NEW DEVELOPMENTS IN ARITHMETIC GEOMETRY

Organizer: Ye Tian (Chinese Academy of Sciences), Shou-Wu Zhang (Princeton and Tsinghua)

	9:30-10:30	11:00-12:00	2:30-3:30	4:00-5:00
June14	Dorian Goldfeld	Hang Xue	Winnie Li	Will Chen
June 15	Wei Zhang	Xin Wan		
June 16	Xinyi Yuan	Xuhua He	Ruochuan Liu	Yongquan Hu
June 17	Alex Smith	Weizhe Zheng		

(1) **Will Chen** (IAS, Princeton)

Noncongruence subgroups of $\mathrm{SL}(2,\mathbb{Z})$ *and nonabelian level structures for elliptic curves*

I will describe how noncongruence modular curves can naturally be viewed as moduli spaces of elliptic curves with nonabelian level structures. Via certain "ASD-type" congruences on the Fourier coefficients of noncongruence modular forms, the moduli interpretations relate these coefficients to the Frobenius actions on the prime-to-p fundamental groups of punctured elliptic curves over finite fields. I will also describe an application to the unbounded denominators conjecture, and some recent work with Deligne on the structure of the moduli spaces for metabelian level structures.

(2) **Dorian Goldfeld** (Columbia University)

Orthogonality relations for the Fourier coefficients of GL(n) cusp forms

(3) **Xuhua He** (University of Maryland)

Some results on affine Deligne-Lusztig varieties

In Linear Algebra 101, we encounter two important features of the group of invertible matrices: Gauss elimination method, or the LU decomposition of almost all matrices, which is an important special case of the Bruhat decomposition; the Jordan normal form, which gives a classification of the conjugacy classes of invertible matrices.

The study of the interaction between the Bruhat decomposition and the conjugation action is an important and very active area. In this talk, we focus on the affine Deligne-Lusztig variety, which describes the interaction between the Bruhat decomposition and the Frobenius-twisted conjugation action of loop groups. The affine Deligne-Lusztig variety was introduced by Rapoport around 20 years ago and it has found many applications in arithmetic geometry and number theory.

In this talk, we will discuss some recent progress on the study of affine Deligne-Lusztig varieties, and some applications to Shimura varieties.

(4) **Yongquan Hu** (Chinese Academy of Sciences)

Asymptotic growth of the cohomology of Bianchi groups

Given a level N and a weight k, we know the dimension of the space of (classical) modular forms. This turns out to be unknown if we consider Bianchi modular forms, that is, modular forms over imaginary quadratic fields. In this talk, we consider the asymptotic behavior of the dimension when the level is fixed and the weight grows. I will first explain an upper bound obtained by Simon Marshall using Emerton?s completed cohomology and the theory of Iwasawa algebras. Then I explain how to improve this bound using the mod p representation theory of GL2($\mathbb{Q}_p$).

(5) **Winnie Li** (U. Penn)

Noncongruence modular forms: advances and challenges

Unlike its counterpart, the arithmetic of modular forms for noncongruence subgroups is much less understood. In this survey talk we shall review the current progress and discuss some open problems.

(6) **Ruochuan Liu** (Beijing University)

Towards a p-adic Riemann-Hilbert correspondence

I'll report the recent progress on the p-adic version of Riemann-Hilbert correspondence.

(7) **Alex Smith** (Harvard University) 2^k*-Selmer groups,* 2^k*-class groups, and Goldfeld's conjecture*
Take $E/\mathbb{Q}$ to be an elliptic curve with full rational 2-torsion (satisfying some extra technical assumptions). In this talk, we will show that 100% of the quadratic twists of E have rank less than two, thus proving that the BSD conjecture implies Goldfeld's conjecture in these families. To do this, we will extend Kane's distributional results on the 2-Selmer groups in these families to 2^k-Selmer groups for any $k > 1$. In addition, using the close analogy between 2^k-Selmer groups and 2^{k+1}-class groups, we will prove that the 2^k-class groups of the quadratic imaginary fields are distributed as predicted by the Cohen-Lenstra heuristics for all $k > 1$.

(8) **Xin Wan** (Chinese Academy of Sciences)
BSD formula for elliptic curves and modular forms
We introduce recent progresses on BSD formulas in the analytic rank 0 and 1 cases for elliptic curves and generalizations to modular forms. If time allows we also explain some main ideas.

(9) **Hang Xue** (University of Arizona)
ϵ*-dichotomy for linear models*
I will explain how to use the Harish-Chandra theory in the relative setting, in particular the germ expansion of spherical characters, to obtain epsilon dichotomy for linear models.

(10) **Xinyi Yuan** (UC Berkeley)
Positivity of Hodge bundles of abelian varieties.
Given a semi-abelian scheme over a projective curve over a field of characteristic 0, the Hodge (vector) bundle is known to be nef. However, in positive characteristics, the Hodge bundle can fail to be nef by an example of Moret-Bailly. Up to some simple operations on the semi-abelian scheme, we can even obtain a semi-abelian scheme with an ample Hodge bundle in some cases. We will also present some applications of this result.

(11) **Wei Zhang** (Columbia and MIT)
Selmer groups for Rankin-Selberg L-functions of $GL(2) \times GL(3)$
Let Π (resp. Σ) be a cohomological (for the trivial coefficient) cuspidal automorphic representation of GL(3) (resp. GL(2)) over a CM number field, and assume that they are base change from unitary groups. We prove the following theorem: if the Rankin-Selberg L-function $L(\Pi \times \Sigma, s)$ does not vanish at its center, then the associated ℓ-adic Bloch-Kato Selmer group vanishes (for primes ℓ where the mod ℓ Galois representations satisfy certain mild conditions). The conditions on ℓ come from Euler system type argument. We will discuss some examples from elliptic curves. This is a joint work with Yifeng Liu, Yichao Tian, Liang Xiao, and Xinwen Zhu.

(12) **Weizhe Zheng** (Chinese Academy of Sciences)
Compatible systems along the boundary
A theorem of Deligne says that compatible systems of ℓ-adic sheaves on a smooth curve over a finite field are compatible along the boundary. I will present an extension of Deligne's theorem to schemes of finite type over the ring of integers of a local field. This has applications to the equicharacteristic case of some classical conjectures on l-independence. I will also discuss the relationship with compatible wild ramification. This is joint work with Qing Lu.