Few- and Many-body Physics near Feshbach Resonance:

Taming Multiple Scatterings in A Unitary Gas

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At IASTU, Beijing, Sept, 2011

Thank: Canadian Institute for Advanced Research
Isszak W. Killam Fund for Advanced Studies
and Institute for Advanced Studies, TsingHua University

2010
West Vancouver
Cypress Snowboarding
2010
Downtown Vancouver
BC Place Opening/Closing
GM Place Ice Hockey
2010
Vancouver & UBC
False Creek Athletes' Quarters
Pacific Colleseum Short Track Speed Skating
Nat Bailey Stadium Curling
UBC Thunderbird Rink Secondary Ice Hockey
2010
Richmond
Richmond Olym. Oval Speed Skating
to Lions' Bay, Whistler
W E S T V A N C O U V E R
Capilano Watershed
Capilano Lake
Lynn Watershed
Mt. Seymour Prov. Park
D I S T R I C T O F N O R T H V A N C O U V E R
Capilano Park
Fish Hatchery
Canyon Heights
Mt Royal
Cypress Rd
Upper Levels Hwy
Upper Delbrook
East Braemar Rd
British Properties
Capilano
North Lonsdale
Queens Rd
29th St
Eagle Harbour
Caulfield
Sandy Cove
Westmount
Dundarave
Marine Dr
Hollyburn
Lighthouse Park
Ambleside
Ambleside Park
Pemberton
Norgate
13th St
Keith Rd
Lynn Valley
Lynn Valley
Deep Cove
Parkway
Mt Seymour Pkwy
Dollarton Hwy
Dollarton
Lions Gate
Lonsdale
English Bay
N. VANCOUVER CITY
Stanley Park
Burrard Inlet
Burrard Inlet
Second Narrows (Ironworkers' Mem.) Bridge
DOWNTOWN
Waterfront
West End
McGill
Powell
Hastings
PNE
Burnaby Heights
Capitol Hill
Burnaby Mountain
V A N C O U V E R
B U R N A B Y
Kitsilano
Chinatown
Prior
Parker St
Lochdale
Museum of Anthropology
NW Marine Dr
Jericho Park
Cornwall
4th Ave
Terminal
Grandview Woodlands
1st Ave
Brentwood
Broadway
5th Av
UBC
10th Ave
12th Ave
16th Ave
16th Av
Broadway
Broadway
Lougheed Hwy
Grandview Hwy
Trans Canada
Canada Way
BCIT
Winston St
Pacific Spirit Park
SW Marine Dr
Shaughnessy
Mt Pleasant
Kingsway
King Edward Ave
22nd Av
29th Ave
Kensington
33rd Ave
33rd Ave
Heritage Village
Deer Lake
Moscrop
Q.E. Park
Oakridge
Collingwood
Buckingham Heights
Deer Lake
41st Ave
41st Ave
41st Ave
Musqueam
Kerrisdale
49th Ave
49th Ave
Southlands
Langara
Imperial
Oakland
Middlegate
Iona Beach
54th Ave
Metrotown
Royal Oak
57th Ave
57th Ave
Champlain Heights
Rumble
70th Av
Fraserview
SE Marine Dr
SW Marine Dr
Fraser River North Arm
Marine Way
North Fraser
Marpole
Sea Island
Bridgeport
Bridgeport Rd
Vancouver Int'l Airport (YVR)
Cambie
Cranberry Farms
Queensborough
South Terminal

Cold-Atom Theory Research in Vancouver

Hyperfine Spin Correlated Physics in Quan. Gases (2003-)

--- Snoek(UvA), Demler (Harvard), Affleck, Semenoff (UBC), Hui Zhai (IASTU), Junliang Song (Innsbruck), Andrew Ji, W. M. Liu. (IoP, CAS)…

Unitary Gas (2006-)

--- Warner, Zhengcheng Gu (KITP), Alex Huang(UBC), Xiaoling Cui (IASTU), Junliang Song, David MacNeill(Cornell), Sizhong Zhang (OSU), Mohammad Mashayekhi, Dmitri Borzov (UBC)

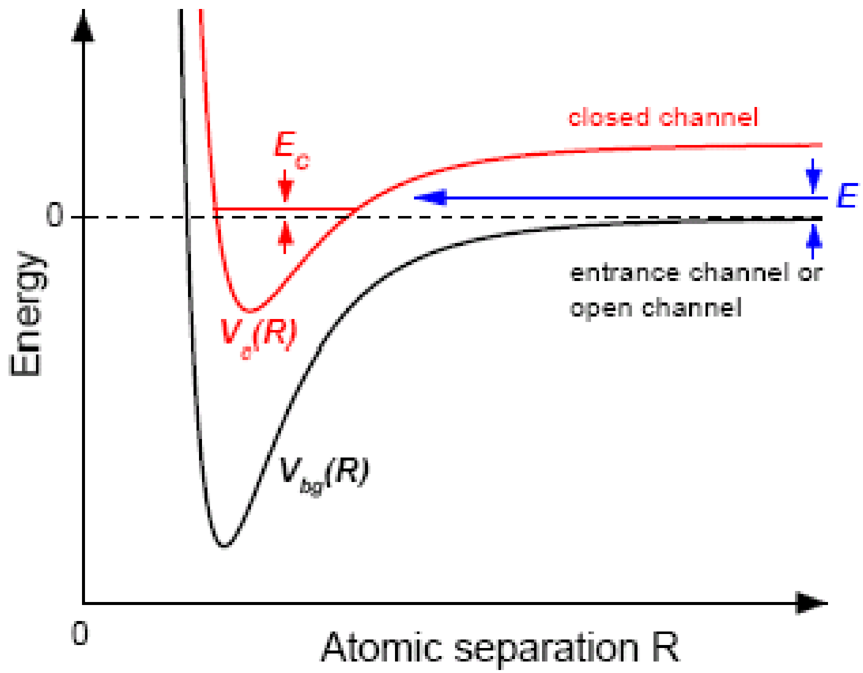
“Spinful” Quantum Gases

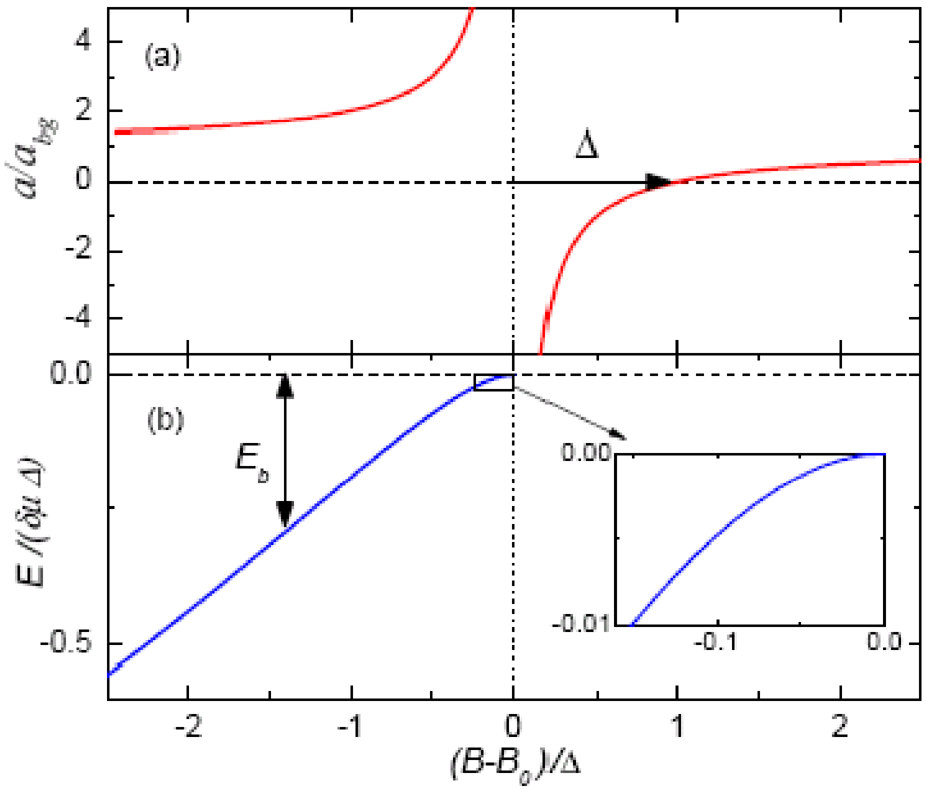
---- Discrete symmetries; fractionalized vortices and abliean/non-Abliean topological excitations in spinor gases; dynamic generation in rotating traps.

---- Field-theoretical models for spin nematic (Z_2-uniaxial, Dih4-biaxial and T-tetrahedral, etc) condensates and Non-MF spin correlated condensates; low-D systems;

---- Quantum-fluctuation-induced order from disorder quantum phase transitions, and spin dynamics;

Atomic Feshbach Resonances



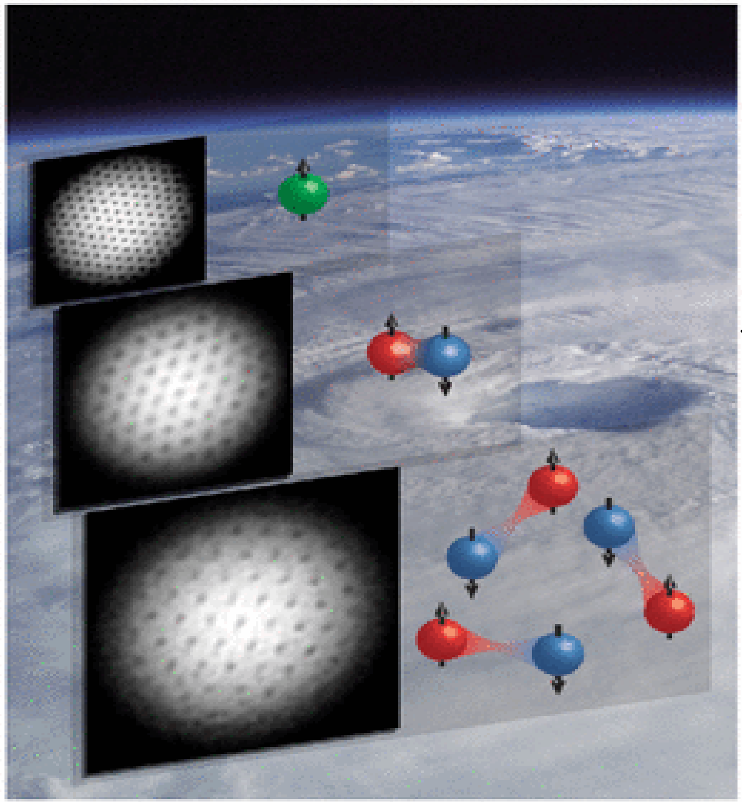


detuning

BEC

Unitary Gas

BCS



$a > 0$

$$\frac{1}{k_F a} \in (-1,+1)$$

$a < 0$

FIGURE 3-5 Vortices in gases. Shown are a vortex pattern in bosonic sodium atoms (green cartoon) in a magnetic trap; vortices in tightly bound lithium molecules (red-blue cartoon); and a vortex lattice in loosely bound fermion pairs on the BCS side of a Feshbach resonance. The background shows a classical vortex (Hurricane Isabel in summer 2003). SOURCE: Wolfgang Ketterle, Massachusetts Institute of Technology.

Outline

Lecture I: Phase shifts in Feshbach resonances: Fundamentals

Phase shifts, scattering lengths and bound states;

Feshbach resonance versus potential scattering;

Lecture II: Resonance Scatterings as a critical phenomenon

RGE approach to resonance: two-body and three-body running coupling constants; limit cycle behavior;

RGE approach to confinement induced resonance and optical lattices.

Lecture III: Few-body phenomena in a quantum gas

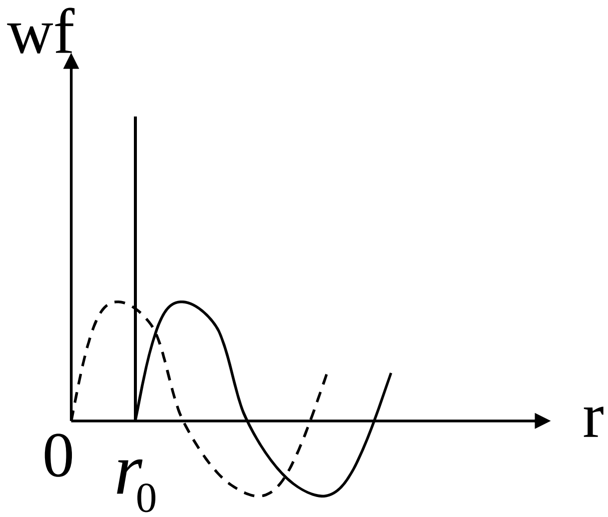
Many-body effects on few-body scattering phase shifts; on few-body structures. Dressed few-body clusters.

Lecture IV: Taming Divergence in the Dilute Gas Theory

Nearly fermionized bosons versus Efimov physics;

A diagrammatic approach recently developed in Vancouver to regularize the theory beyond LYH limit.

S-wave Phase shifts and scattering amplitude



$$\psi_0 = \frac{\sin kr}{r},$$

$$\psi_{hs} = \frac{\sin(kr+\delta)}{r}, \delta = -kr_0$$

$$\psi_{out} \propto e^{ikz} + \frac{f(k)}{r} e^{ikr} = (\frac{e^{ikr} - e^{-ikr}}{2kr}) + \frac{f(k)}{r} e^{ikr}.$$

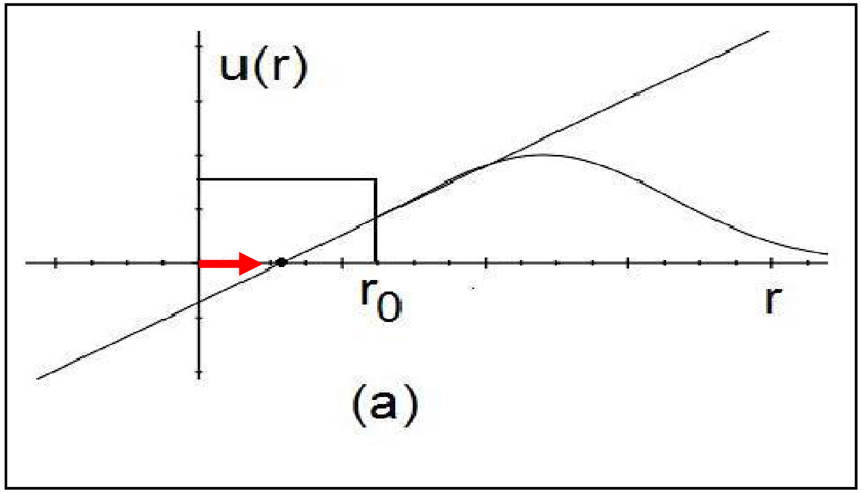
$$f(k) = \frac{e^{i2\delta} - 1}{2ik} = \sin\delta \quad \frac{e^{i\delta}}{k}$$

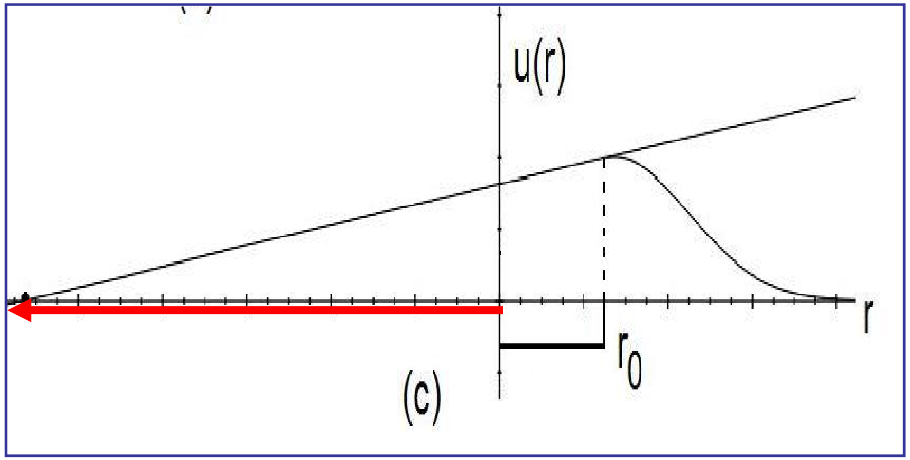
Phase shifts and scattering lengths: derivatives of the wf outside a potential

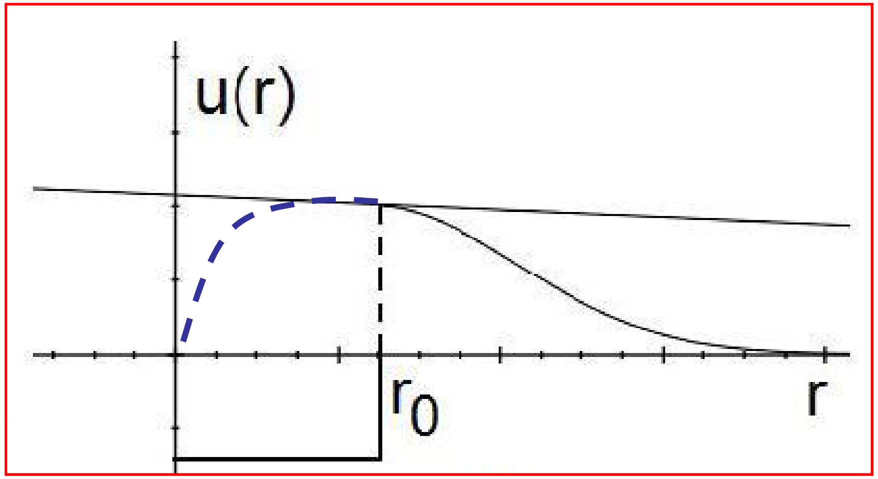
$$a = -\lim_{k\to 0} \frac{\tan\delta(k)}{k}.$$

$$\psi = \frac{\sin(kr+\delta(k))}{r}, \rightarrow \frac{(r\psi)'}{r\psi} = k \lim_{r=0} \frac{\cos(kr+\delta)}{\sin(kr+\delta)} = -\frac{1}{a}$$

Phase shifts, Scattering lengths and bound states







Scatter length diverges when the phase shift becomes pi/2 when a bound state appears.

Scattering length versus amplitude

$$f(k) = \frac{e^{i2\delta} - 1}{2ik} = \sin\delta \; \frac{e^{i\delta}}{k}$$

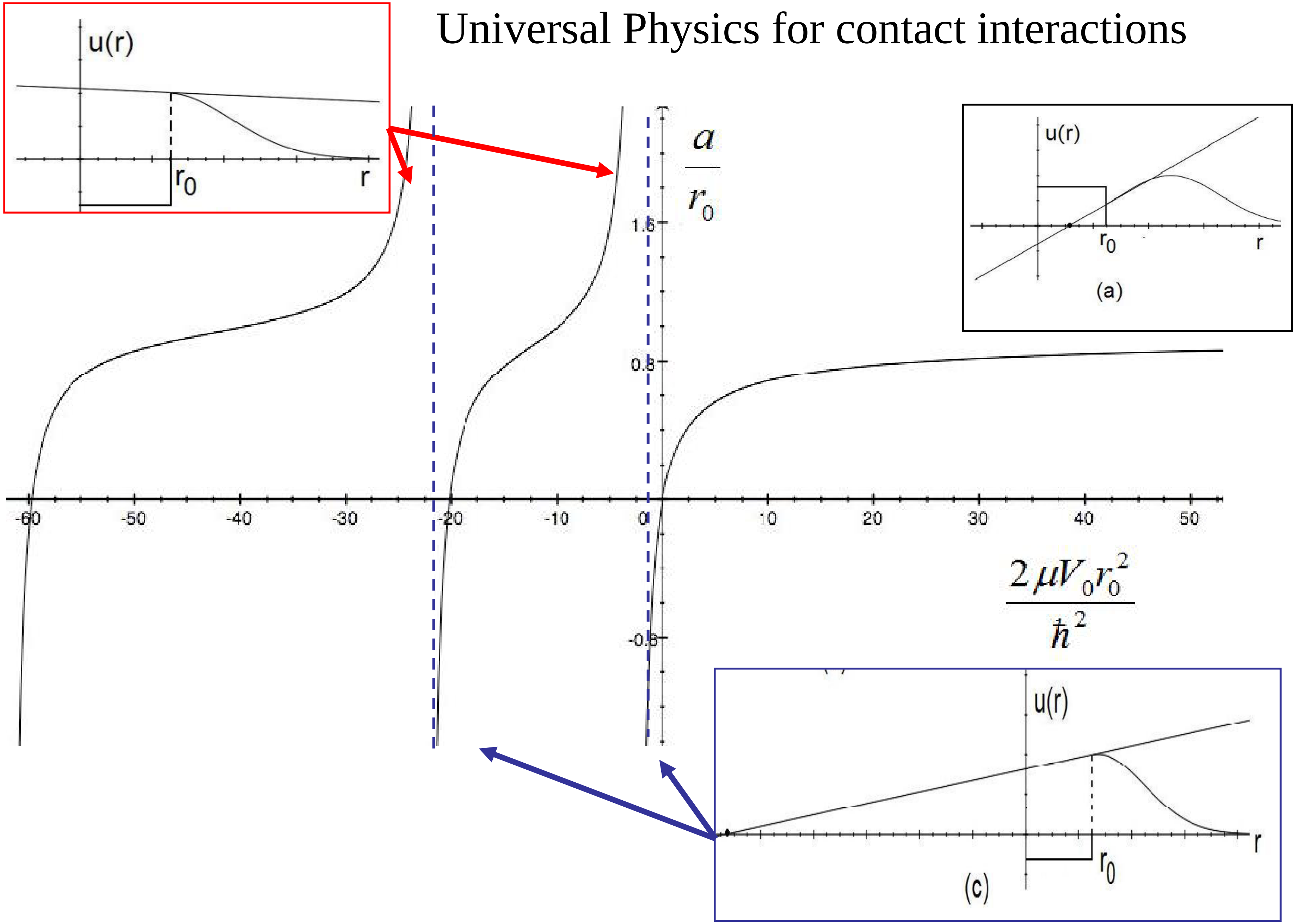
$$f(k) = | f | e^{i\delta(k)} \Leftrightarrow \frac{1}{\frac{1}{a} + ik - C\frac{r_0}{2}k^2}.$$

Scattering cross-section area

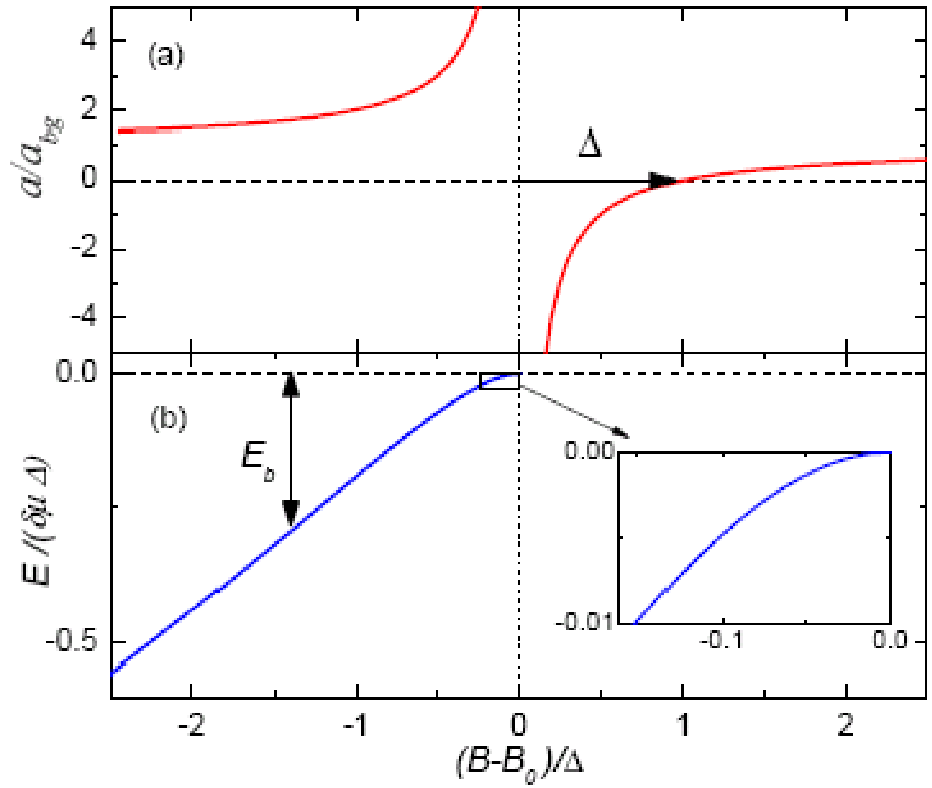
$$\tan\delta(k \to 0) \approx -ka, \quad or\; a = -\lim_{k\to 0}\frac{\tan\delta(k)}{k}.$$

$$\sigma(r_0 = 0) = 4\pi\frac{\sin^2\delta(k)}{k^2} = 4\pi\frac{a^2}{1+k^2a^2}.$$

Universal Physics for contact interactions



Binding energy versus scattering length

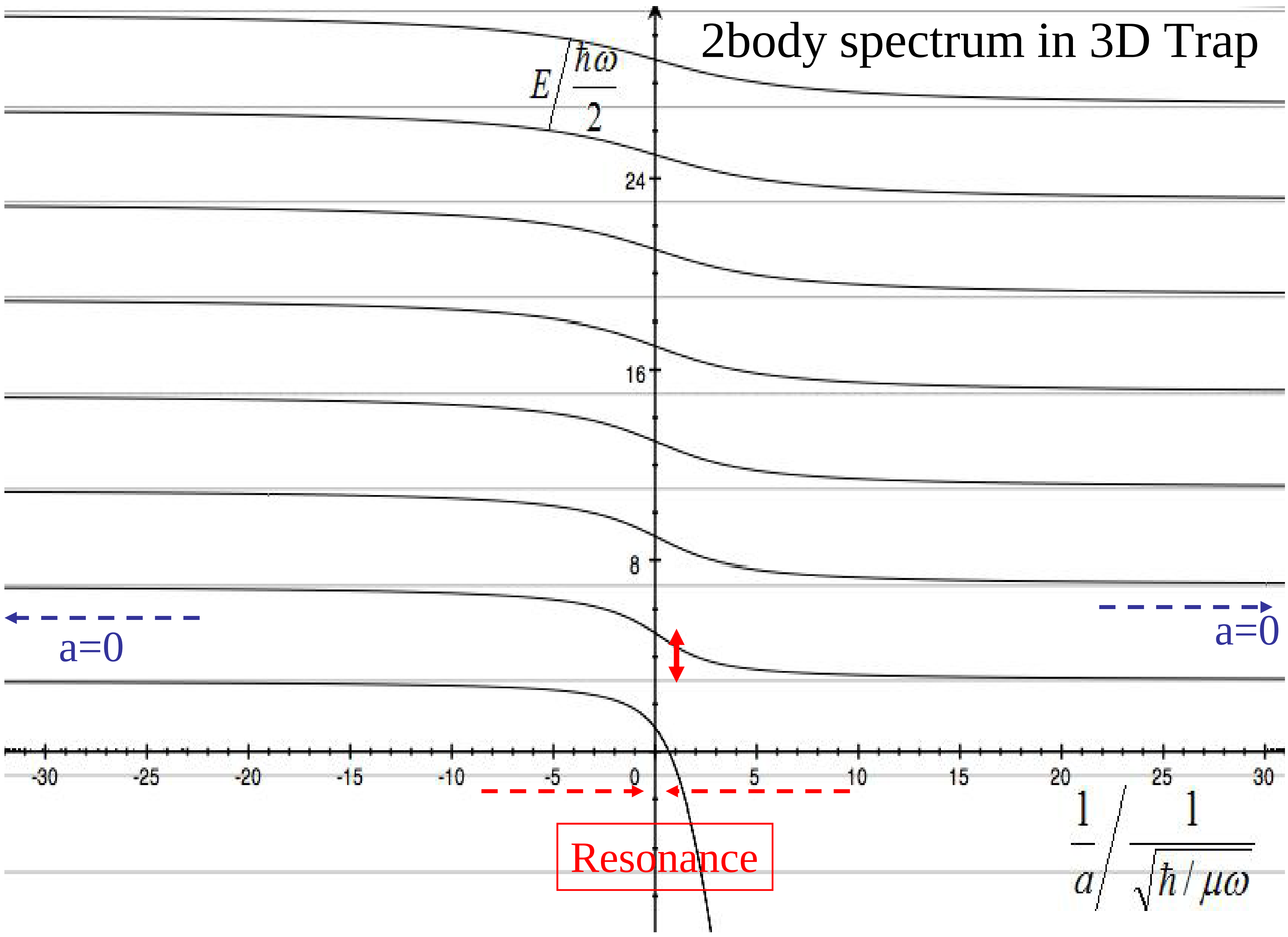


$$E_b = -\frac{1}{2\mu a^2} = -\frac{1}{ma^2}, \quad \mu = \frac{m}{2}.$$

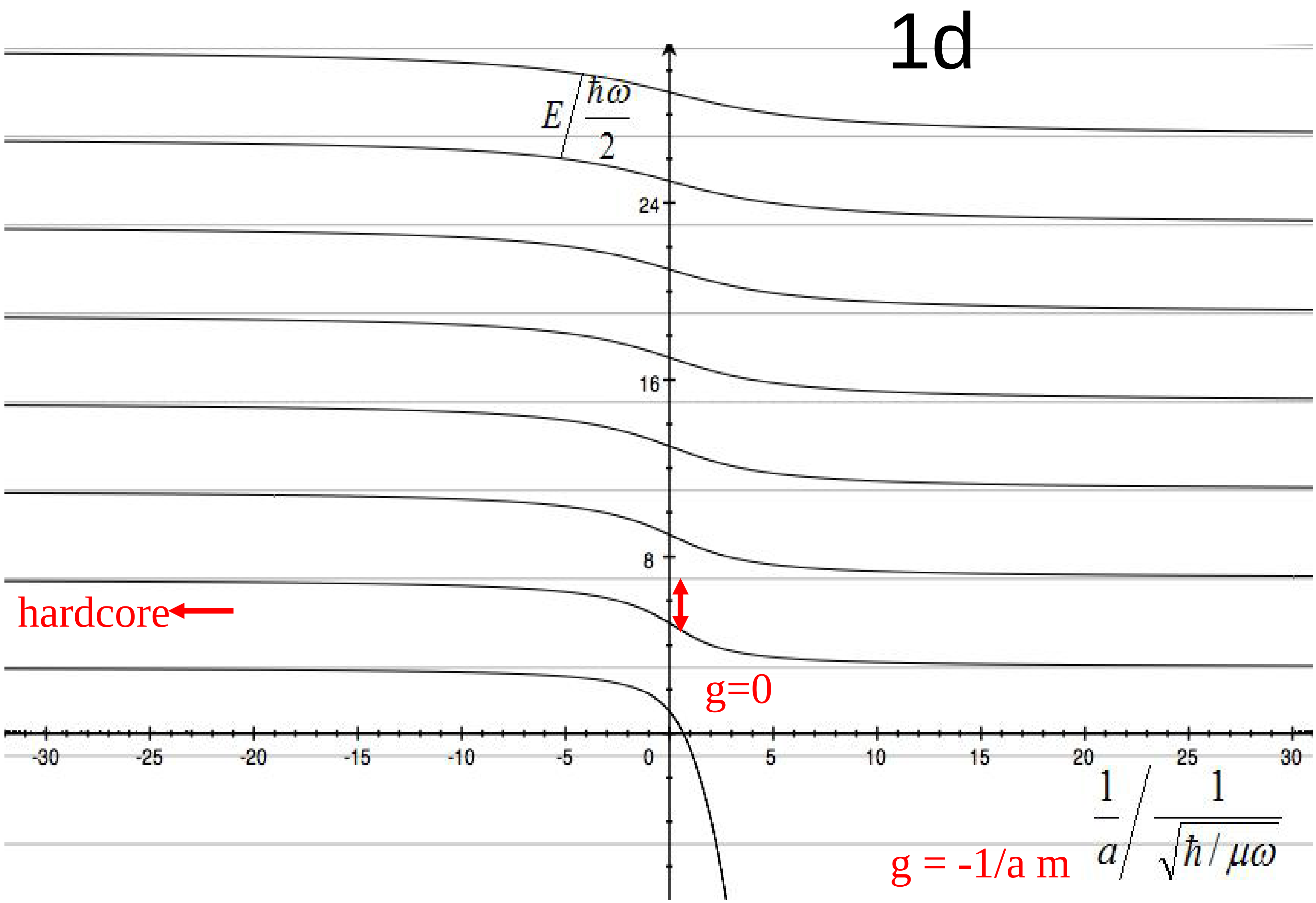
Scaling dimension of interaction energy

versus phase shift/scattering length

(Alex Huang thesis at UBC, 2008)



2body spectrum in 3D Trap
$E \Big/ \frac{\hbar\omega}{2}$
24
16
8
a=0
a=0
-30
-25
-20
-15
-10
-5
0
5
10
15
20
25
30
Resonance
$\frac{1}{a} \Big/ \frac{1}{\sqrt{\hbar/\mu\omega}}$

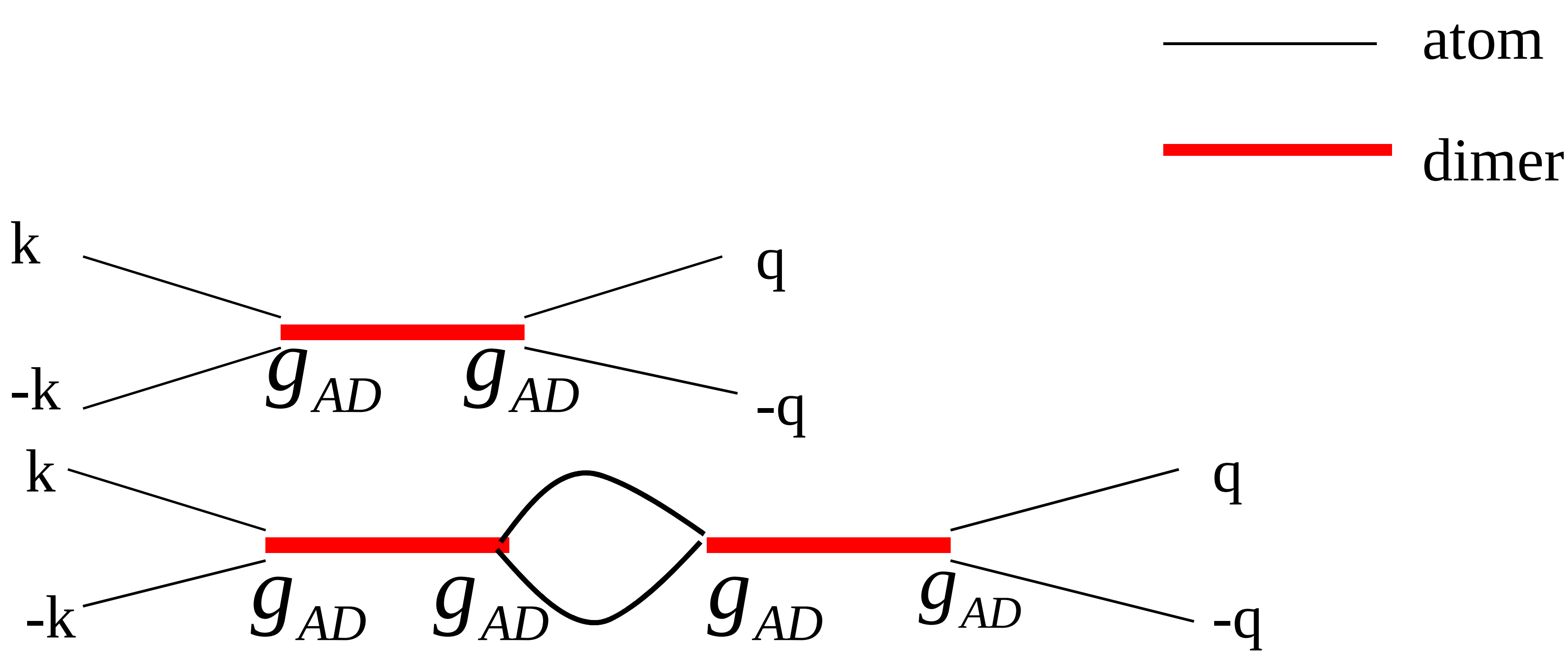

1d
E / ħω/2
24
16
8
hardcore
g=0
-30
-25
-20
-15
-10
-5
0
5
10
15
20
25
30
1/a / 1/√(ħ/μω)
g = -1/a m

Life time of Feshbach molecules

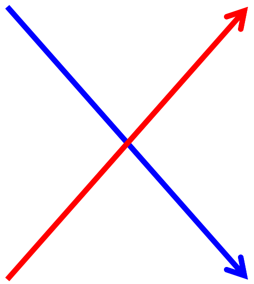
$$G^0(\omega) = \frac{1}{\omega - \delta_T + i\varepsilon} \rightarrow$$

$$G^D(\omega) = \frac{1}{\omega - (\delta_T - \frac{m g_{AD}^2 \Lambda^*}{2\pi^2}) + i m^{3/2} g_{AD}^2 \frac{\omega^{1/2}}{2\pi}}$$

Feshbach Resonance—a diagrammatic S-matrix approach
(Dyson series of S-matrix)



Effective interaction



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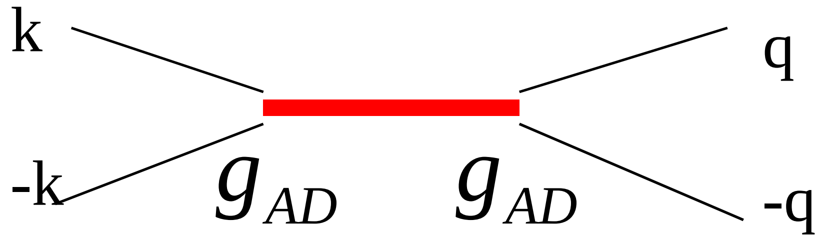
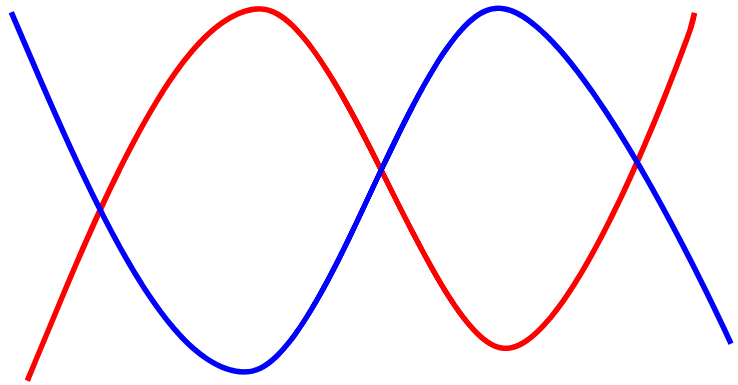
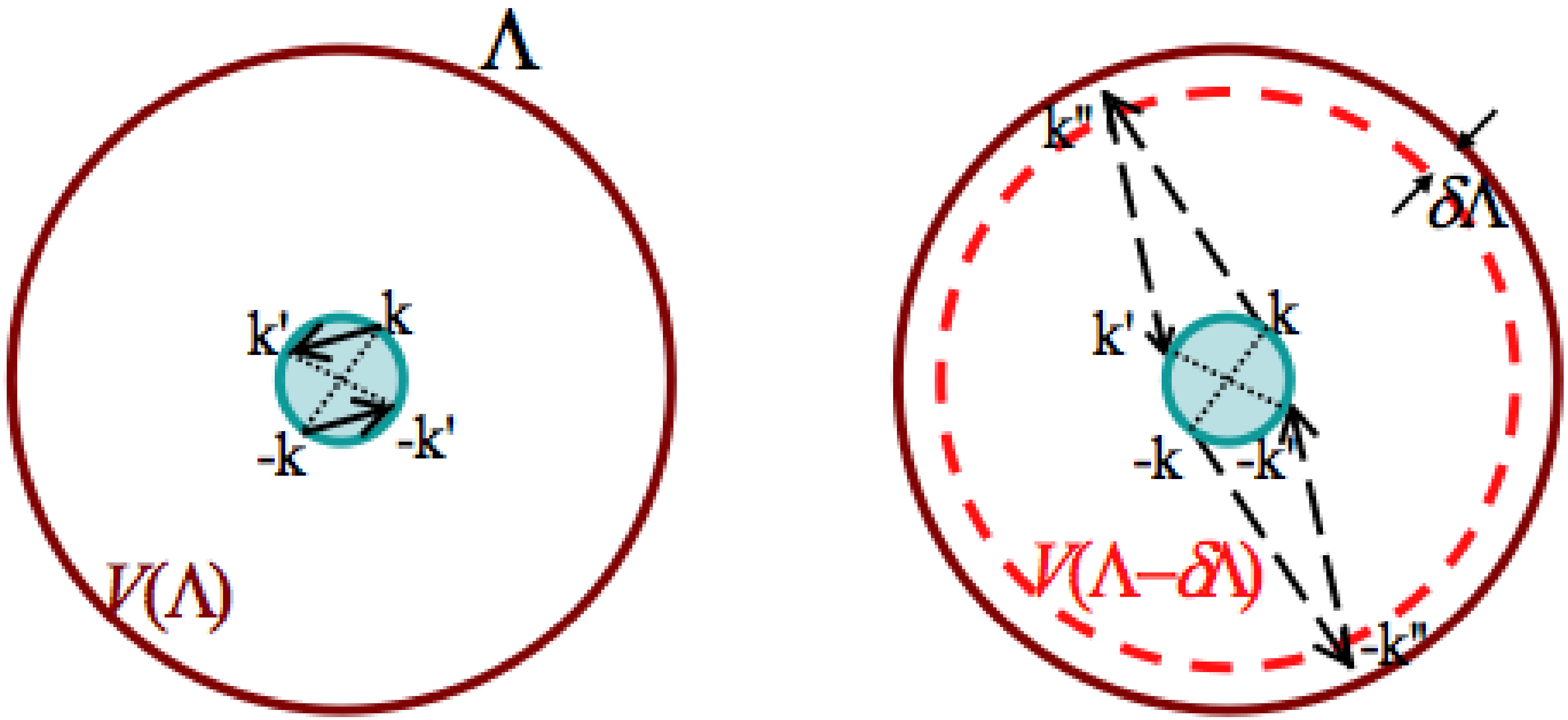


$$g_{AA}(\omega) = \frac{g_{AD}^2}{\omega - \delta_T + i\delta}$$



Summing up the virtual processes



Properties of Feshbach Resonances

$$f = \frac{1}{\frac{1}{a} + ik - \frac{1}{2} r_e k^2}, \quad a \propto -\frac{m g_{AD}^2}{\delta_D}, r_e \propto -\frac{1}{m^2 g_{AD}^2}.$$

$$[g_{AD}] = [E][V]^{1/2}$$

The scattering amplitude depends on the atom-dimer coupling and detuning which physically is the tunable magnetic field.

When the coupling is large, the scattering amplitude is universal.

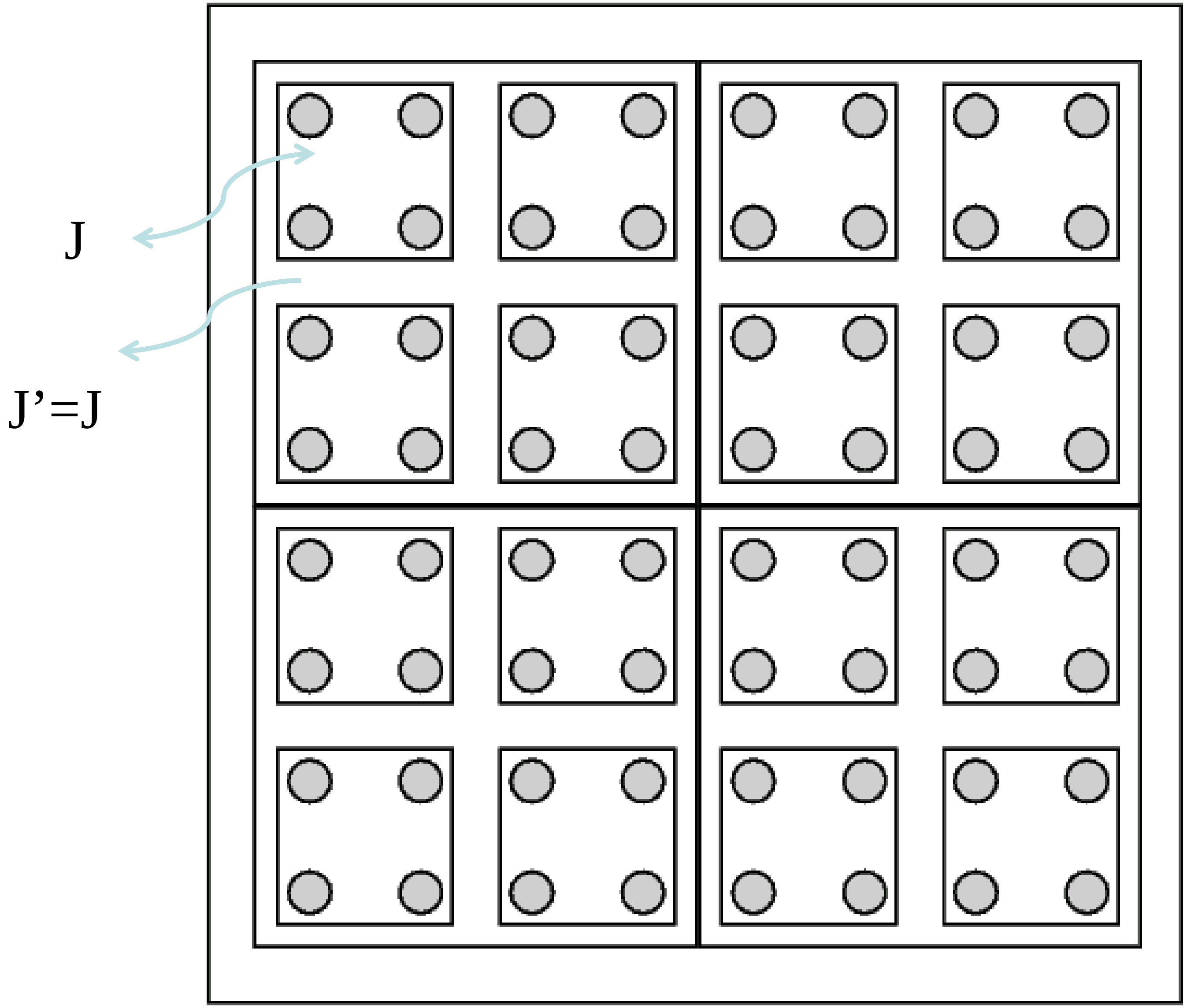
Estimate of the effective range

$$a = (1 - \frac{\Delta B}{B - B_C})a_{bg} \propto \frac{mg_{AD}^2}{\delta_D}$$

$$\Rightarrow | r_e | \propto \frac{1}{a_{bg} \Delta \mu_B \Delta B m}$$

Lecture II
Resonance scatterings as a critical phenomenon
Infrared fixed points and limit Cycle

Kadanoff scaling at critical points

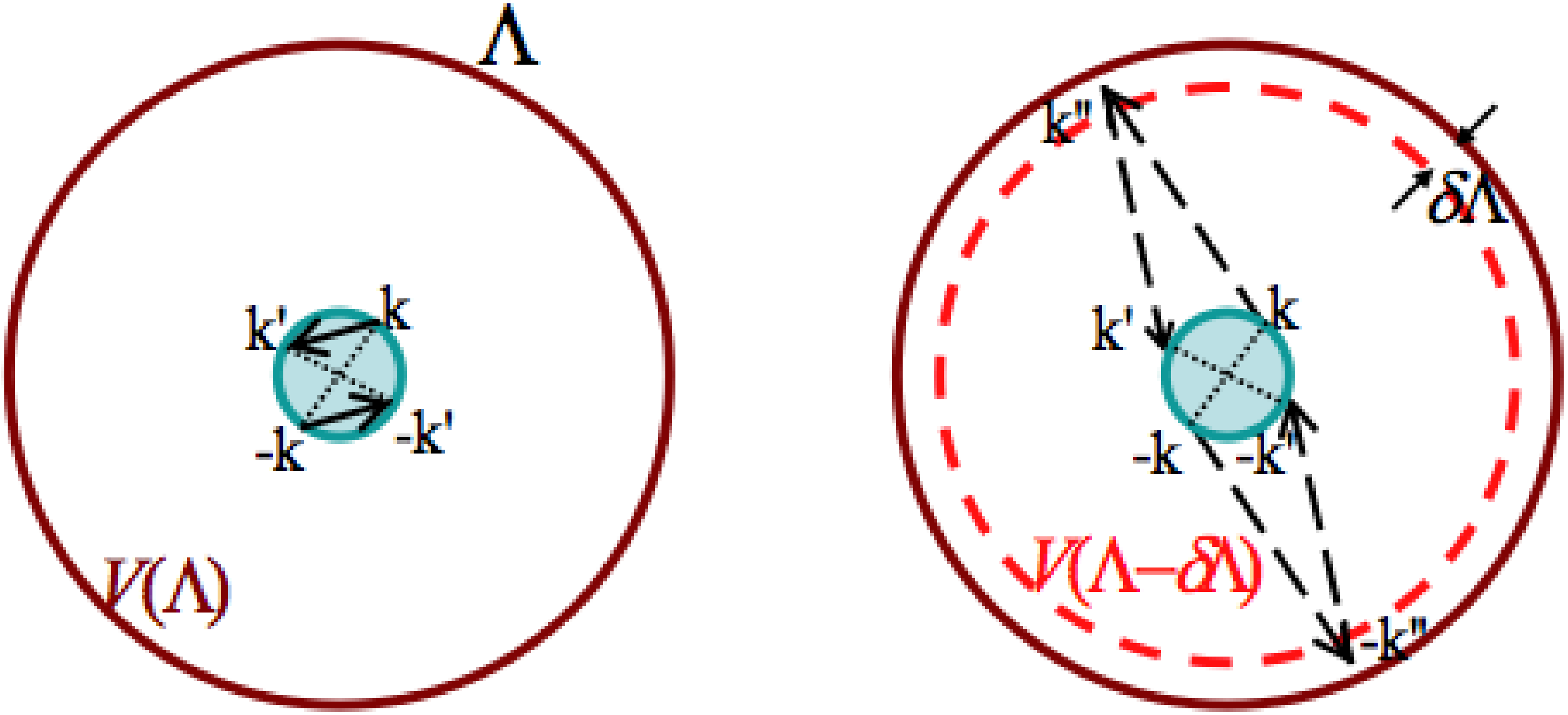


Resonance scattering state

$$\psi(r) = \frac{\sin(kr+\delta)}{kr} \propto 1 - \frac{a}{r};$$

$$r << a, \quad \psi(\lambda r) \to \lambda^{-1}\psi(r)$$

Summing up k-space multiple scatterings



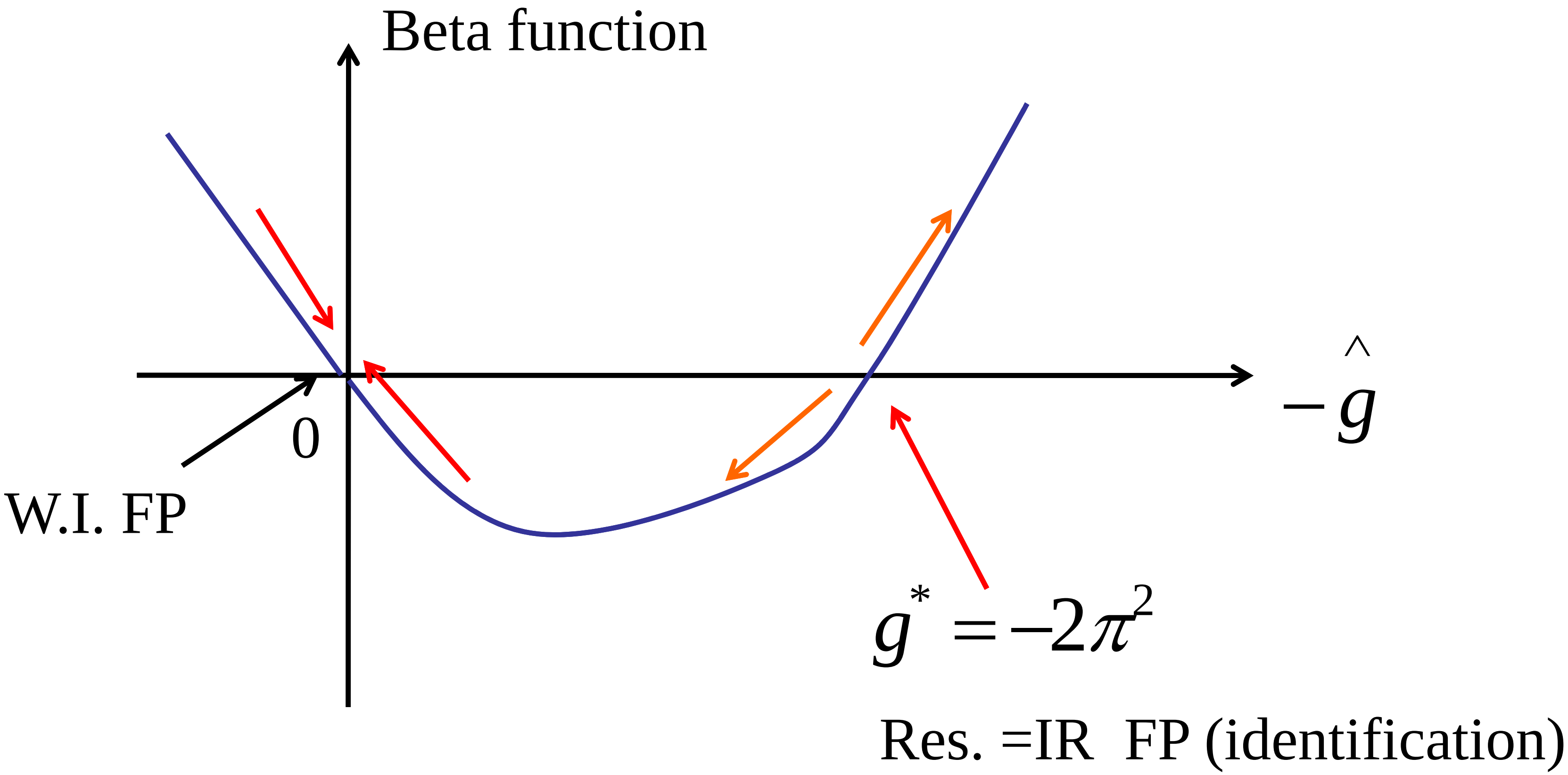
RG equation

(Kaplan et al 98; Nishida and Son et al, 2006; Nikolic and Sachdev et al, 2007)

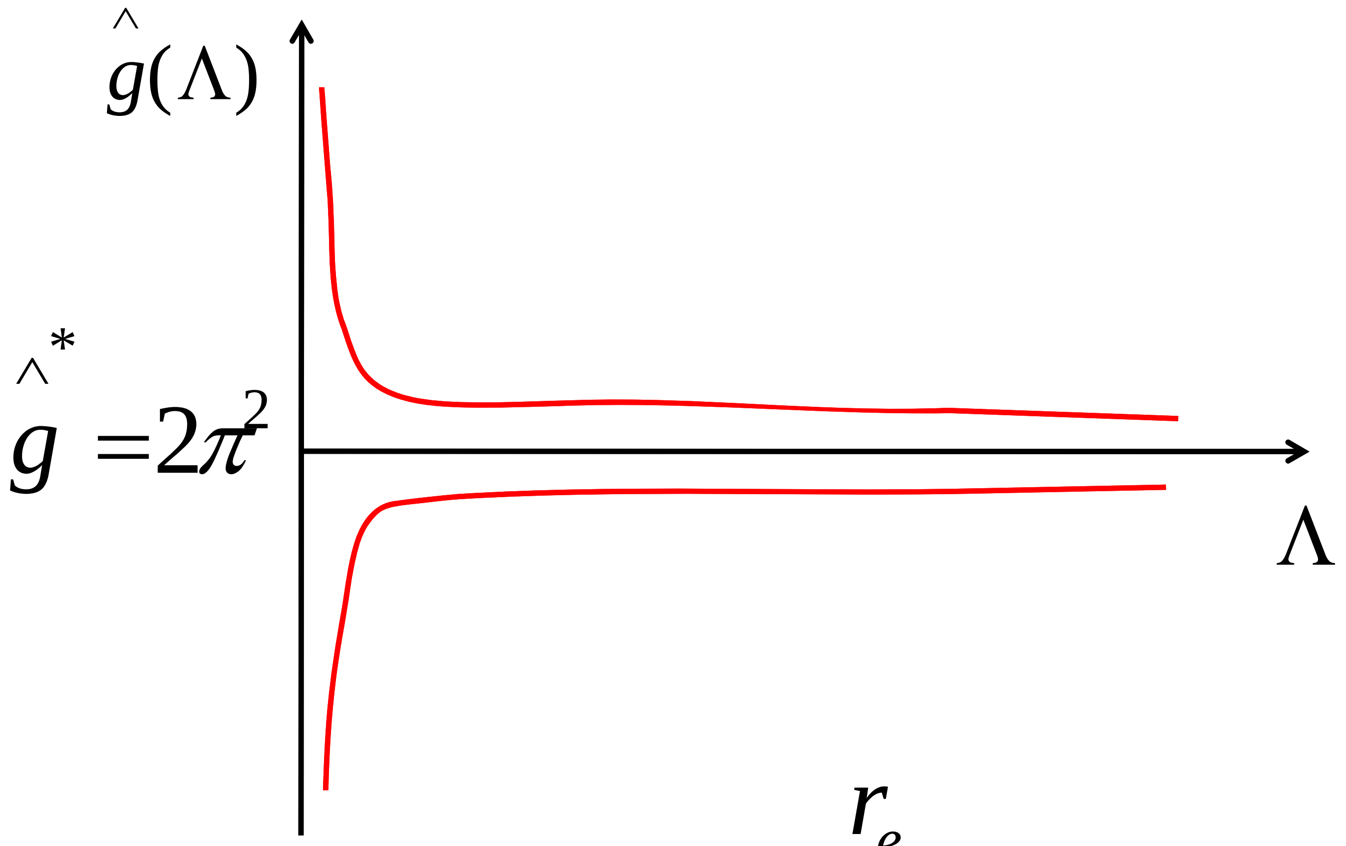
$$\hat{g}(\Lambda) = g_2 * \Lambda,$$

$$\frac{d\hat{g}}{d\log\Lambda} = \beta(\hat{g}), \quad \beta(\hat{g}) = \hat{g} + \frac{1}{2\pi^2}\hat{g}^2$$

Resonance as a unstable FP (3D)



RG flow of the coupling const. and critical exp.



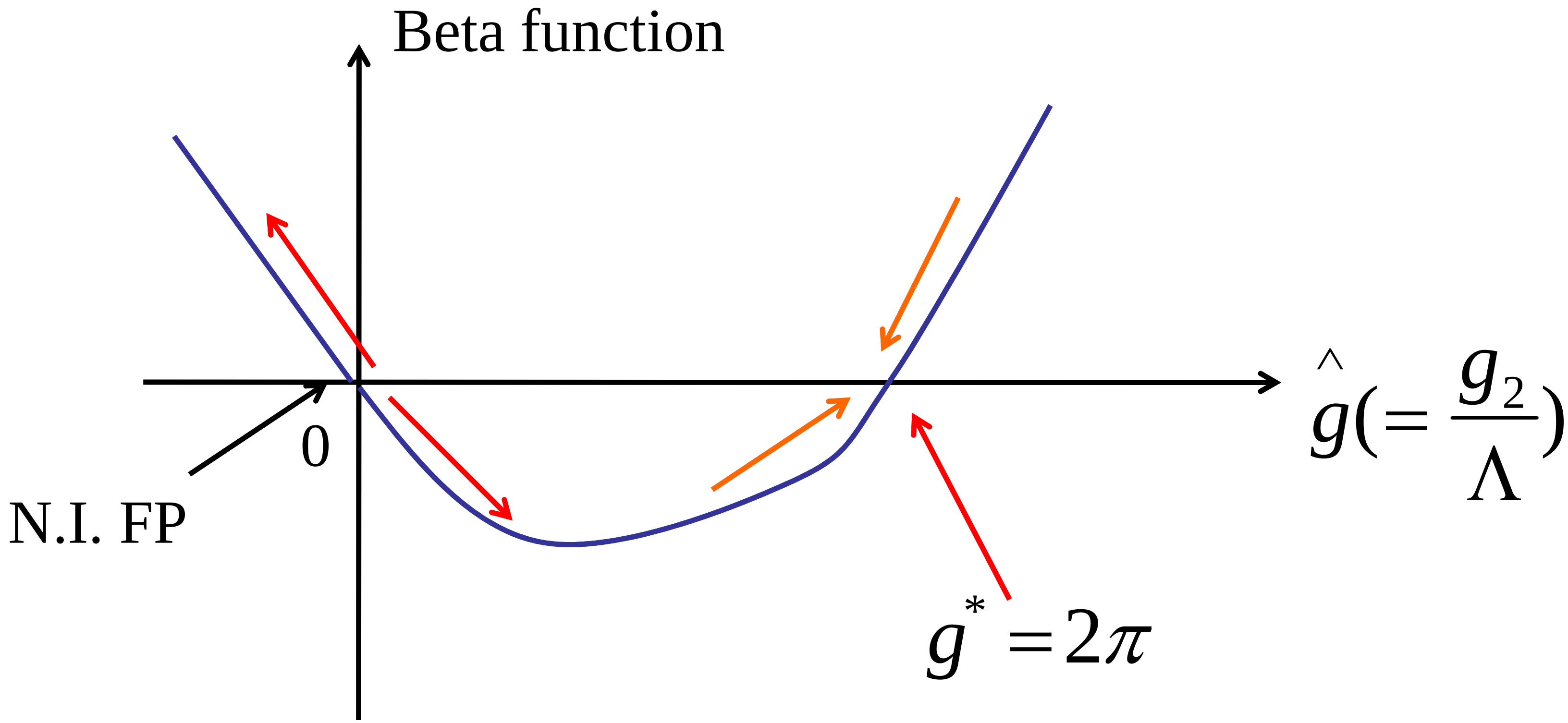
$$a \propto \frac{r_e}{g - g^*}$$

1D

$$\hat{g}(\Lambda) = \frac{g_2}{\Lambda}$$

$$\frac{d\hat{g}}{d\log\Lambda} = \beta(\hat{g}), \quad \beta(\hat{g}) = -\hat{g} + \frac{1}{2\pi}\hat{g}^2$$

Resonance as a unstable FP (1D)

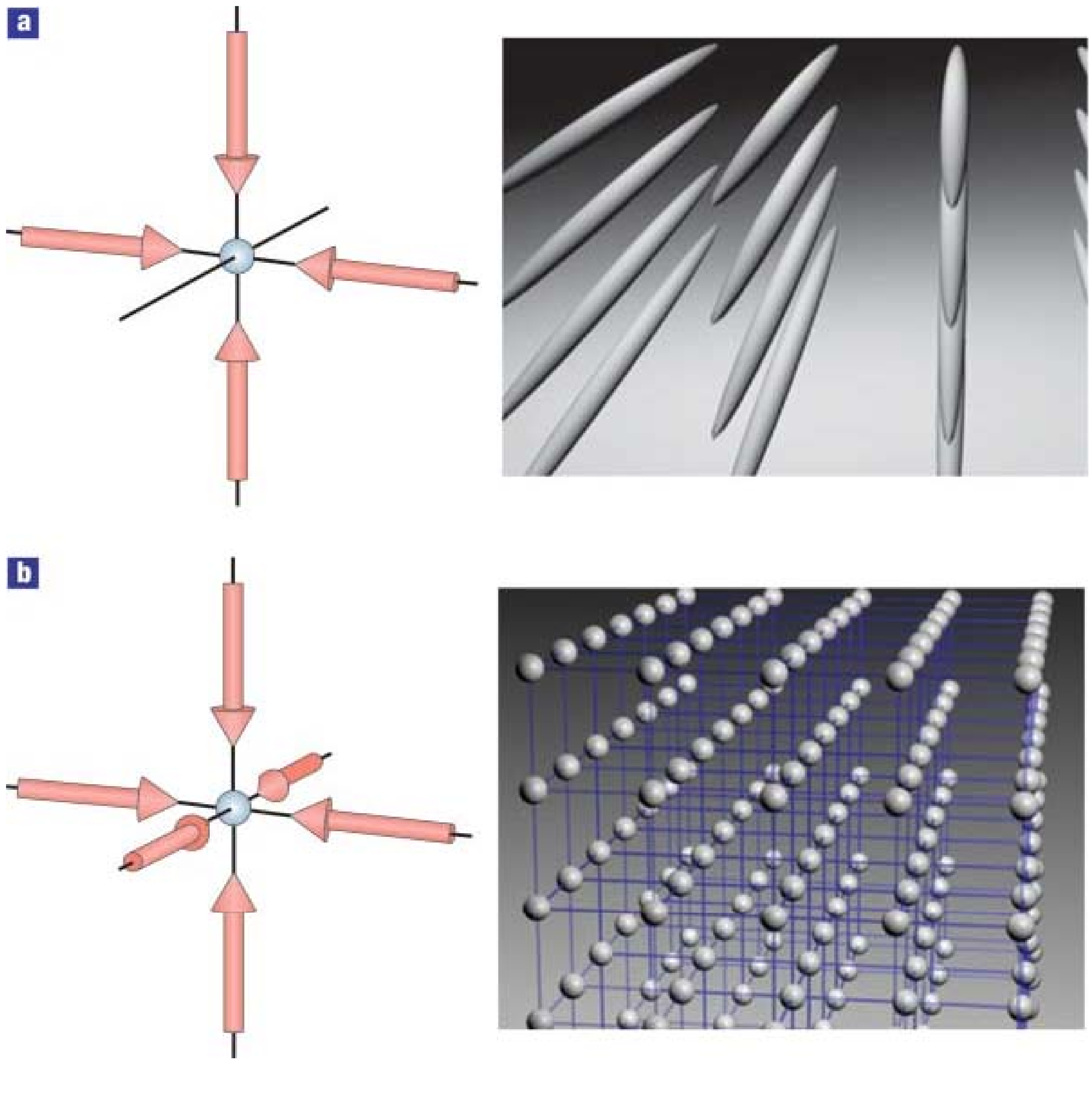


IR FP= Tonk-Girardeau Gas
(Fermionized Bose gas, identification)

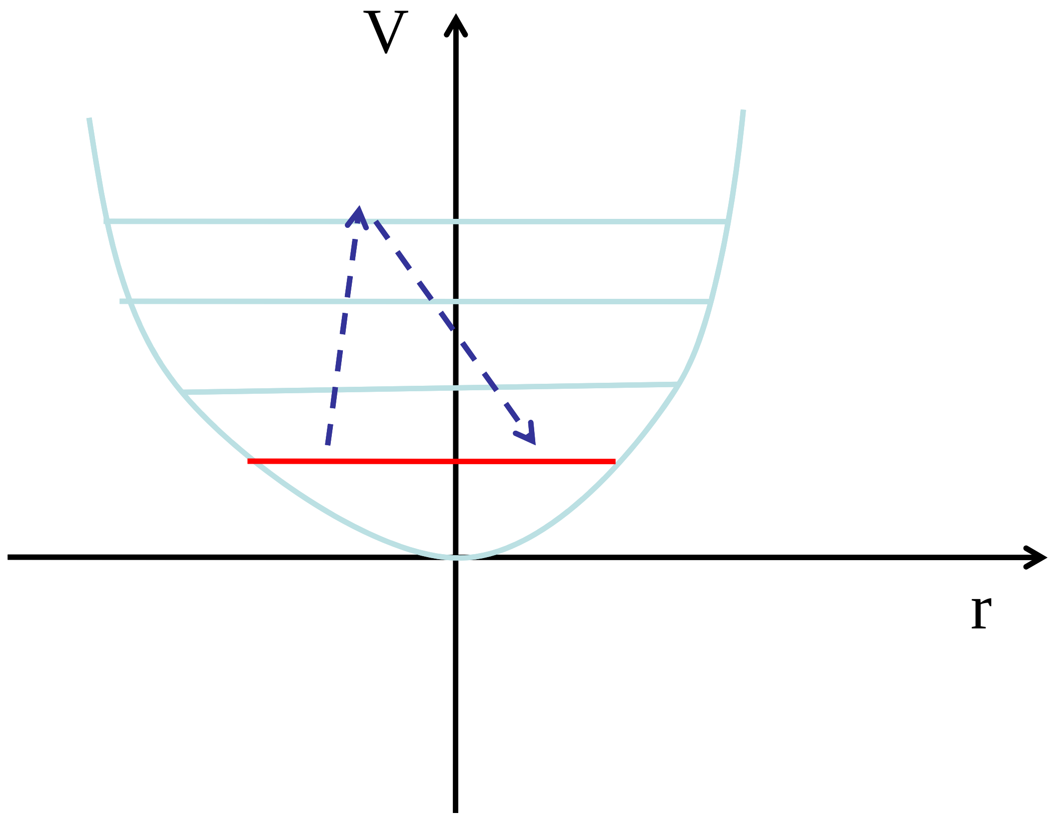
Two applications of RG

1) Confined geometry:

2) Implications on upper branch physics:


a
b

RG in a confined space---a tube

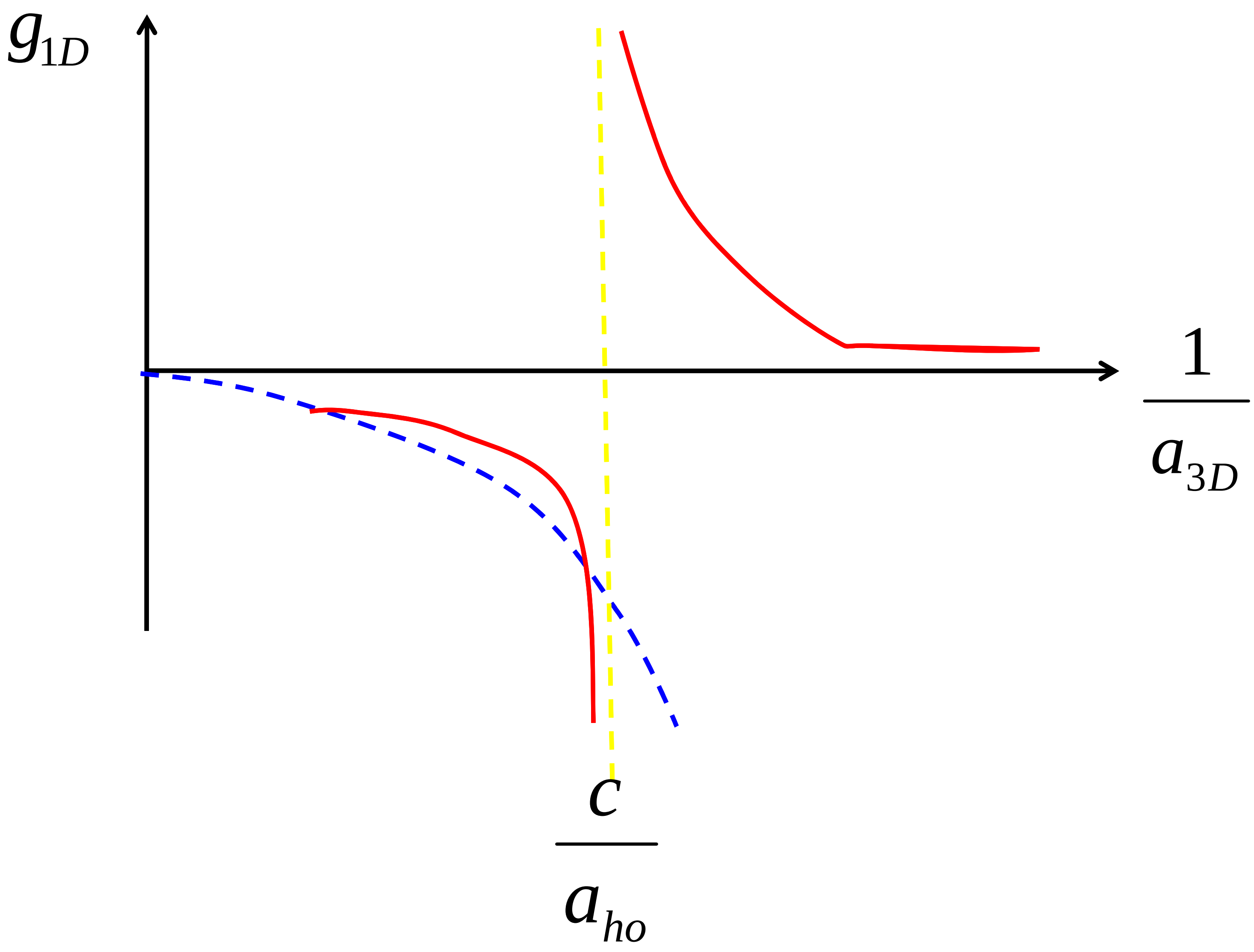


$$g_{1D} \propto \frac{a_{3D}}{a_{ho}^2}\left[\frac{1}{1 - C\frac{a_{3D}}{a_{ho}^2}}\right]$$

$$\varepsilon(n_x, n_y, k_z) = (n_x + n_y + 1)\hbar\omega + \frac{k_z^2}{2m}$$

Olshanii, 98;
Cui, Wang, FZ, 2009.

Confinement induced “hardcore limit”

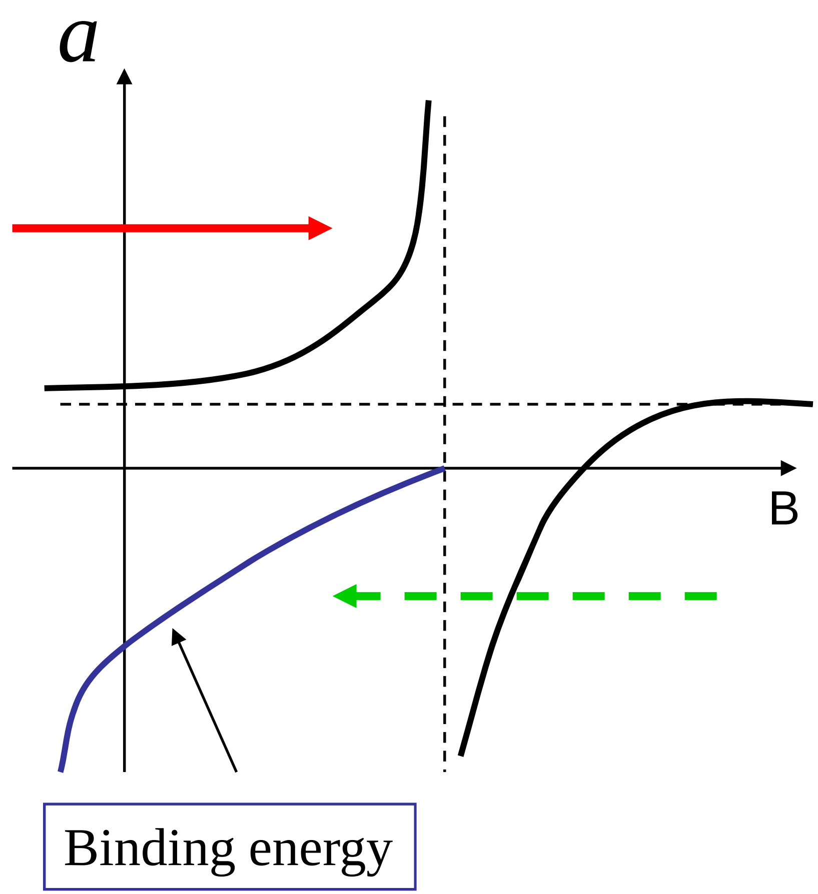


Two applications of RG

1) Confined geometry:

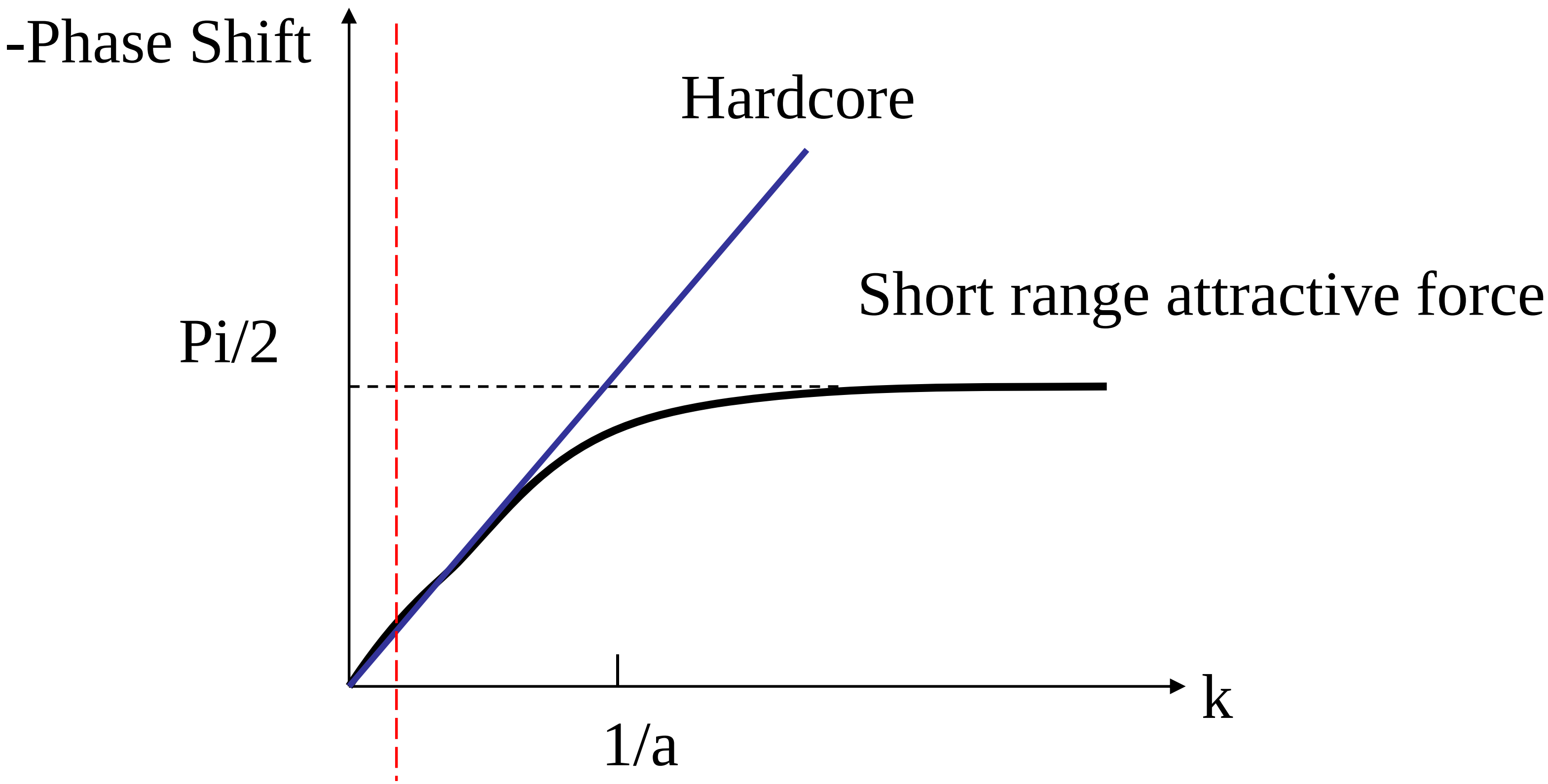
2) Implications on upper branch physics:

Quantum Gas near Feshbach Resonance
(Upper Branch)



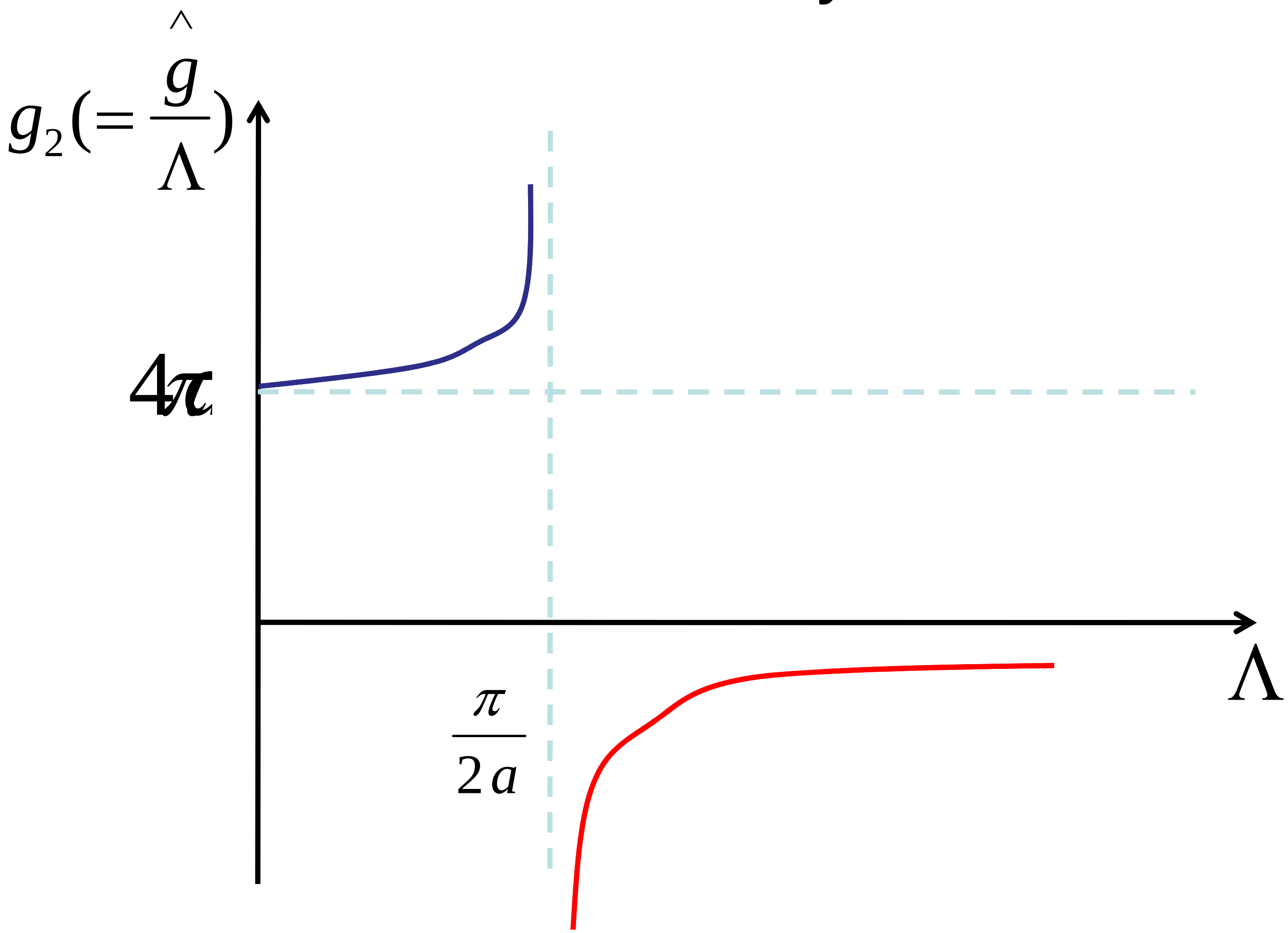
Is positive "a" equivalent to repulsive interactions?

Positive scattering lengths versus hardcore interactions



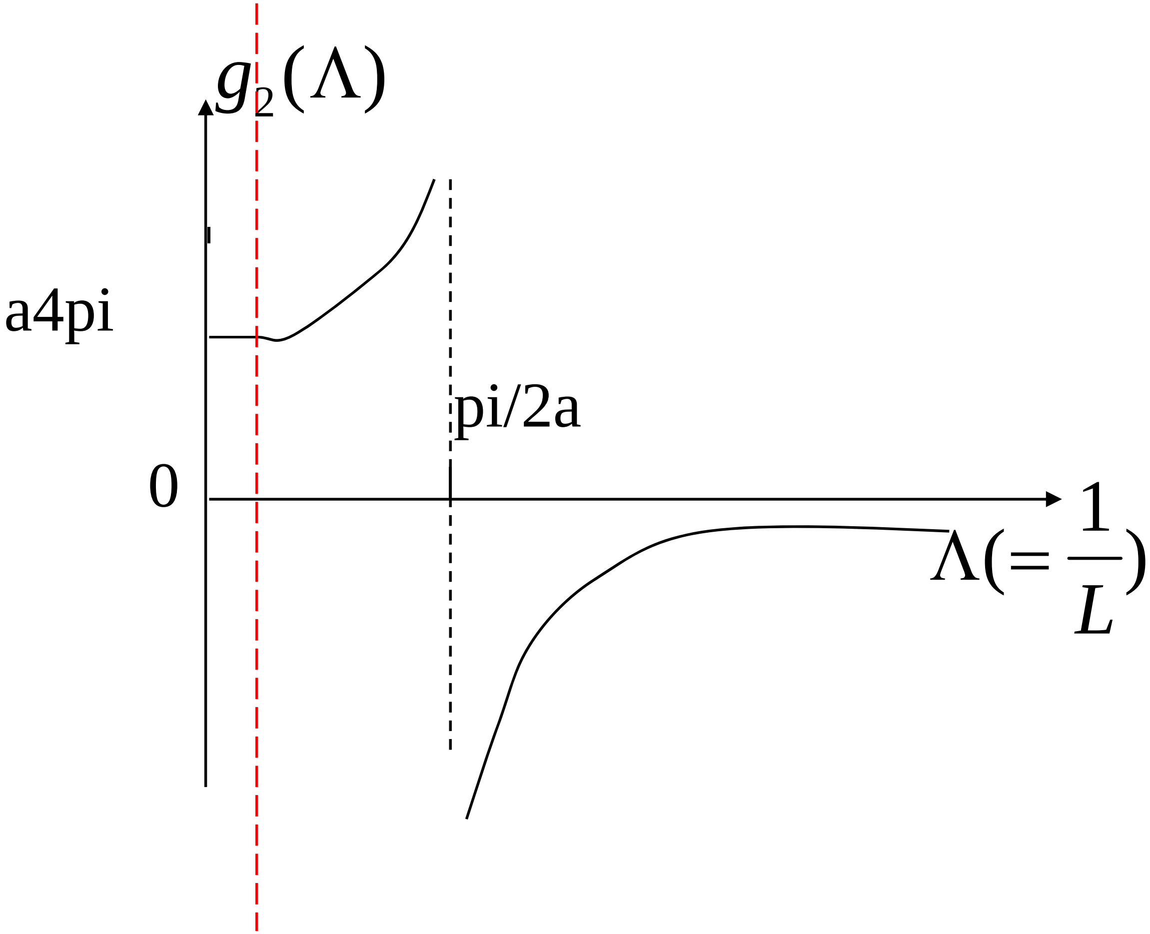
Not equivalent near resonance when k_F a is much large than unity.

Effective 2-body forces



Borzov, Mashayekhi, Zhang, Song and FZ, to appear 2011

Running 2-body forces



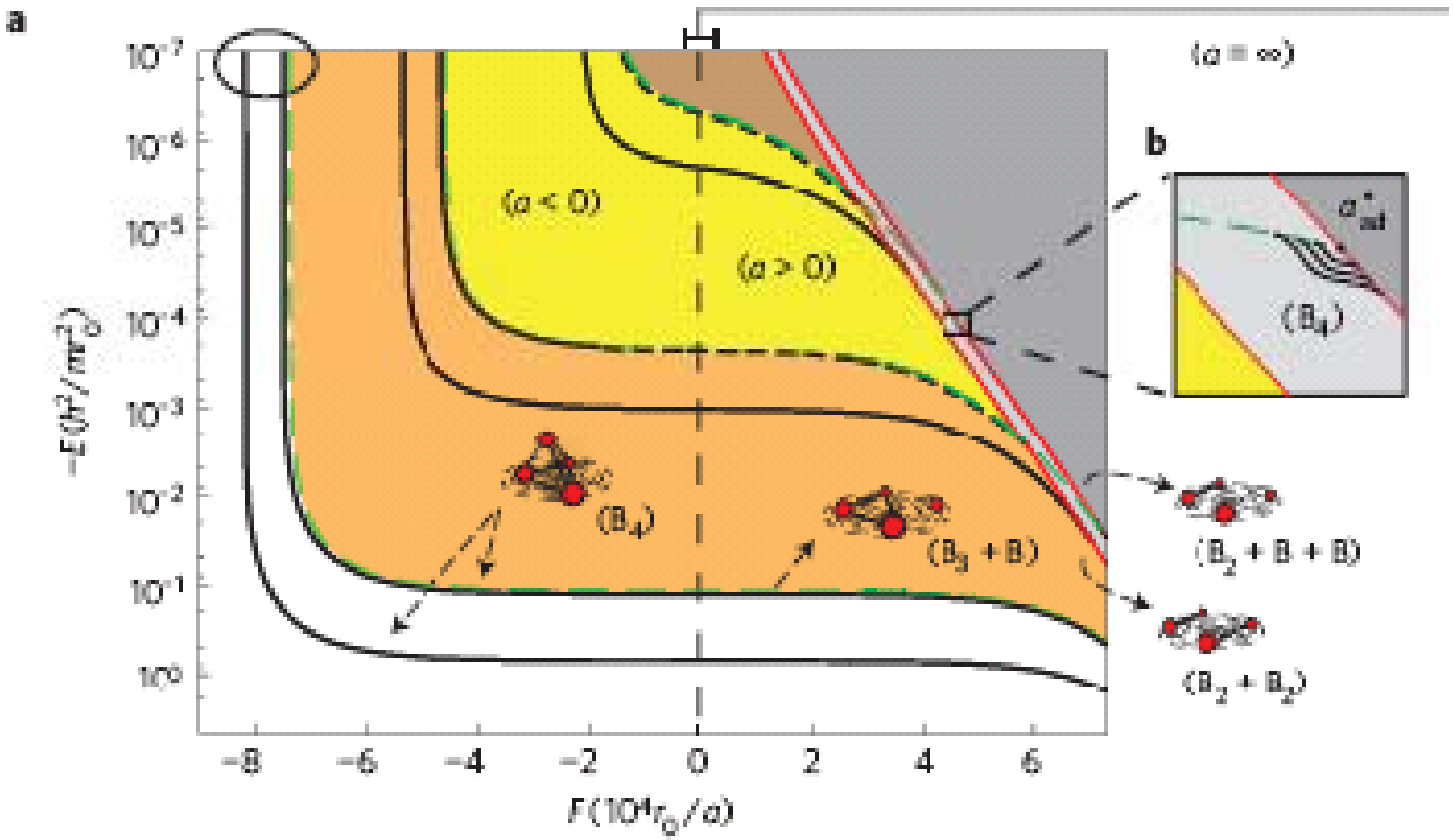
Lecture III

More on RGE

Multiple scatterings in a quantum gas:

Many-body effects

Efimov Trimers (Efimov, 1970-73)



Discovery of Tetramers:
Hammer and Platter Eur.Phys. 2007;
Stecher, D'Incao and C.Greene, Nature Phys 2009.

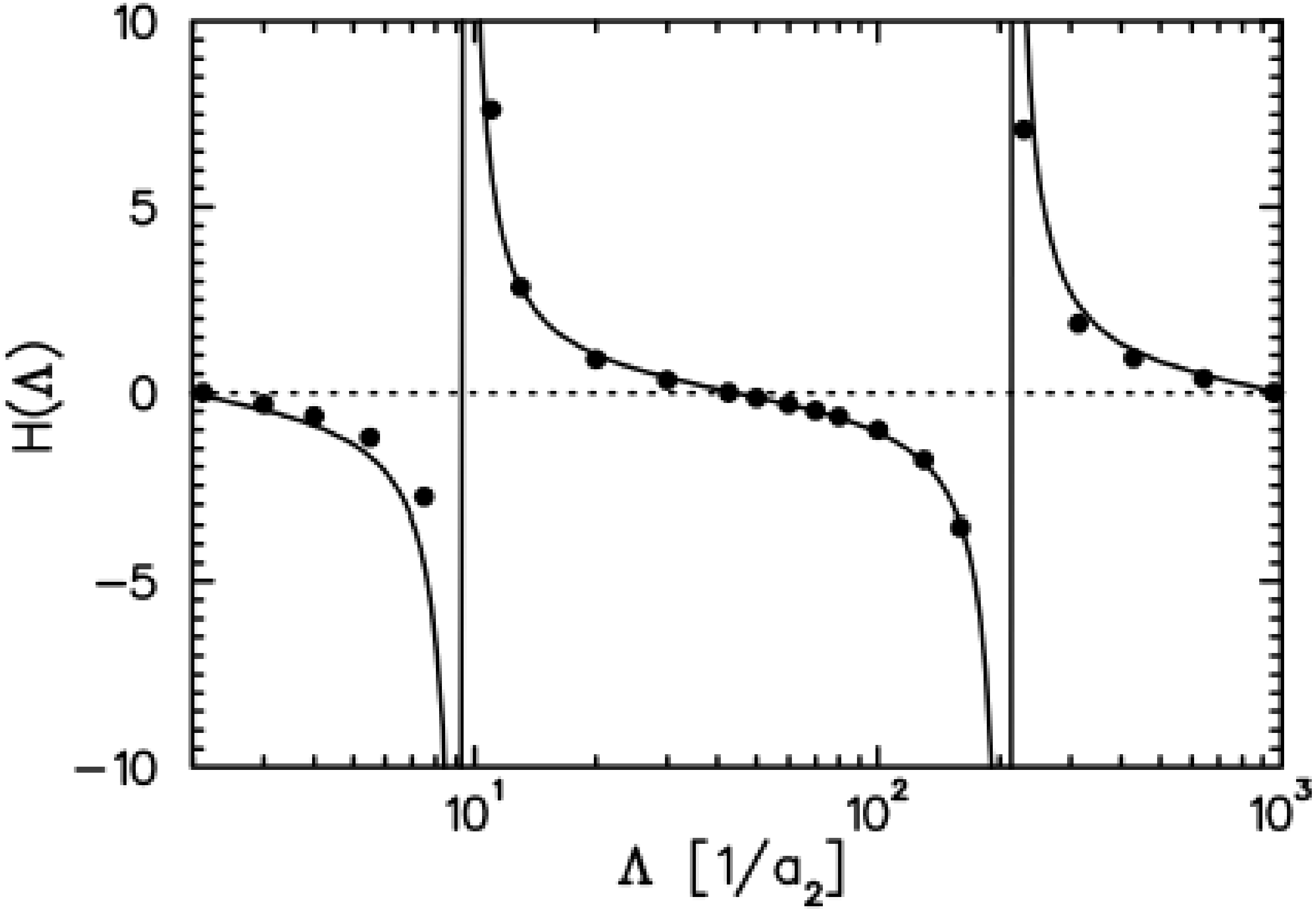
Bedaque-Hammer-van Kolck three-body Force (Nucl. Phys. A, 98)

$$\mathcal{L} = \psi^\dagger (i\partial_0 + \frac{\vec{\nabla}^2}{2M})\psi + \Delta T^\dagger T - \frac{g}{\sqrt{2}}(T^\dagger \psi\psi + \text{h.c.}) + hT^\dagger T\psi^\dagger\psi + \ldots \quad (2)$$

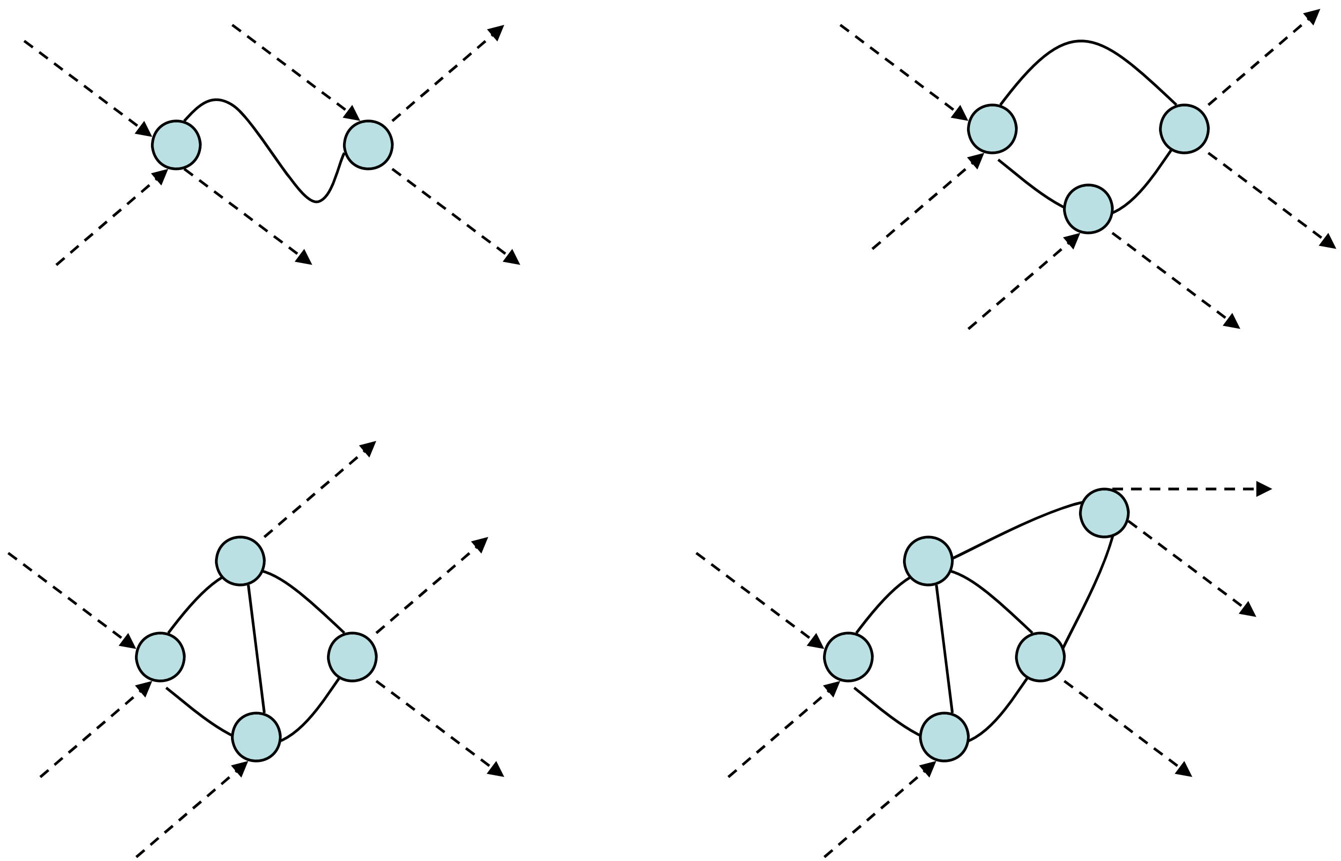
$$H(\Lambda) = -\frac{\sin(s_0 \ln(\frac{\Lambda}{\Lambda_*}) - \text{arctg}(\frac{1}{s_0}))}{\sin(s_0 \ln(\frac{\Lambda}{\Lambda_*}) + \text{arctg}(\frac{1}{s_0}))}.$$

Zero energy atom-dimer scattering length is independent of the cut-off .

Limit cycle RG solution

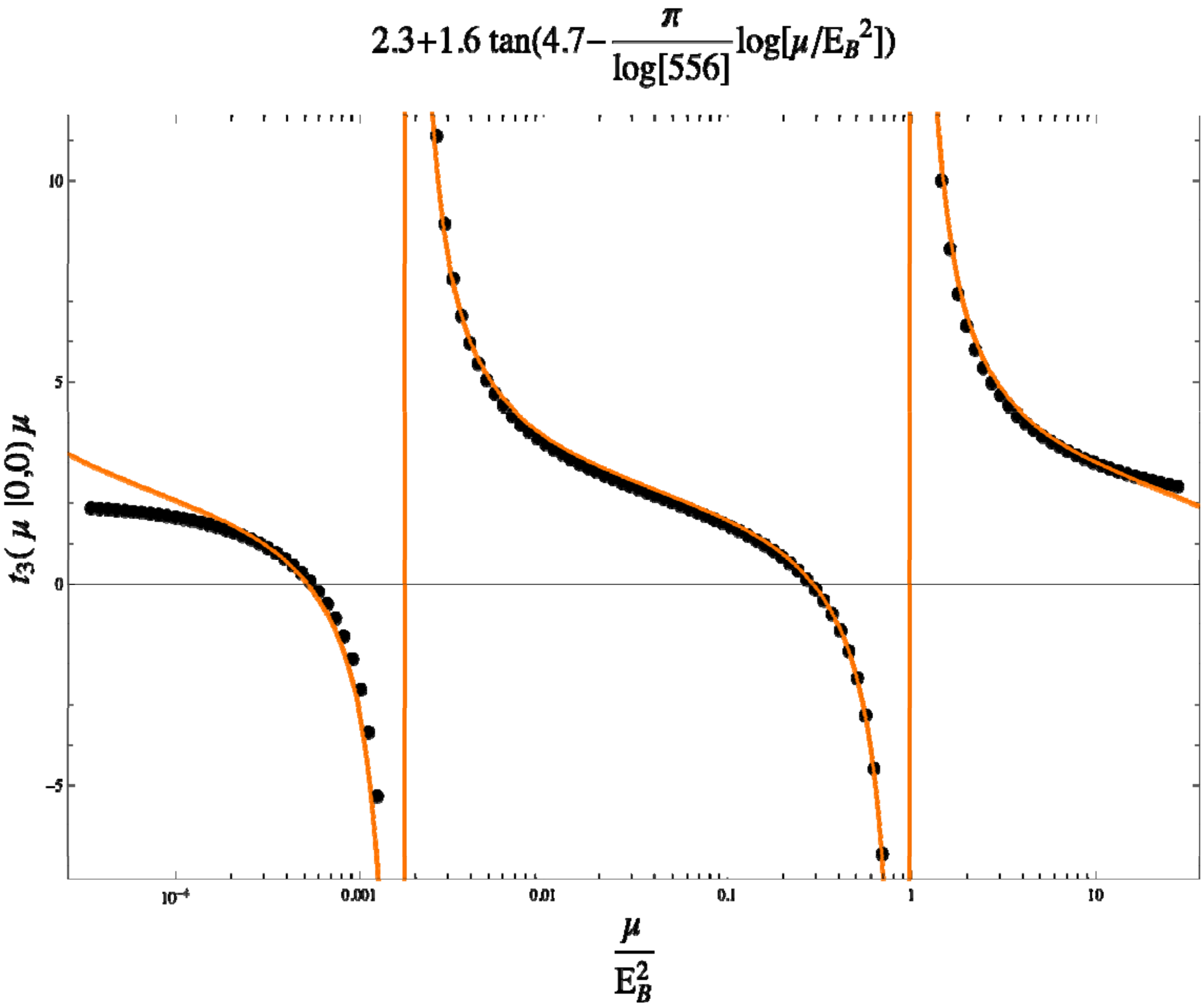


6-pt correlators in BECs: Generalization in a quantum gas



N-loop six-pt correlations (Borzov et al, to appear)

Regularized 6-pt correlator in BECs



A discrete symmetry

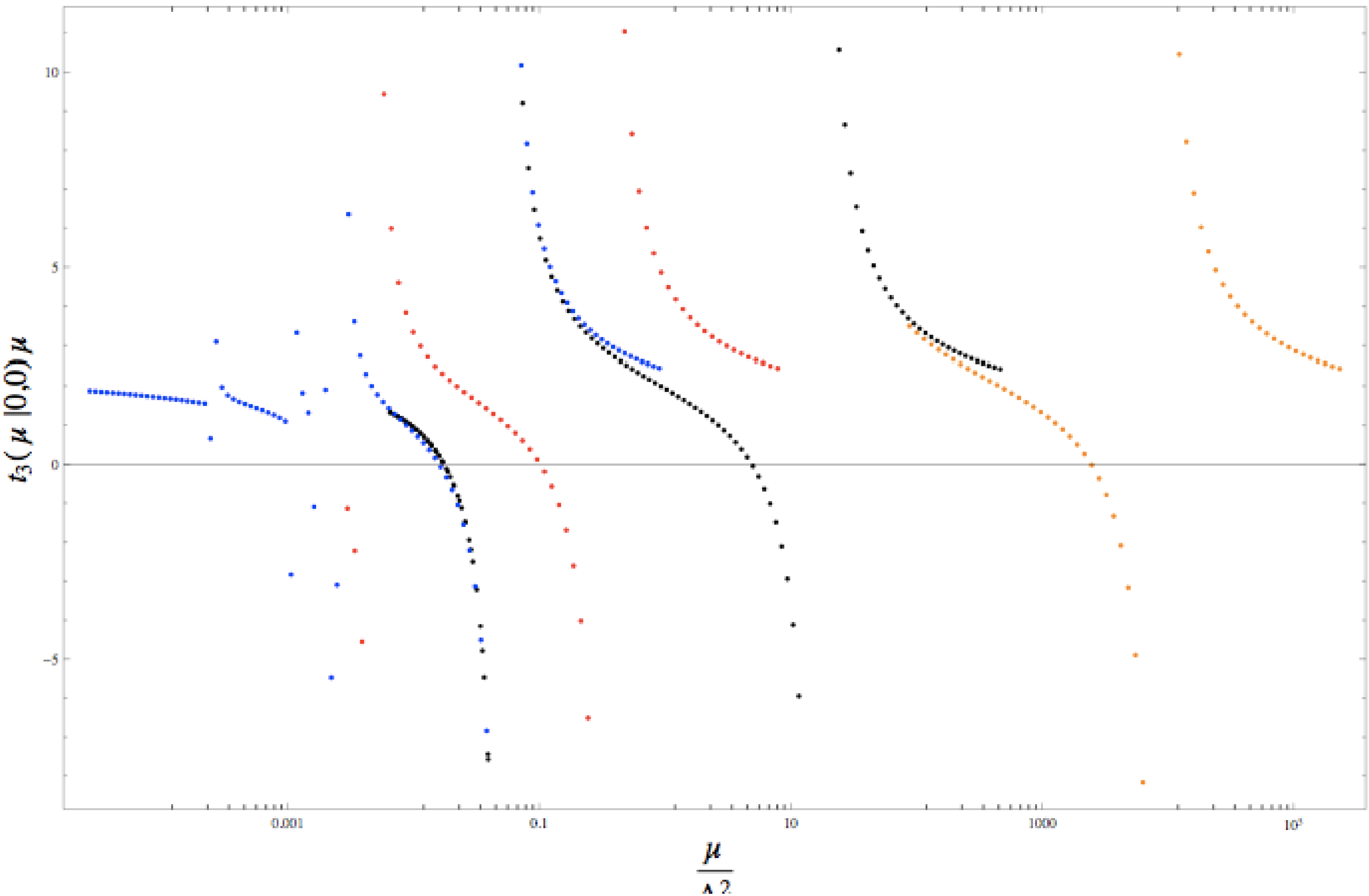
$$G_3(-3\mu\lambda^2;\ 1/a_2\lambda\ ,\Lambda_3^*) = G_3(-3\mu;\ 1/a_2,\Lambda_3^*),$$

$$\lambda = e^{\pi/s_0} = 22.7.$$

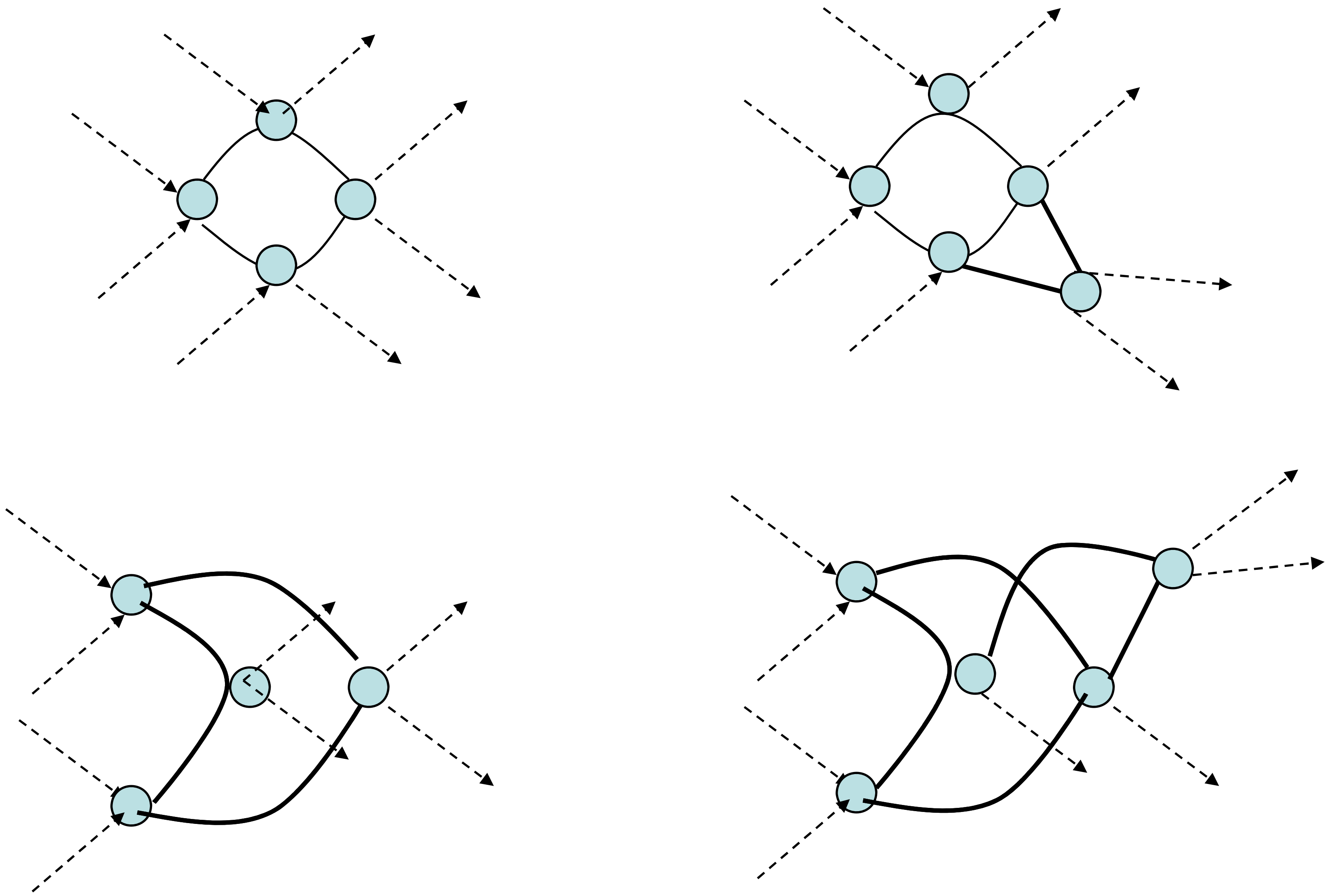
$$G_3(-3\mu\lambda^2;\ 0,\Lambda_3^*) = G_3(-3\mu;\ 0,\Lambda_3^*),$$

$$G_3(-3\mu;\ 0,\lambda^2\Lambda_3^*) = G_3(-3\mu;\ 0,\Lambda_3^*),$$

$1/a=0.5\ t(\mu)$ (Blue) and $1/a=0.5/\lambda\ t(\lambda^2\ \mu)$ (Black) and $1/a=0.5/\lambda^2\ t(\lambda^4\ \mu)$ (Orange) and $1/a=0.5/3\ t(3^2\ \mu)$
(Red, to illustrate that the symmetry is discrete)



4-particle correlators



Issue of multiple scattering in quantum gases

MacNeill and FZ, PRL, 2010

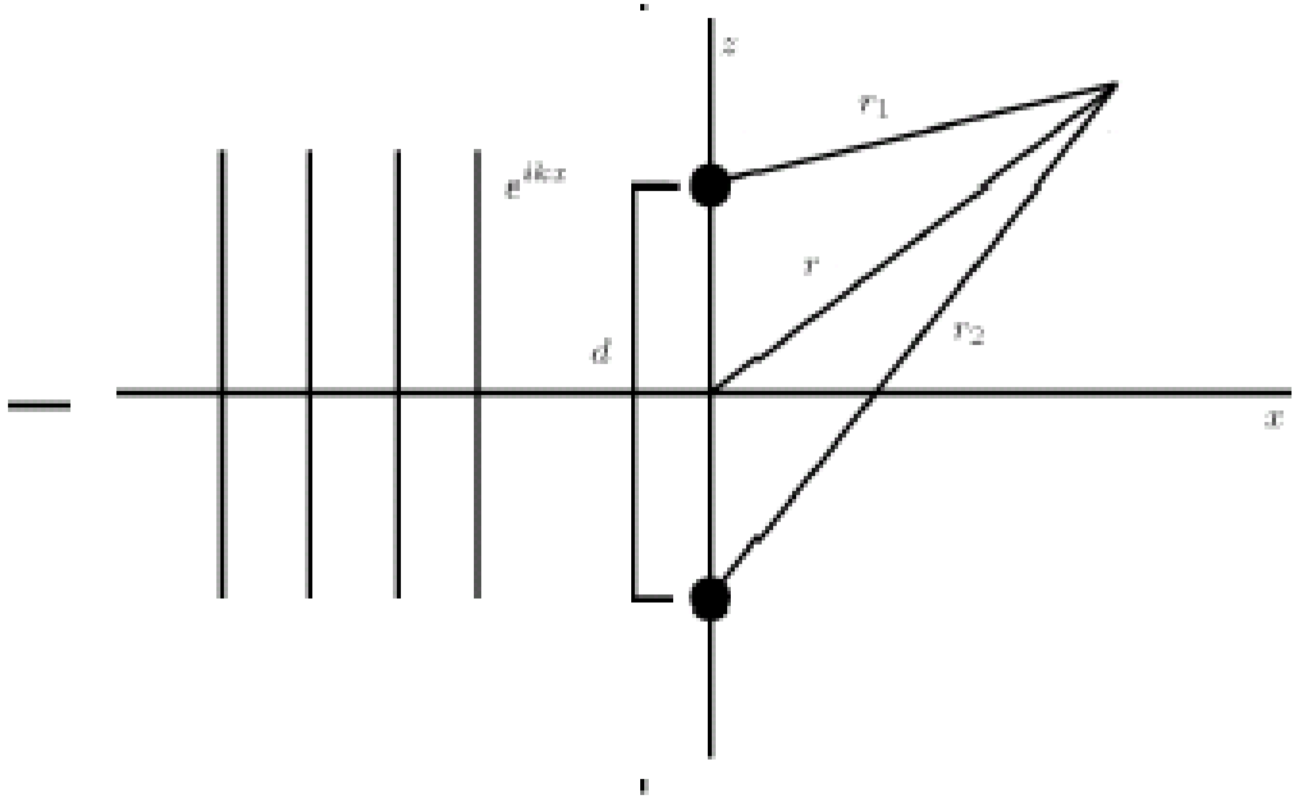
Song and FZ, arXiv. 1104.2924; Phys. Rev. A, 2011;

Song and FZ, 2011, to appear.

1) K-space multiple scattering between two atoms when other identical particles are by-standers or as a (statistical) static background.

2) Multiple scattering between one of few-body structures and a 3rd particle in quantum gases---dynamical background.

Multiple Scattering (d << a): From 2 to N ---MacNeill test



Total Cross-section $\quad 1)0; \quad 2)2\sigma; \quad 3)4\sigma; \quad 4)\sigma\sqrt{\frac{d}{a}}.$

$$\sigma = 4\pi a^2$$

Effective scattering scattering length ~ d

“Three-body” bound States