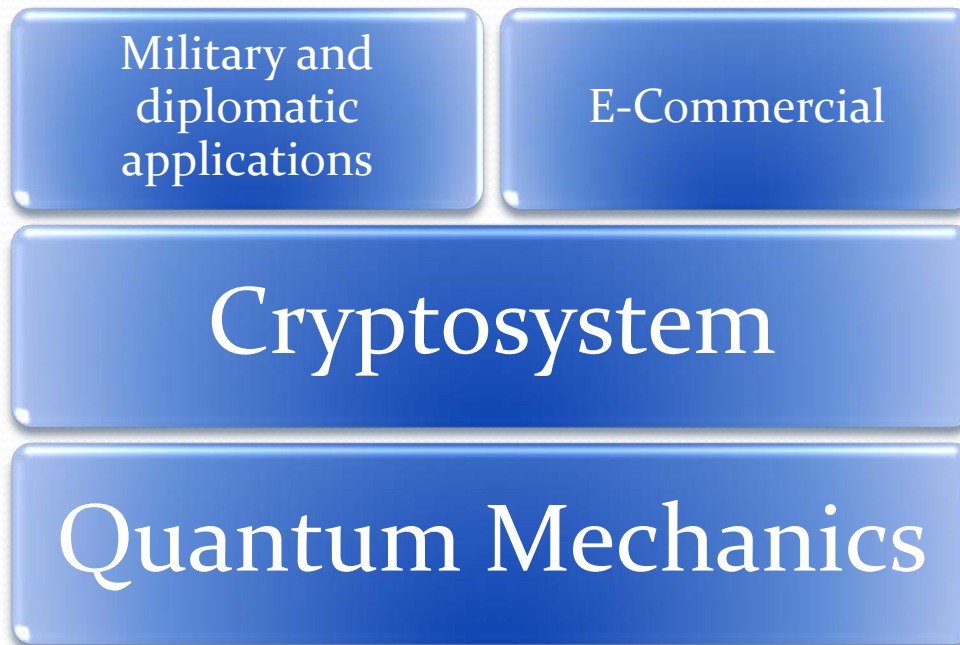


# Cryptographic Key

- Identical
  - Faithful decryption
- Private
  - Secure communication
- At least as long as the message
  - An efficient way to distribute key

# Quantum Cryptography

- Application framework





**Bob**

**15km Fiber**

**CG**

**DG**

**Jr. Alice**

**DA**

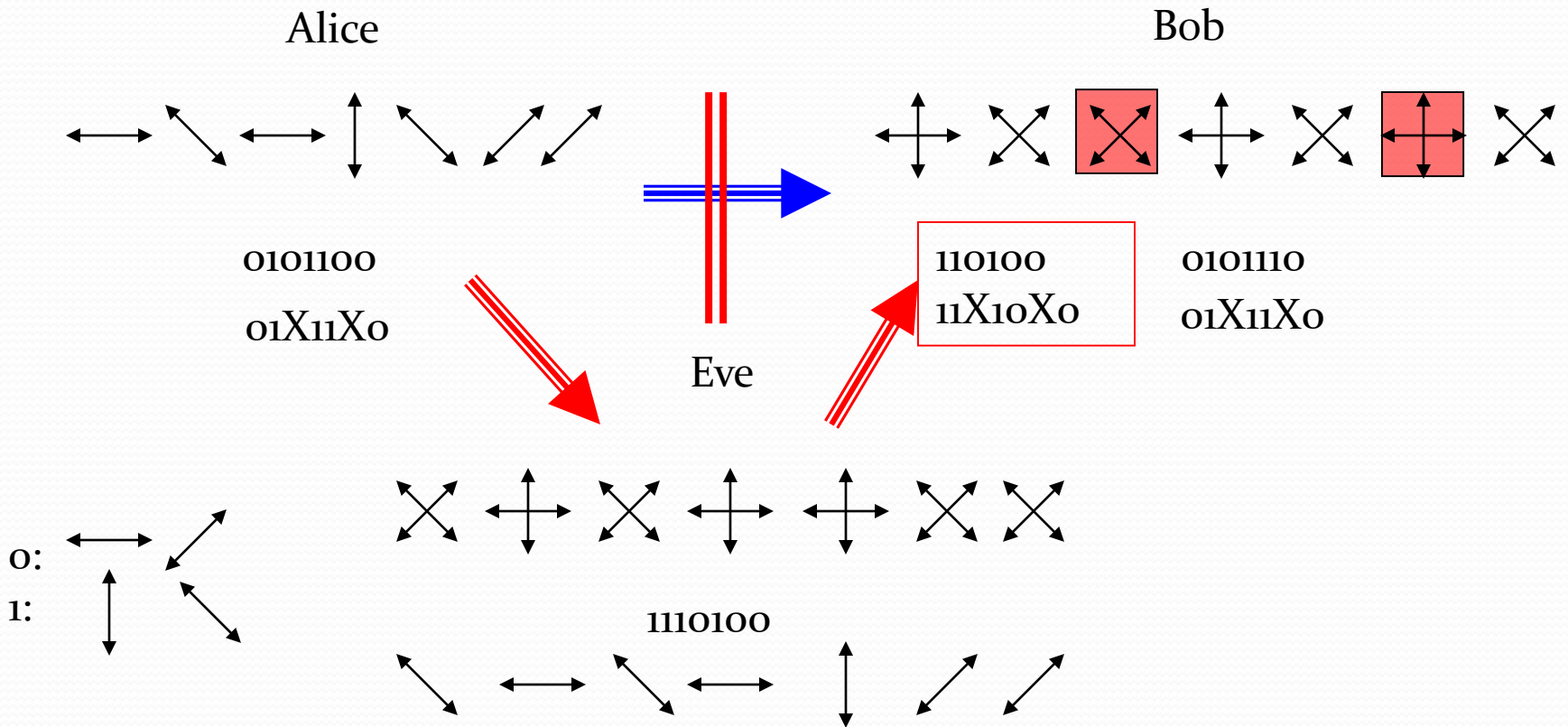
**CA**

**100MHz  
Driver**

Newport  
High Performance  
Laminar Flow Isolator

# Quantum key distribution (QKD)

- BB84 (Bennett & Brassard 1984)



# A few observations

- Random keys are distributed via QKD systems
  - Alice does not send messages directly
  - Users can safely discard keys if they feel the channel is insecure without causing any security problem
  - Secure keys are used in later cryptosystems: composable
- A successful attack by Eve
  - Eve obtains non-trivial amount of information about the final key without Alice and Bob's notice
  - If Alice and Bob do not end up with any secure key, the attack is a failure
- Channel is totally insecure
  - Classical channel: authentication

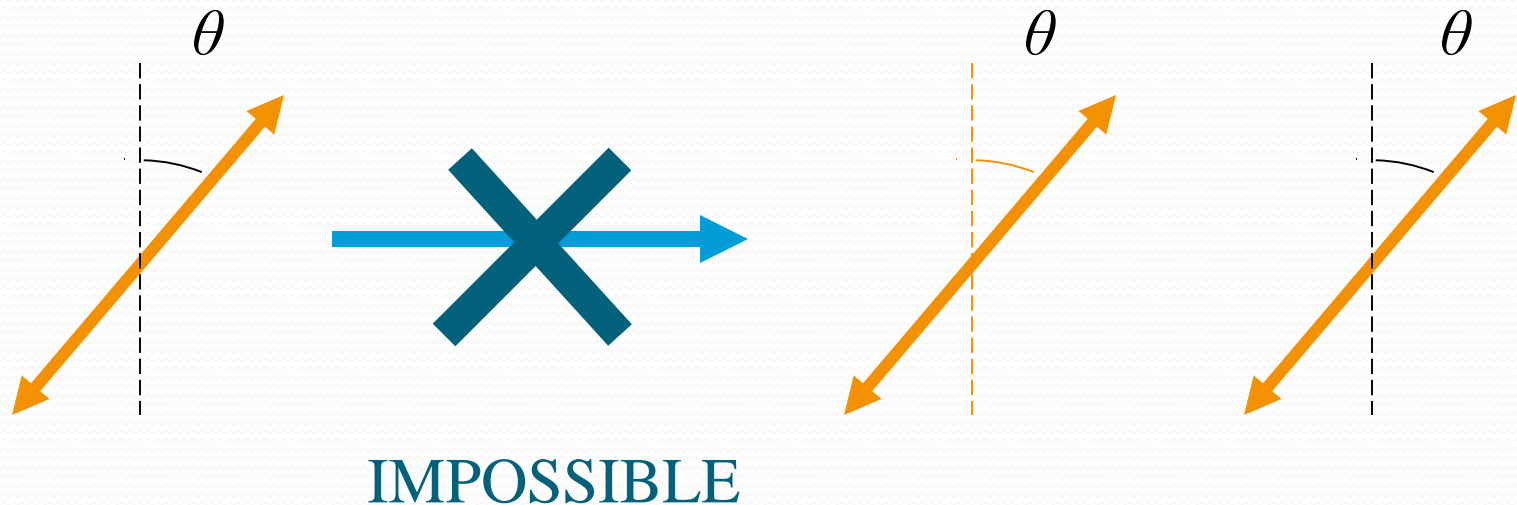


# Summary (take home message)

- QKD systems distribute keys
- QKD does not replace all the current cryptosystems
- QKD does not replace current communication systems
- An ideal private key
  - **Identical**: Alice and Bob share the same key
  - **Private**: Eve has no (or up to trivial amount of) information about the final key
- Post processing
  - Error correction: all classical, e.g., Cascade, LDPC
  - Privacy amplification: shorten the key so that Eve's information is eliminated; the focus of all security proofs

# Quantum No-Cloning Theorem

- The polarization of a single photon cannot be cloned. Therefore, eavesdropper **CANNOT** have the same quantum information that Bob has.







- Information gain means disturbance

# A quick review

- Prepare-and-measure protocols
  - BB84, six-state...
- Entanglement based protocols
  - Ekert91, BBM92
- Unconditional security proof
  - Mayers (1996)
  - Lo and Chau (1999)
  - Shor and Preskill (2000)
  - Devetak and Winter (2003), Renner (2005)
- Security analysis for QKD with imperfect devices
  - E.g. Mayers, Lütkenhaus, ILM
  - Koashi-Preskill
  - Gottesman-Lo-Lütkenhaus-Preskill (GLLP)

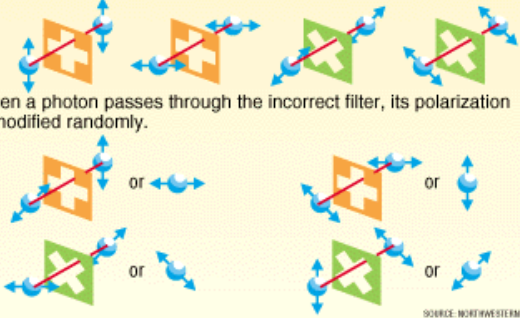
**SINGLE PHOTONS ENCODE BITS**  
*Quantized polarization used to build secure optical nets*

The value of each bit, its polarization for example, is encoded on the property of a photon. The polarization of a photon is the oscillation direction of its electric field. It can be, for example, vertical, horizontal or diagonal (+45° and -45°). Sender and receiver agree that:

'0' =  or   
'1' =  or 

A filter can be used to distinguish between horizontal and vertical photons; another one between diagonal photons (+45° and -45°).  
When a photon passes through the correct filter, its polarization does not change.

When a photon passes through the incorrect filter, its polarization is modified randomly.

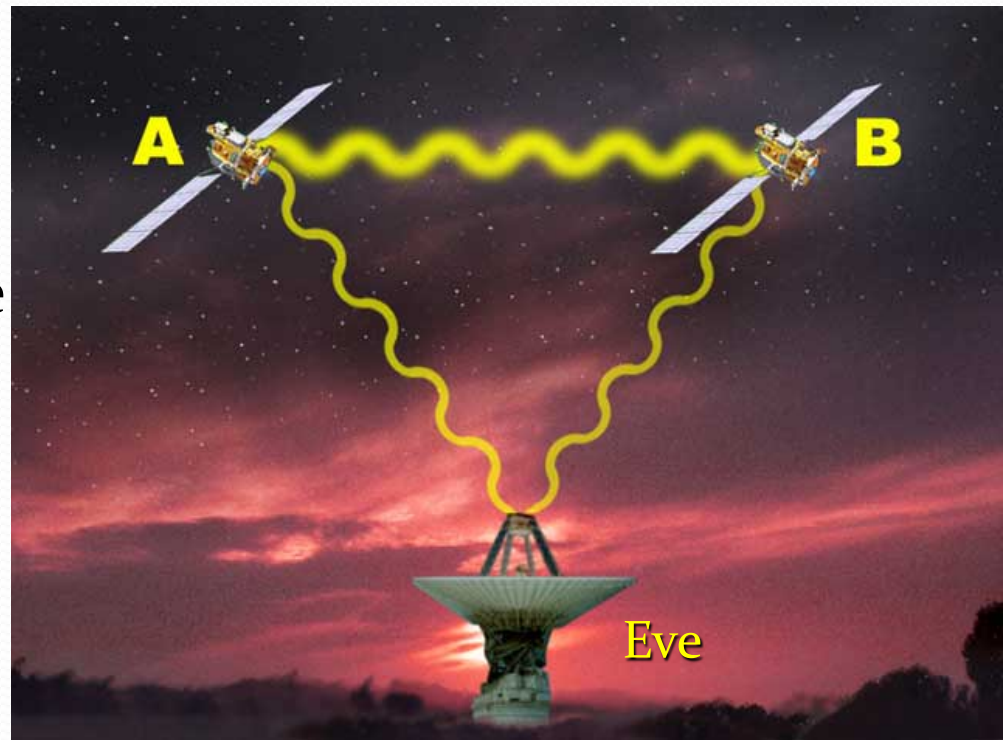


SOURCE: NORTHWESTERN UNIVERSITY

# Entanglement-based protocols

## BBM92

- Alice (or Eve) prepares an EPR pair
  - $|\Psi\rangle_{AB} = |00\rangle + |11\rangle$
  - Alice and Bob each measures one half of the pair
- Z basis measure (bit)
  - $|0\rangle$  or  $|1\rangle$
- X basis measure (phase)
  - $|0\rangle + |1\rangle$  or  $|0\rangle - |1\rangle$





# Lo-Chau security proof I

- Lo and Chau, 1999
- Entanglement distillation
  - Distill perfect EPR pairs from imperfect ones
  - Bell basis:  $|00\rangle + |11\rangle$ ,  $|01\rangle + |10\rangle$ ,  $|00\rangle - |11\rangle$ ,  $|01\rangle - |10\rangle$
  - Objective:  $|00\rangle + |11\rangle$
- Bit errors (Z)
  - $|01\rangle + |10\rangle$
- Phase errors (X)
  - $|00\rangle - |11\rangle$
- Both bit and phase errors (Y)
  - $|01\rangle - |10\rangle$

# Lo-Chau security proof II

- Bit error correction (Z: 0,1)
  - Bit errors:  $|01\rangle + |10\rangle$  and  $|01\rangle - |10\rangle$
  - After bit error correction:  $|00\rangle + |11\rangle$  or  $|00\rangle - |11\rangle$
- Phase error correction (X: +,-)
  - Phase errors:  $|00\rangle - |11\rangle$  or  $|01\rangle - |10\rangle$
  - After phase error correction:  $|00\rangle + |11\rangle$  or  $|01\rangle + |10\rangle$
- Share (almost) **pure** EPR pairs
  - $|00\rangle + |11\rangle$
- Measure in Z basis to get final key
  - Almost perfect privacy (randomness)
- Security: think of teleportation

# Shor-Preskill security proof I

- Shor and Preskill, 2000
- Problem with Lo-Chau proof
  - Requires quantum computers
- Reduce to prepare-and-measure schemes
- Put the final key measurement ahead before error corrections
  - **Commute** operations in quantum mechanics
- Error correction: classical methods
- Privacy amplification: key point of all security proofs

# Shor-Preskill security proof II

- Bit error correction becomes regular error correction
  - Enables Alice and Bob shares **identical** keys
  - $H(\text{bit error rate})$ : Shannon entropy
- Phase error correction becomes privacy amplification
  - Enables Alice and Bob shares **private** keys
  - $H(\text{phase error rate})$ : Shannon entropy
- Final key rate
  - $1 - H(\text{bit error rate}) - H(\text{phase error rate})$
- Symmetry of BB84
  - Between X and Z basis
  - Basis-independent source: bit error rate = phase error rate
- $1 - 2H(\text{QBER})$ 
  - Not optimal: can be enhanced by two-way LOCC



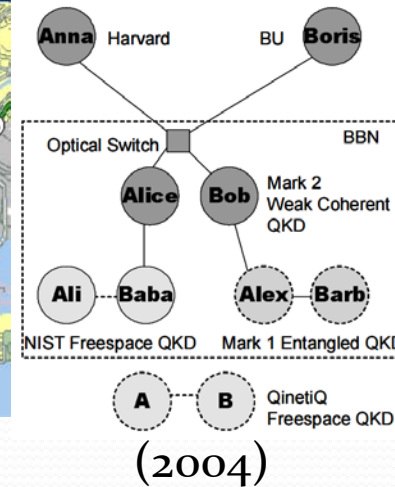
# Some developments in quantum crypto

# USA QKD network

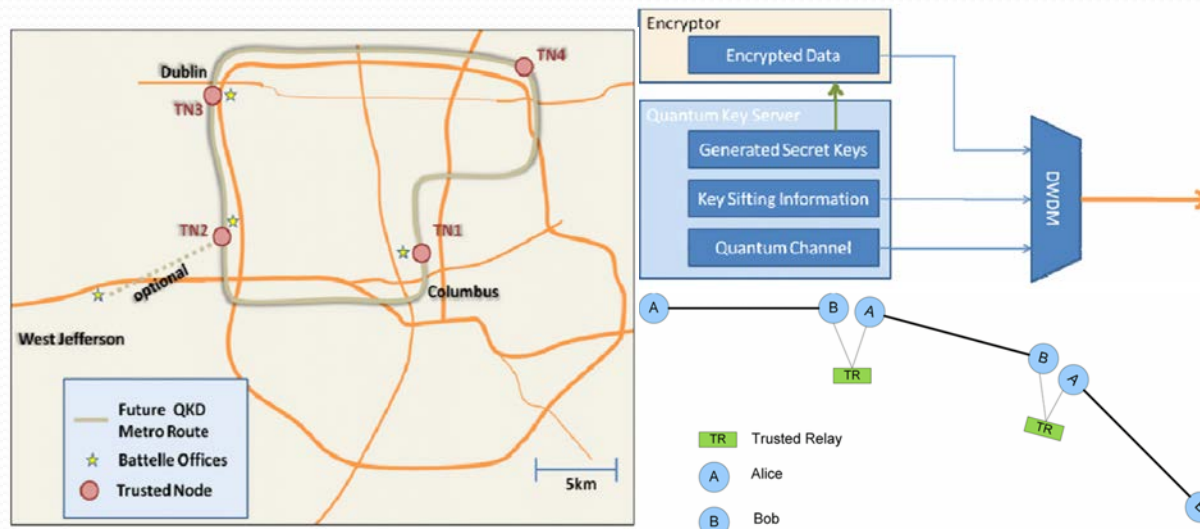
- DARPA
- Battelle
- Smart grid



First field test of QKD network



(2004)



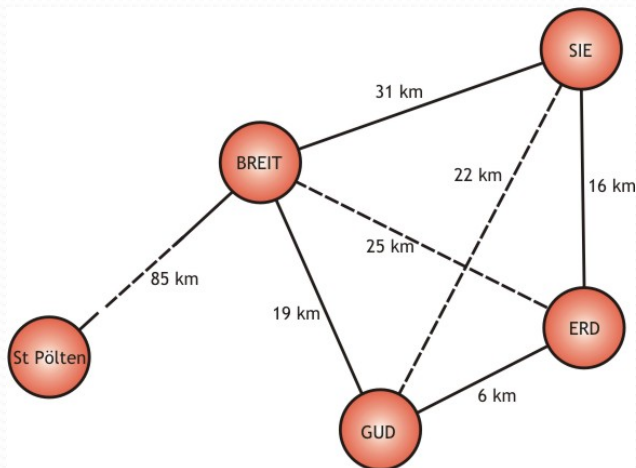




# Europe SECOQC



- Focused on development of a global network for secure communication based on quantum cryptography (2008)





# Swiss election uses quantum cryptography

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## Science & Technology

### Quantum cryptography

## Heisenberg's certainty principle

Oct 18th 2007

From *The Economist* print edition

### The Swiss are using quantum theory to make their election more secure

HANGING chads. Ballot stuffing. Gerrymandering. Such dirty tricks enfeeble democracy. But the security of the votes cast in Geneva during Switzerland's general election on October 21st is guaranteed. The authorities will use quantum cryptography—a way to transmit information that detects eavesdroppers and errors almost immediately—to ensure not only that votes are kept secret but also that they are all counted.

# Switzerland Geneva network

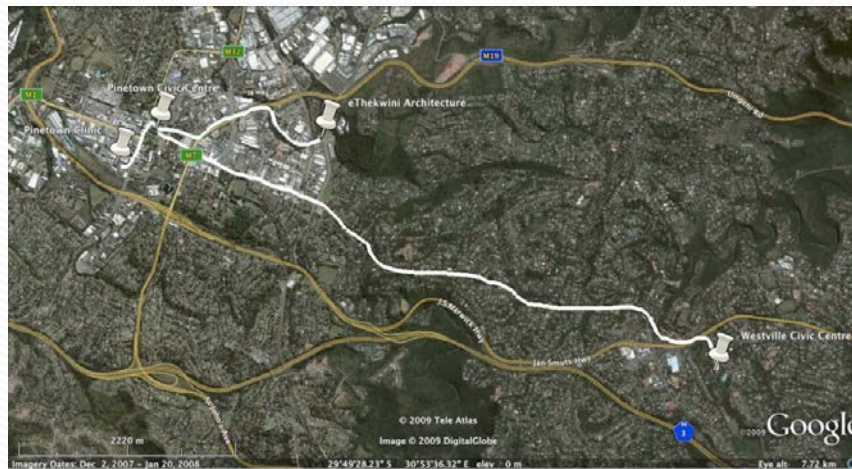
- N. Gisin



(2009)

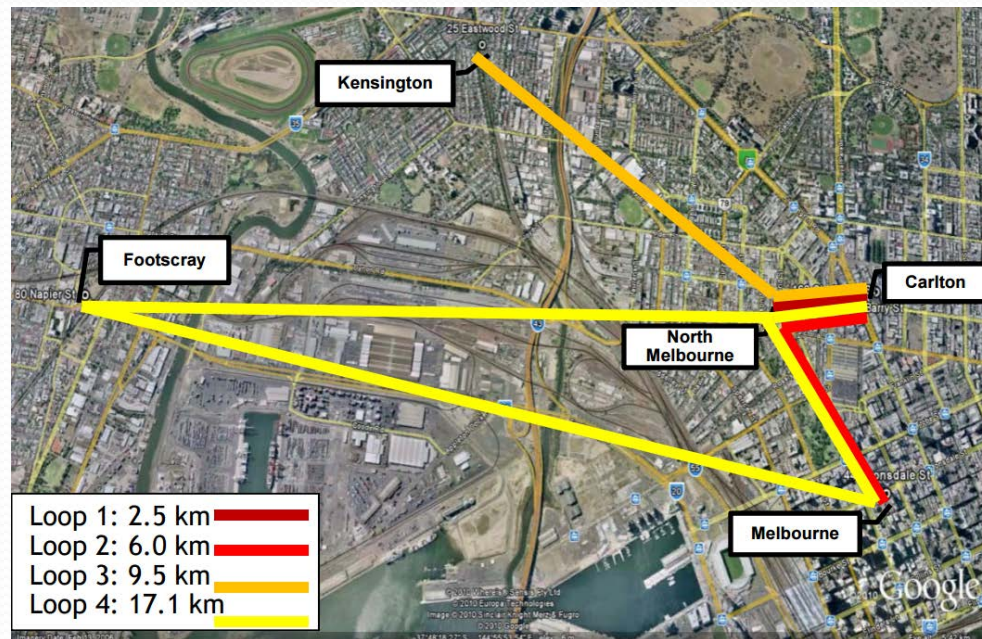
# South Africa Durban Quantum Network

- Distance 2.6 km~27 km
- First Quantum Network in Africa (2009)



# Melbourne Quantum Network

- Melbourne established commercial Quantum Network
- Establish the network between space and ground  
Collaborate with NASA / JPL



(2010)

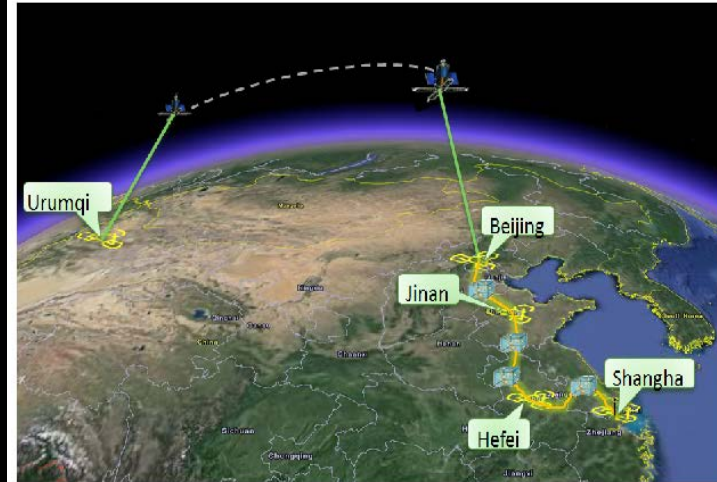
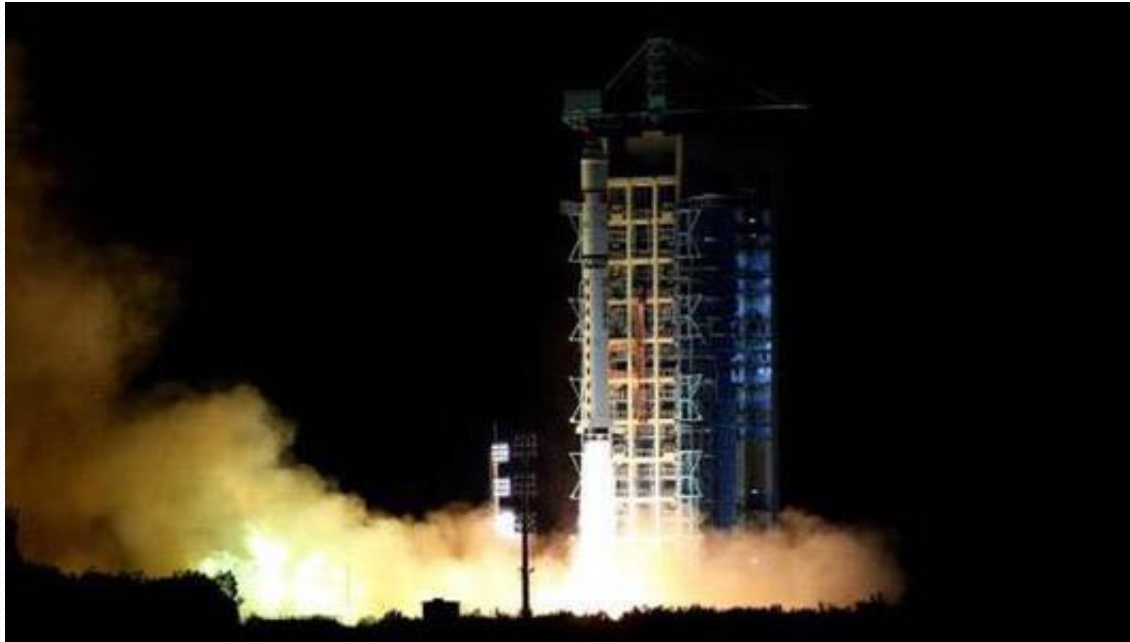
# China's Quantum Secure Backbone project

- Total Length 2000 km
- 2013.6-2016.12
- 32 trustable relay nodes  
31 fiber links
- Metropolitan networks  
Existing: Hefei, Jinan  
New: Beijing, Shanghai
- Customer: China Industrial & Commercial Bank; Xinhua News Agency; China Banking Regulatory Commission ...
- GDP 35.6% (\$3 trillion)
- Population 25.8% (0.3 billion)



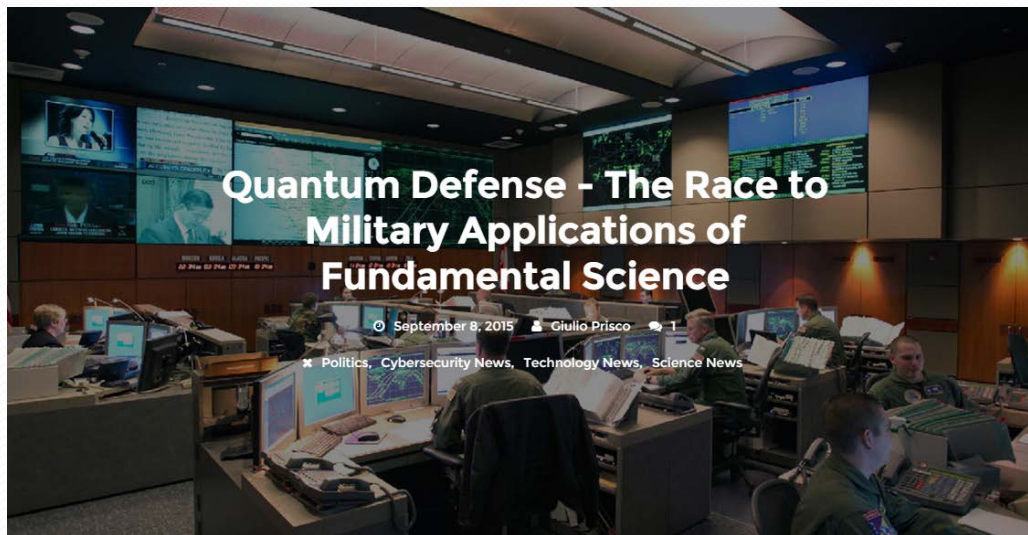


# Quantum Satellite



First quantum satellite, “Mozi”  
1:40am, August 16, 2016

# The race between US and China



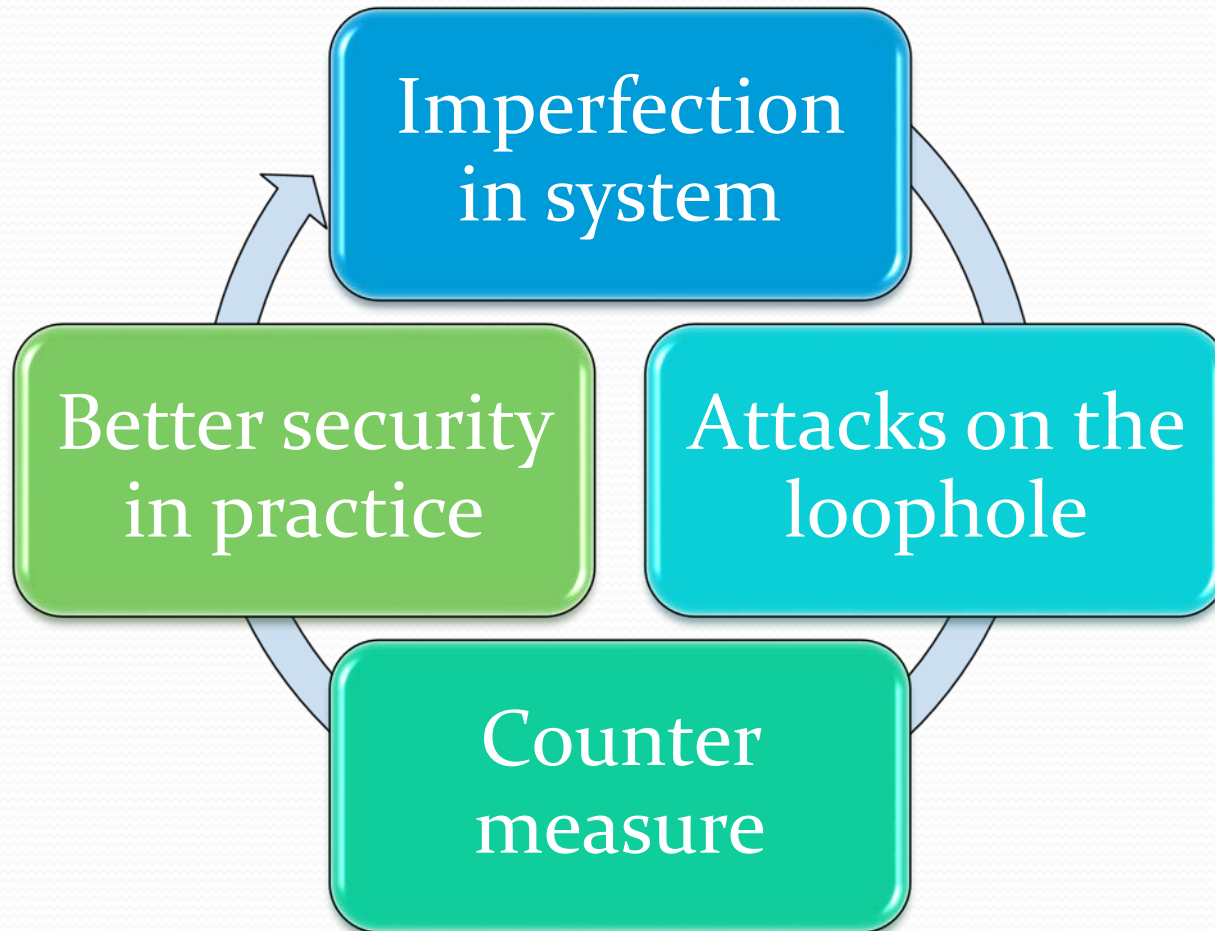
**Unbreakable communication security, and quantum supercomputers much more powerful than today's machines. As usual, the race is between the US and China.**



# Measurement-device-independent QKD (MDIQKD)



# Objective for hacking



# Side information

- Basis (or bit) information may be contained in other degrees of freedom
  - Timing of the pulse
  - Frequency
- Source
  - Multi photons
  - Decoy state
  - Phase randomization
- Loopholes in detection
  - Gated detectors
  - Polarization sensitive
  - Deadtime



*... was unconditionally secure against any eavesdropper who happened to be deaf!*

# QKD scenario

- The channels are not sure, but Alice and Bob's labs are
  - Alice and Bob characterize their devices well and trust them

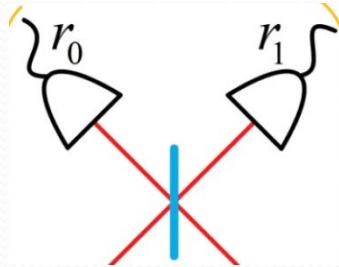


- What if Alice and Bob are not too sure about the devices?
  - For instance, the products are bought from the market and built up by unknown manufacture



# Question from practical side

- Detection efficiency mismatch
  - The numbers of 0's and 1's are different
- Time-shift attack



A typical measurement with two detectors

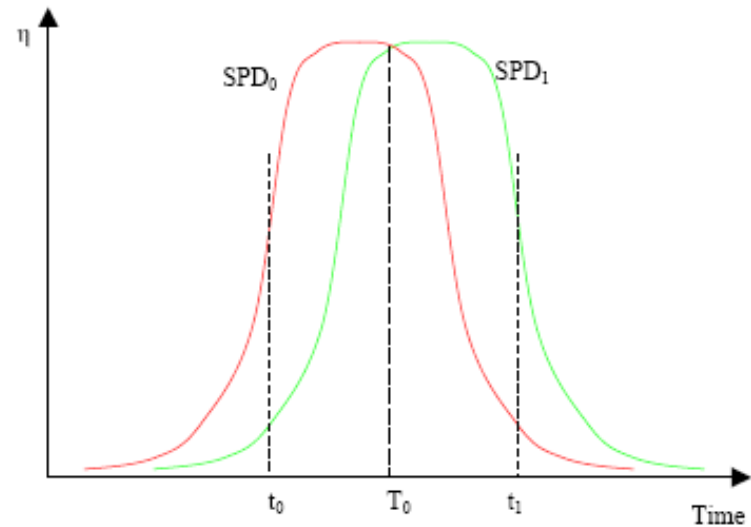


Fig. 1. The time-dependence efficiencies of SPDs.

# Detection device-independent QKD

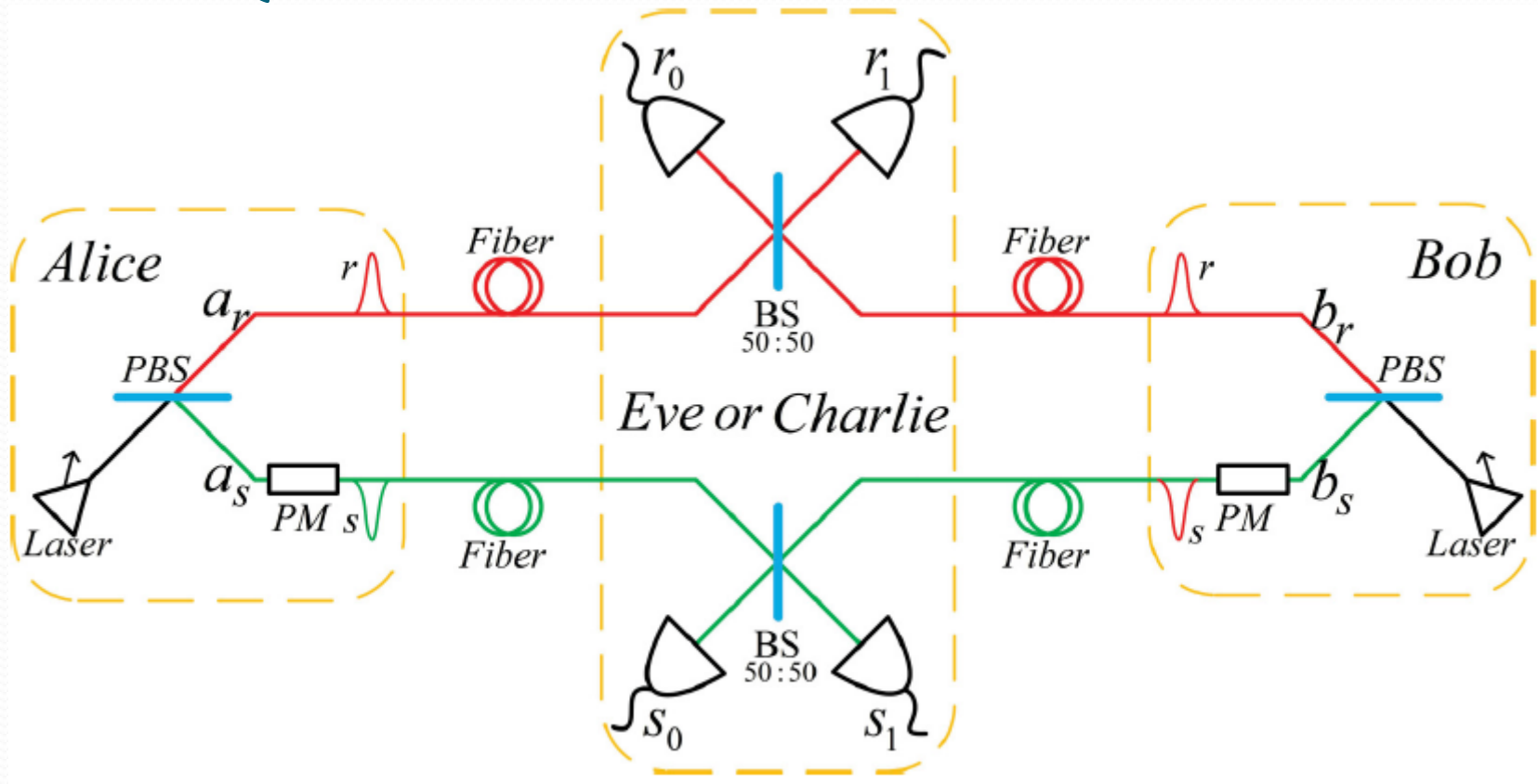
- Random assignment for no clicks
  - Detection efficiency: 65.9%
  - Error tolerance: 11%
- Partially self-testing schemes
- Experimental realization
  - Minimum requirements

X. Ma, T. Moroder and N. Lütkenhaus, arXiv:0812.4301, (2008).  
X. Ma and N. Lütkenhaus, Quant. Info. Comp. 12, 0203 (2012).

# MDIQKD

- Alice randomly chooses bit  $\{0,1\}$  and basis  $\{X, Z\}$  and sends the state to an untrusted party, could be Eve
  - The source is the same as BB84
- Bob does the same
- Eve projects the two qubits into one of four Bell states
  - Bell state measurement (BSM)
- Eve announces the BSM results
- Alice and Bob run postprocessing
- Security relies on a “time-reversed” EPR distribution QKD protocol (BBM92)

# MDIQKD



H.-K. Lo, M. Curty and B. Qi, PRL 108, 130503 (2012).

X. Ma, C.-H. F. Fung and M. Razavi, PRA 86, 052305 (2012)

X. Ma, and M. Razavi, PRA 86, 062319 (2012)

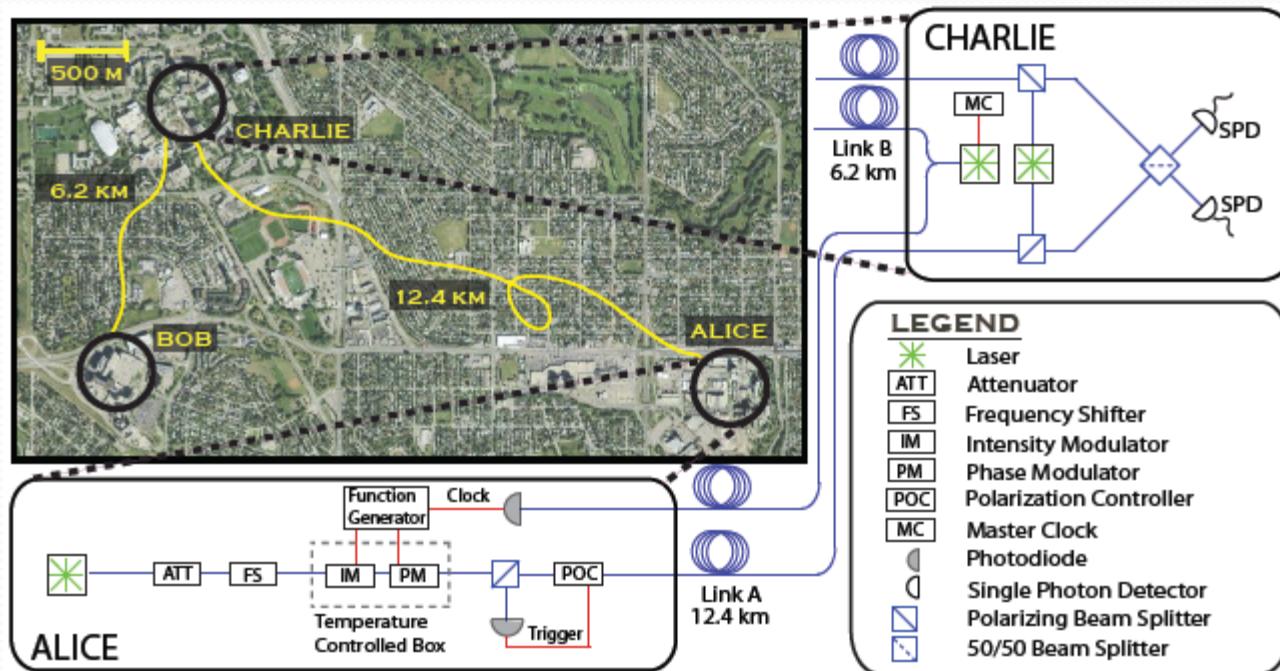


# Key advantages

- Measurement device independent
  - The measurement devices are assumed to be held by an untrusted party
- Two quantum channels
  - Like entanglement based protocol, the effects of background counts can be reduced

# Field test

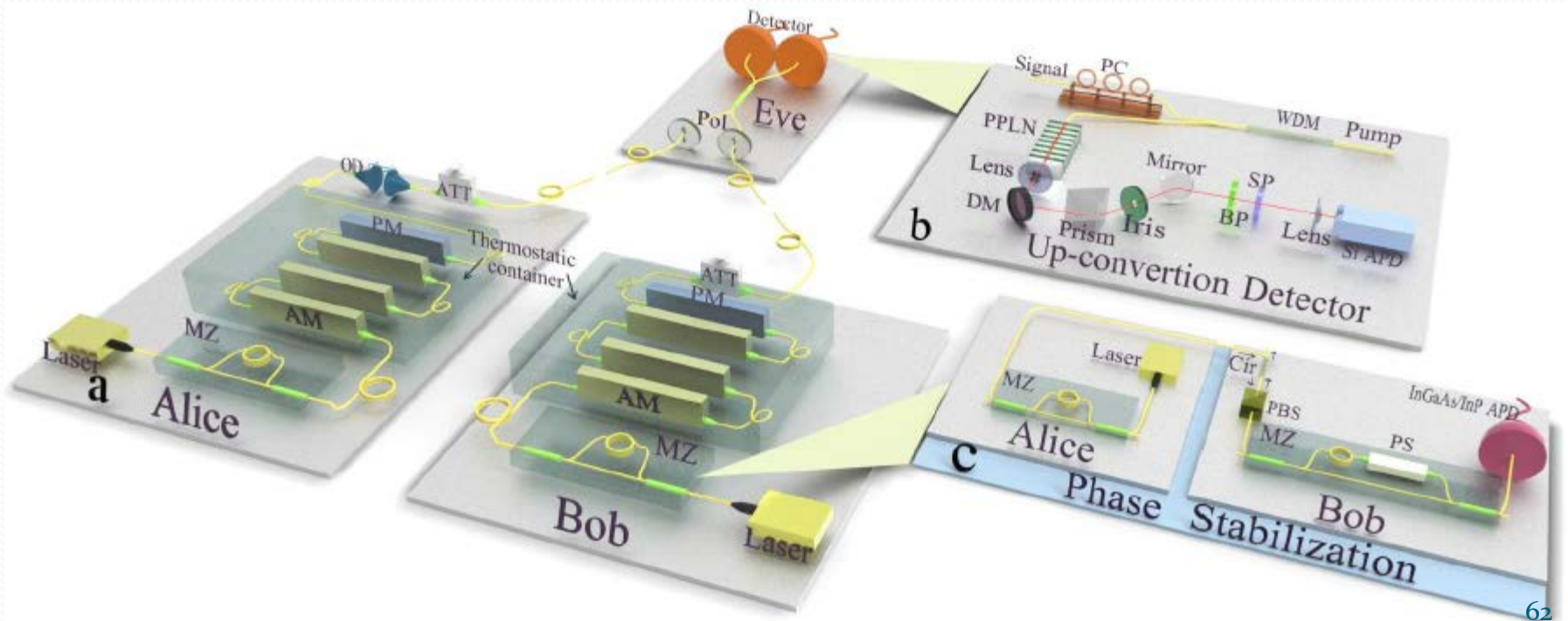
- Calgary: W. Tittel's group
  - Rubenok et al., Phys. Rev. Lett. 111, 130501 (2013).



# Implementation

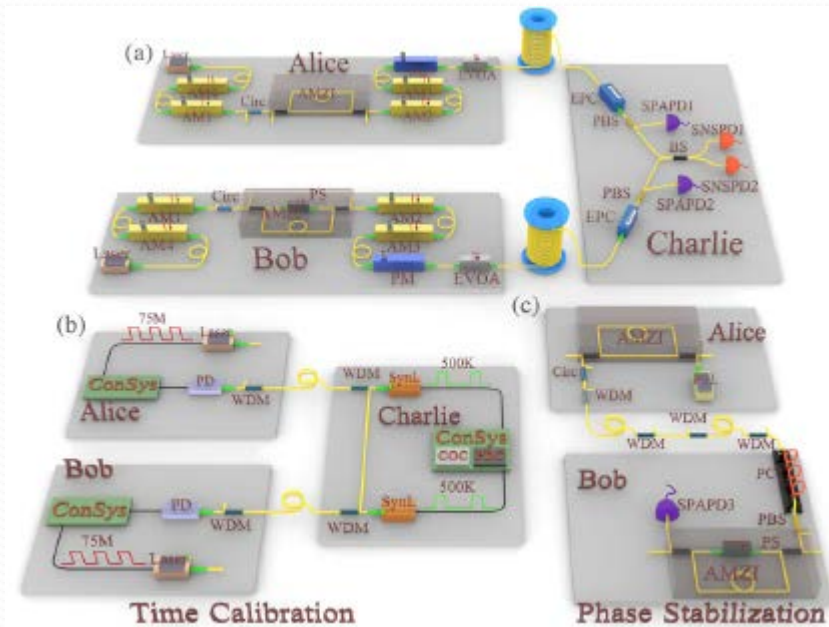
APS Highlights of the Year 2013

- Shanghai
  - Liu et al., Phys. Rev. Lett. 111, 130502 (2013)
  - 60 hour experiment: 25 kbit secure key over a 50 km



# MDIQKD over 200km fiber

- Y.-L. Tang, et al., “Measurement-Device-Independent Quantum Key Distribution over 200 km”, PRL 113, 190501 (2014)



# Full field test of MDIQKD

- Y.-L. Tang, et al., “Field Test of Measurement-Device-Independent Quantum Key Distribution”, IEEE Journal of Selected Topics in Quantum Electronics, 21, 1, 1077-260X (2014)





# Untrusted relay network

- Detection devices are normally much more expensive than sources
  - Users only need to purchase cheap source devices
  - All users hold the same source devices
- Memories can be added to the relay to enhance the performance
  - C. Panayi, M. Razavi, X. Ma, N. Lütkenhaus, *New Journal of Physics* 16 (2014) 043005
  - S. Abruzzo, H. Kampermann, D. Bruß, *Phys. Rev. A* 89, 012301 (2014)
- Implemented for regular QKD
  - Fröhlich, et al., *Nature* 501, 69-72 (2013)
- Measurement-Device-Independent Quantum Key Distribution over Untrustful Metropolitan Network
  - Yan-Lin Tang, et al., *Phys. Rev. X* 6, 011024 (2016)

