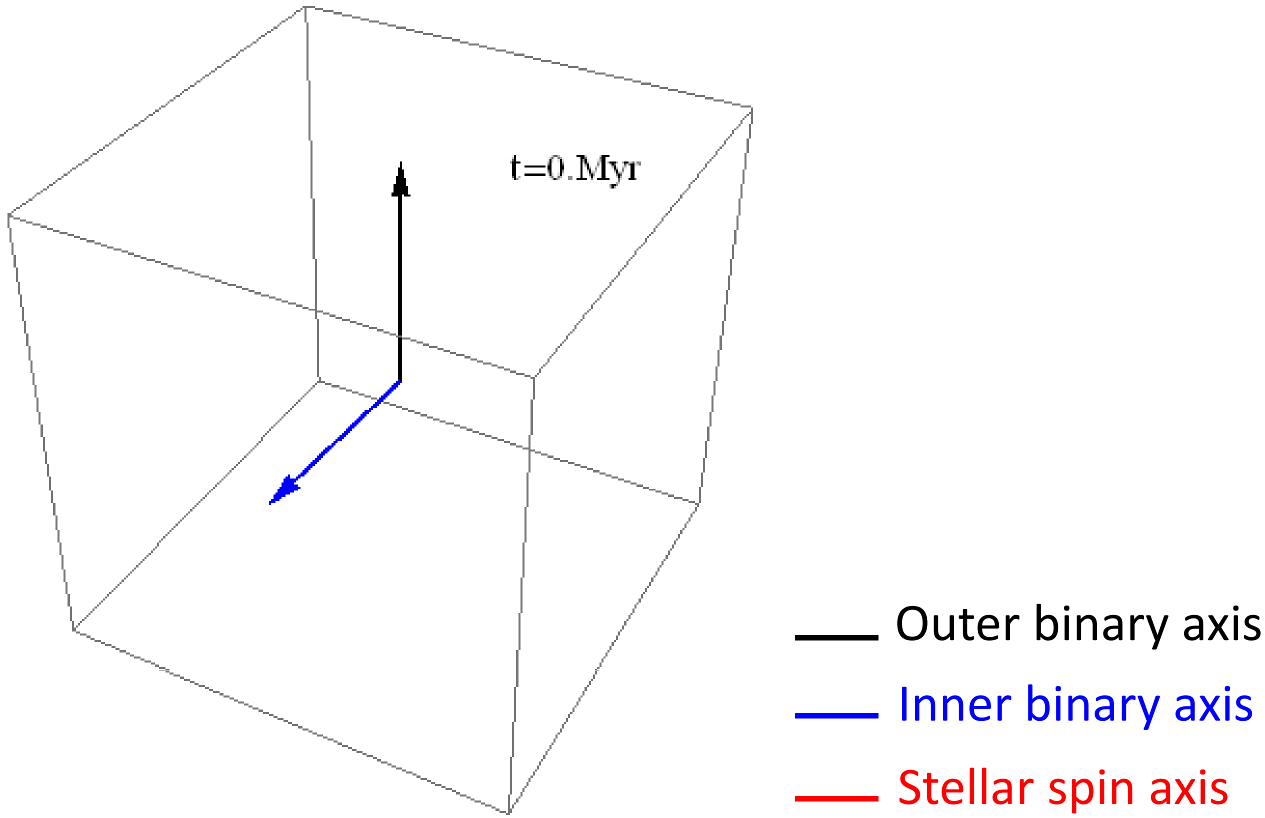
Secular Planetary Dynamics: Kozai, Spin Dynamics and Chaos

Dong Lai
Cornell University

5/17/2014 Tsinghua IAS

Chaotic Dynamics of Stellar Spin in Binaries and the Production of Misaligned Hot Jupiters

Natalia Storch, Kassandra Anderson & Dong Lai



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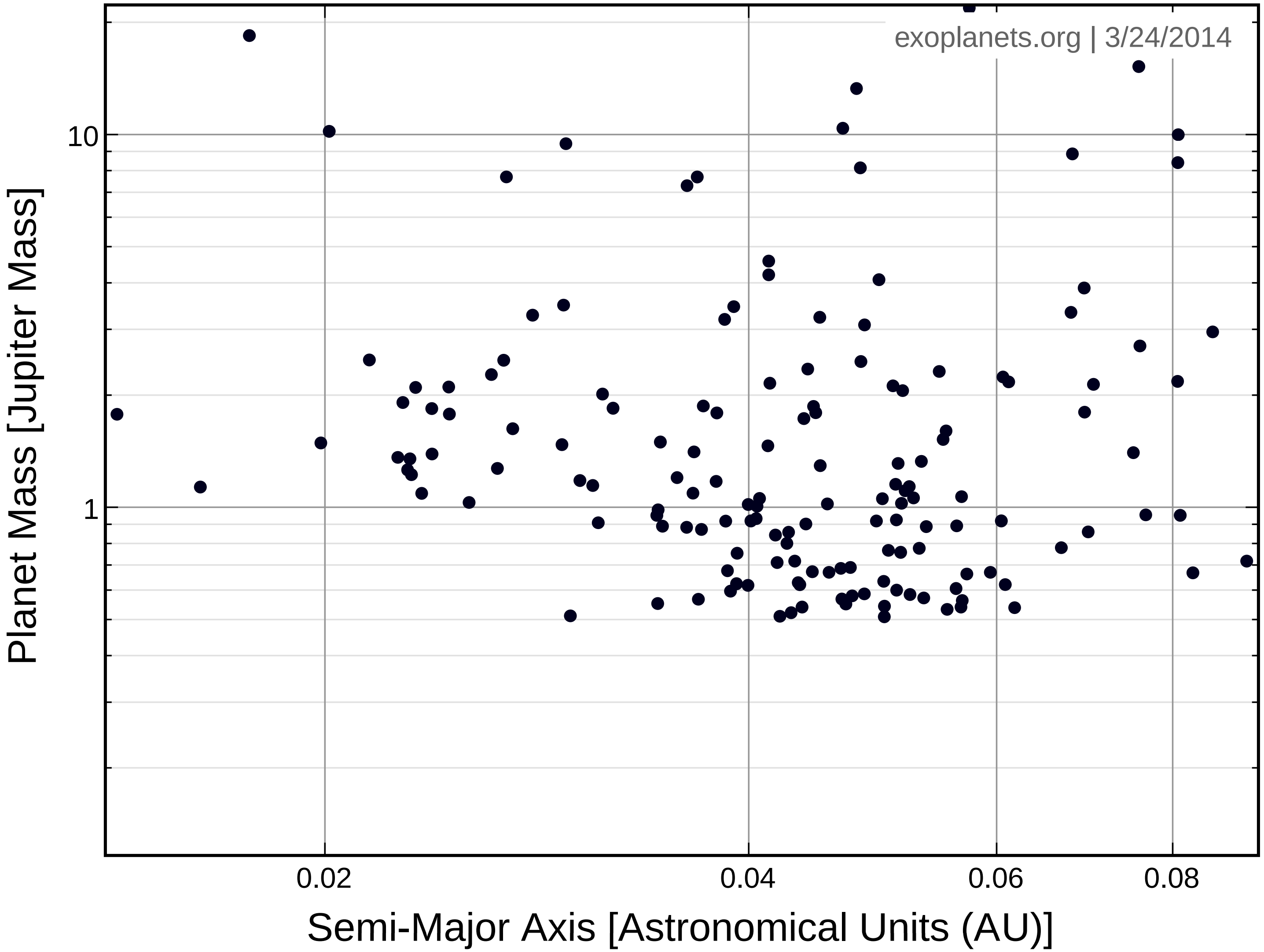
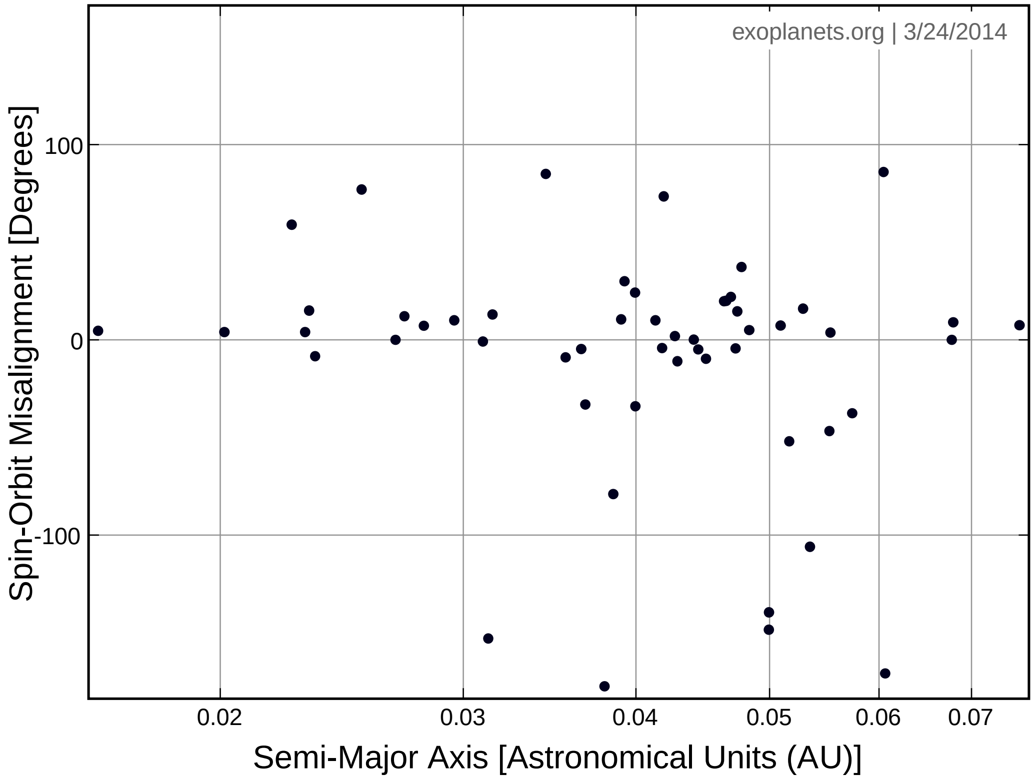
Hot Jupiters



Gas giants, with orbital periods < 10 days

Hot Jupiter Spin-Orbit Misalignment

~ 70 observed planets with projected misalignment angle between stellar spin axis and planet orbital angular momentum



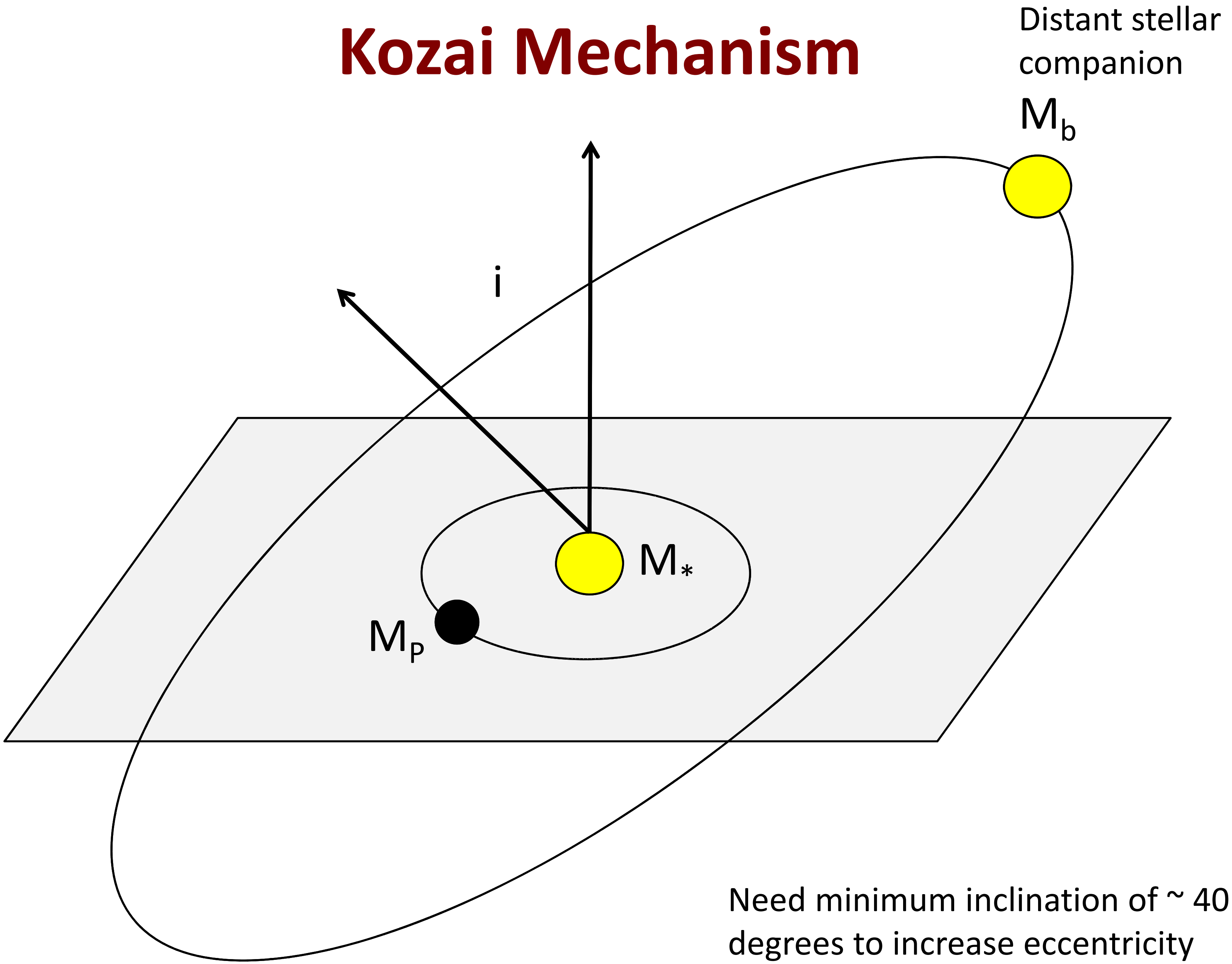
Possible causes:

- Primordial disk misalignment (Bate+10; Lai, Foucart & Lin '11; Batygin '12,13; Lai '14)
- Ang. Momentum transfer from gravity waves (Rogers & Lin '12)
- High-e migration:

  -- Planet-planet interactions/scattering (e.g. Ford & Rasio '08, Wu & Lithwick '11)

  -- **Kozai oscillations** due to binary companion (Wu & Murray '03, Fabrycky & Tremaine '07)

Kozai Mechanism



Need minimum inclination of ~ 40 degrees to increase eccentricity

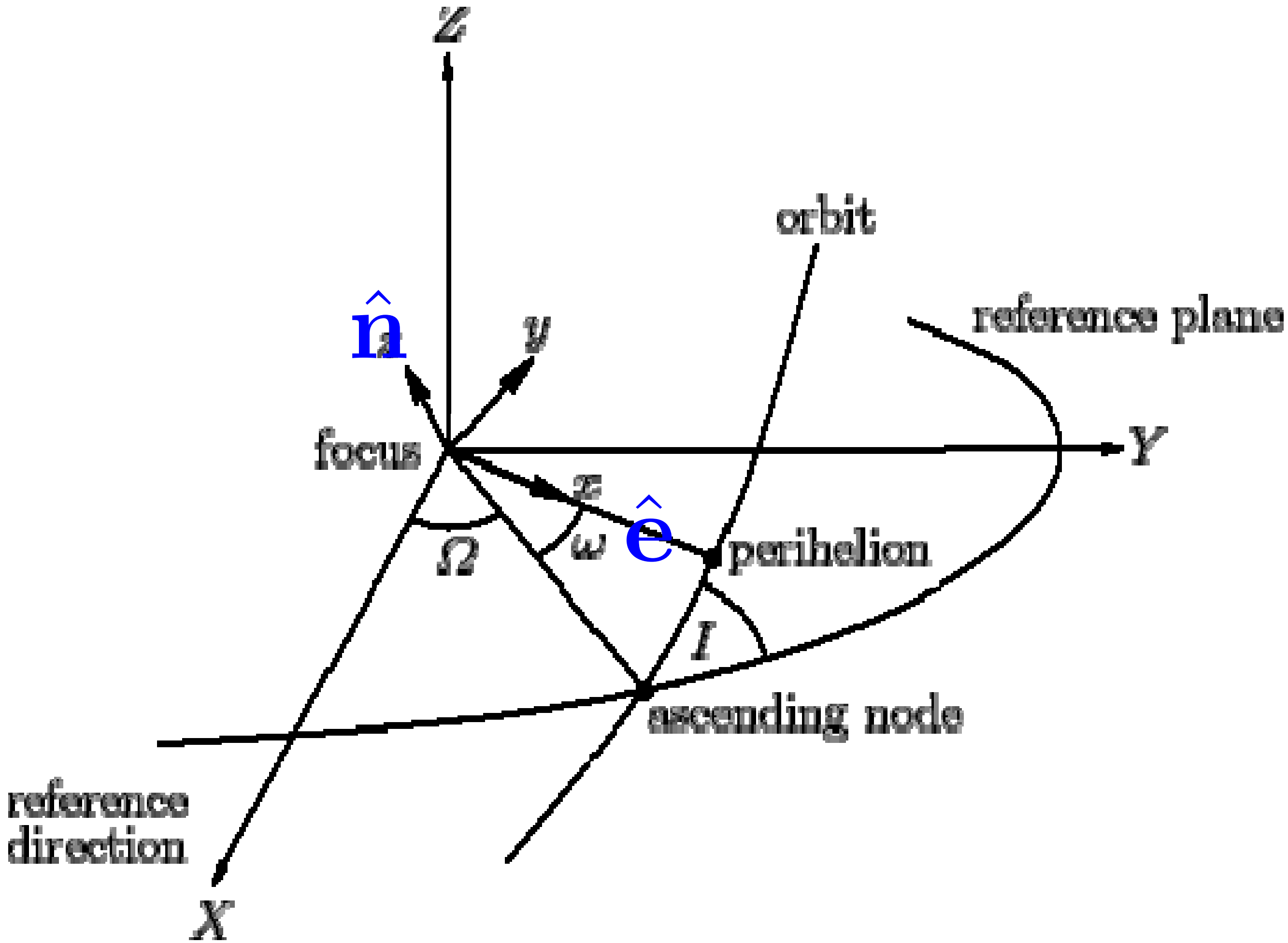
Kozai Mechanism

Conserved quantity $\Theta = (1 - e^2)\cos^2(I)$

Eccentricity is a minimum when inclination is maximum

$$\text{Kozai frequency } \dot{\omega}_K \sim \frac{M_b}{M_\star}\left(\frac{a}{a_b}\right)^3 \Omega_p$$

Digression:
Secular Dynamics of Planetary Motion



$$a, e, \hat{\mathbf{n}}, \hat{\mathbf{e}}$$

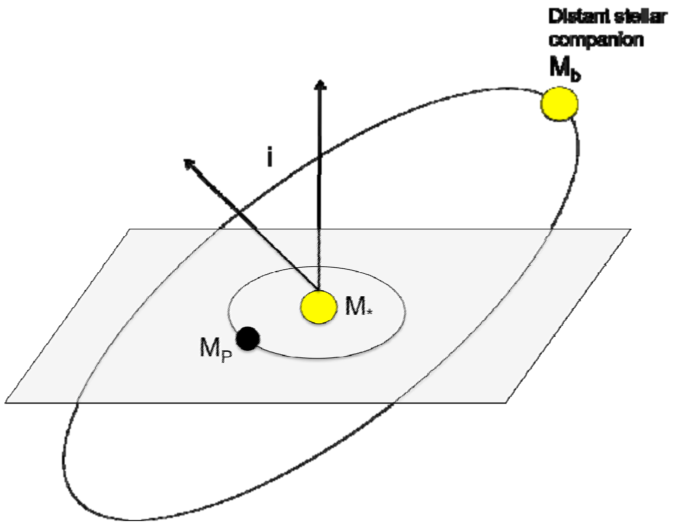
$$a,\ \mathbf{j} = \sqrt{1-e^2}\hat{\mathbf{n}},\ \mathbf{e} = e\hat{\mathbf{e}}$$

$$a, e, I, \Omega, \omega$$

Kozai-Lidov Oscillations

$$\langle\langle\Phi_2\rangle\rangle = \frac{GM_b a^2}{8a_b^3(1-e_b^2)^{3/2}}\left[1 - 6e^2 - 3(\mathbf{j}\cdot\hat{\mathbf{n}}_b)^2 + 15(\mathbf{e}\cdot\hat{\mathbf{n}}_b)^2\right]$$

Two secular constants of motion:



$$(1-e^2)^{1/2}\cos i = \text{const}$$

$$\langle\langle\Phi_2\rangle\rangle = \text{const}$$

➔ $$e^2(5\sin^2 i \sin^2\omega - 2) = \text{const}$$

$$e_{\max} \simeq \left(1 - \frac{5}{3}\cos^2 i_0\right)^{1/2}$$

Kozai Mechanism

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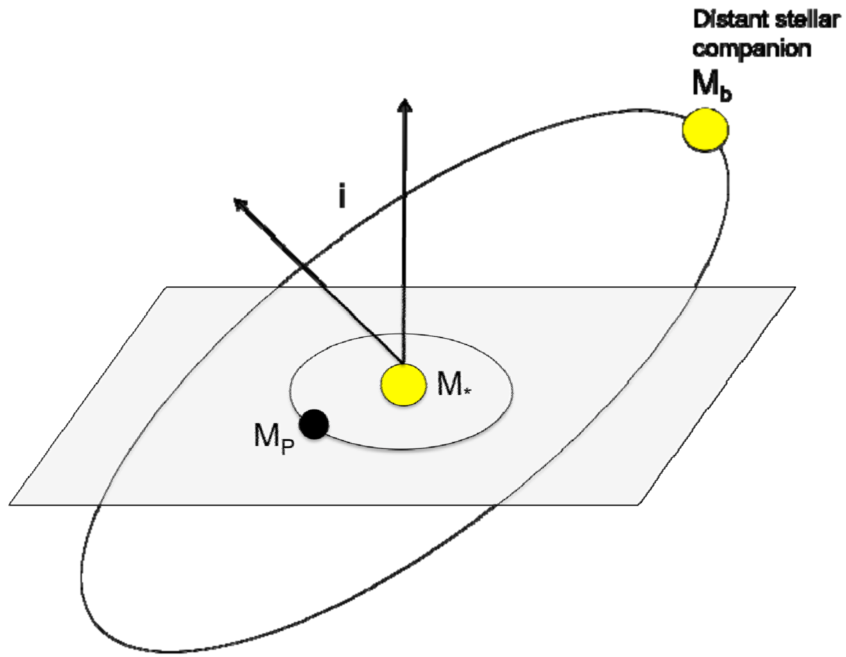
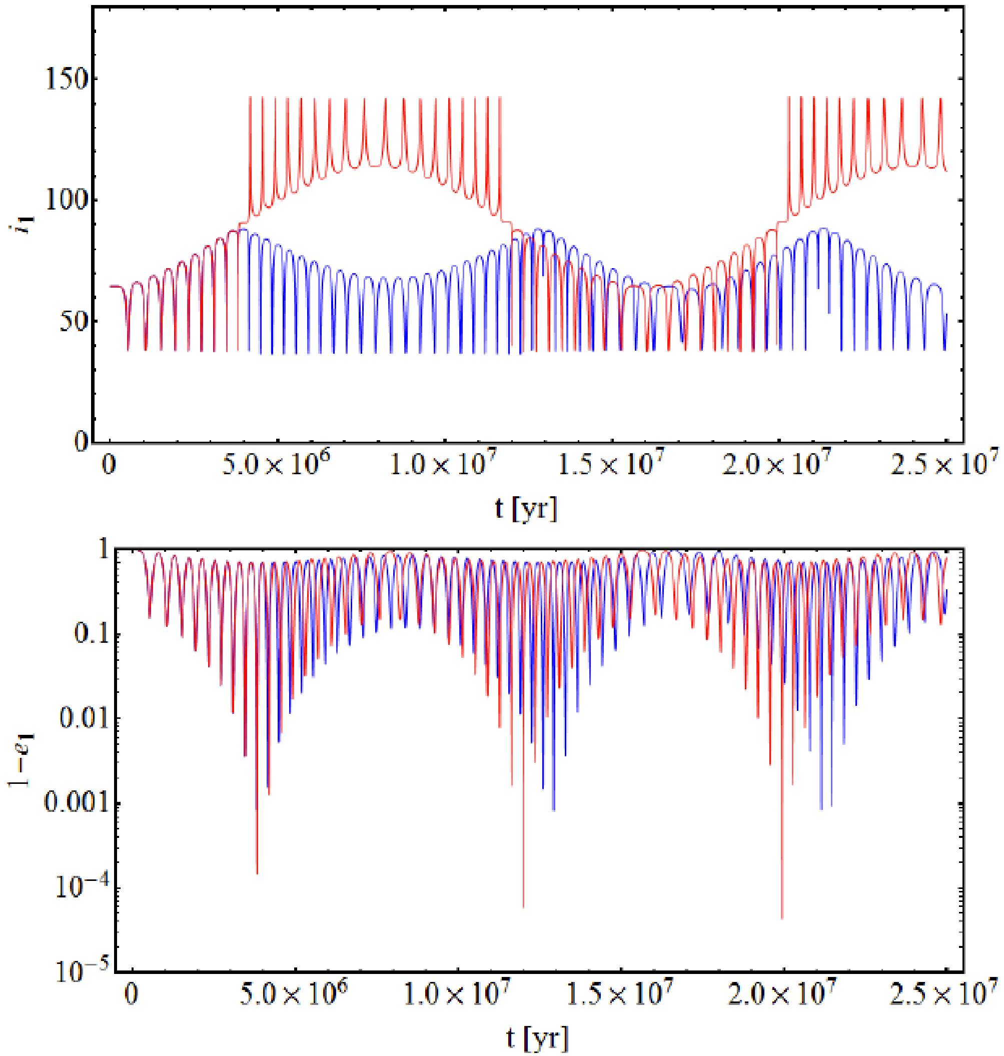
Corrections to Kozai

- Additional periastron precession due to GR, static tides, oblateness
- Octupole-order effects





$$\varepsilon_{\rm Oct} = \left(\frac{a}{a_b}\right) \frac{e_b}{1-e_b^2}$$

Figure 2. Results of numerical integrations for initial conditions $m_0 = 1M_\odot$, $m_1 = 1M_J$, $m_2 = 40M_J$, $a_1 = 6$AU, $a_2 = 100$AU, $e_1 = 0.001$, $e_2 = 0.6$, $i_1 = 64.7°$, $i_2 = 0.3°$, $\omega_1 = 45°$, $\omega_2 = 0°$. The red lines are from the integration of pure Kozai in octupole order, while the blue lines are the results of integration including short-range forces.

B.Liu+2014

Hot Jupiter formation through Kozai Oscillations + Tide

- Kozai oscillations pump planet into high-e orbit

- Tidal dissipation during high-e phases causes orbital decay

- Combined effects can result in planets in ~ few days orbit from host star (a hot Jupiter is born!)

Final planet orbit not necessarily aligned with stellar spin axis

The spin in spin-orbit misalignment

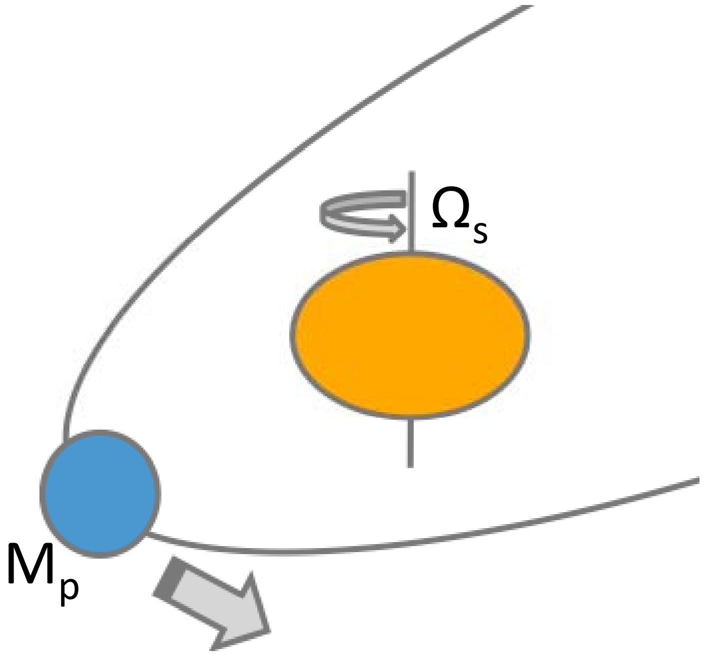
During Kozai cycle, planet orbit undergoes large variation in both **eccentricity** and **inclination** relative to the outer binary axis.

The spin in spin-orbit misalignment

During Kozai cycle, planet orbit undergoes large variation in both **eccentricity** and **inclination** relative to the outer binary axis.

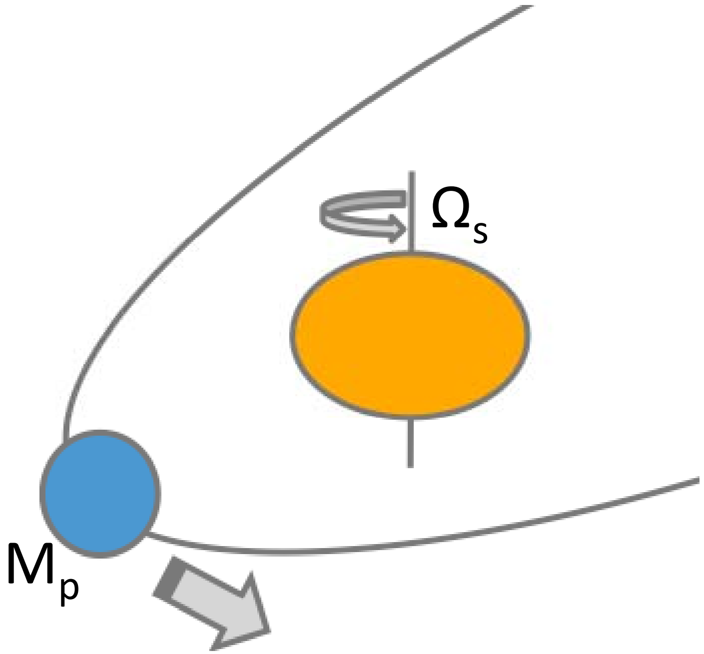
Question: what happens to stellar spin during this time?

The Stellar Spin Dance



- Star is spinning - anywhere from 3 to 30 days. → oblate → will precess

The Stellar Spin Dance



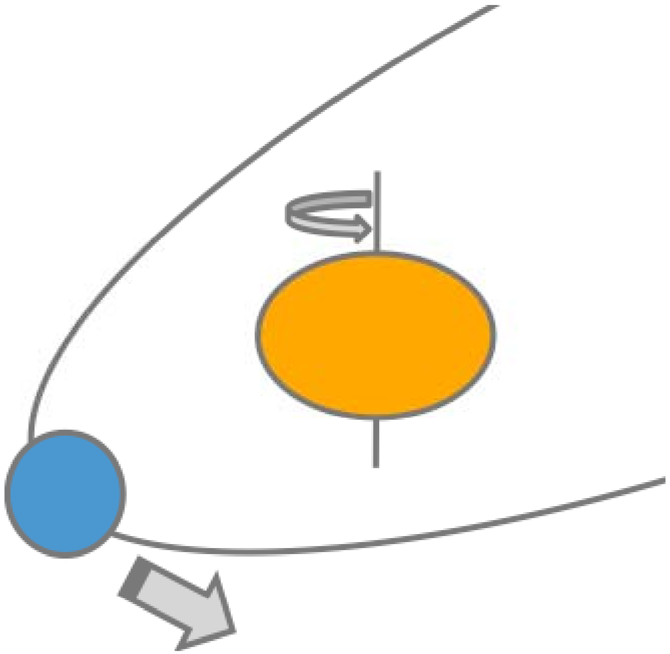
- Star is spinning - anywhere from 3 to 30 days. → oblate → will precess

$$\frac{d\hat{\mathbf{S}}}{dt} = \Omega_{\rm ps}\hat{\mathbf{L}} \times \hat{\mathbf{S}}$$

$$\Omega_{\rm ps} = -\frac{3GM_p(I_3 - I_1)}{2a^3(1-e^2)^{3/2}}\frac{\cos\theta_{\rm sl}}{S}$$

$$\propto \frac{\Omega_{\rm s} M_{\rm p}}{a^3(1-e^2)^{3/2}}$$

The Stellar Spin Dance



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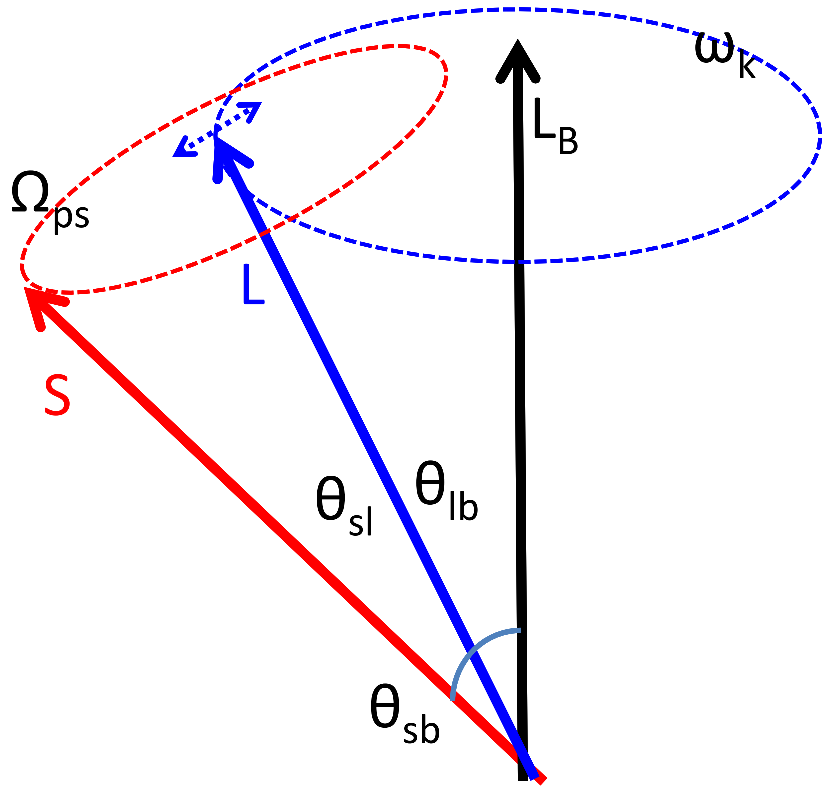
It takes two to tango

Just as the planet torques the star, the star torques the planet's orbit – so really we have mutual precession of L and S

So the ingredients are…

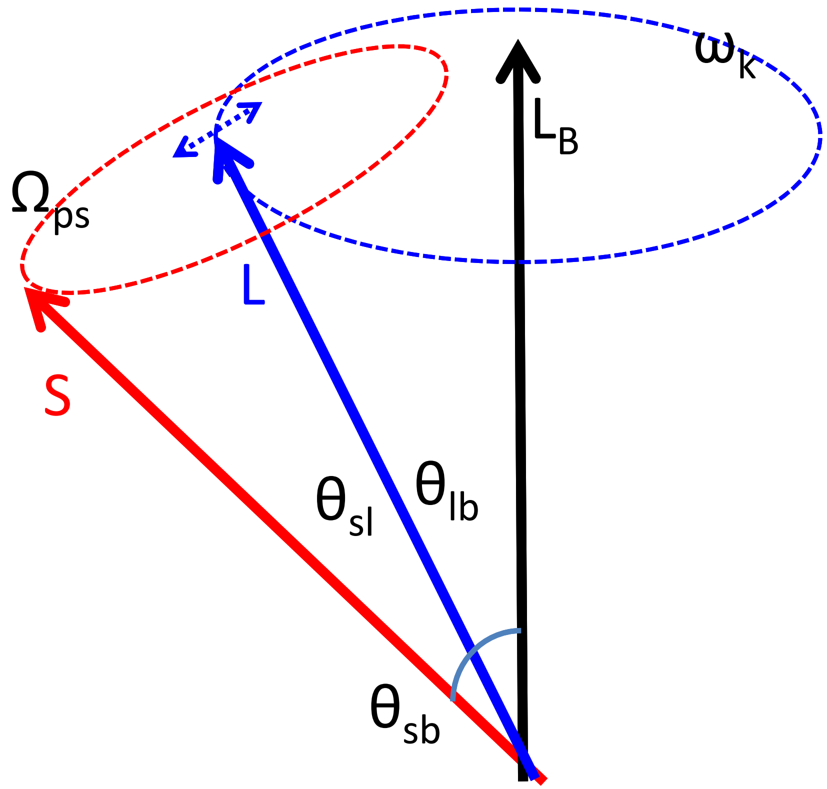
- Kozai: eccentricity and inclination oscillations at frequency ω_k
- Mutual spin and orbital precession at frequency Ω_{ps}



So the ingredients are…

- Kozai: eccentricity and inclination oscillations at frequency ω_k
- Mutual spin and orbital precession at frequency Ω_{ps}

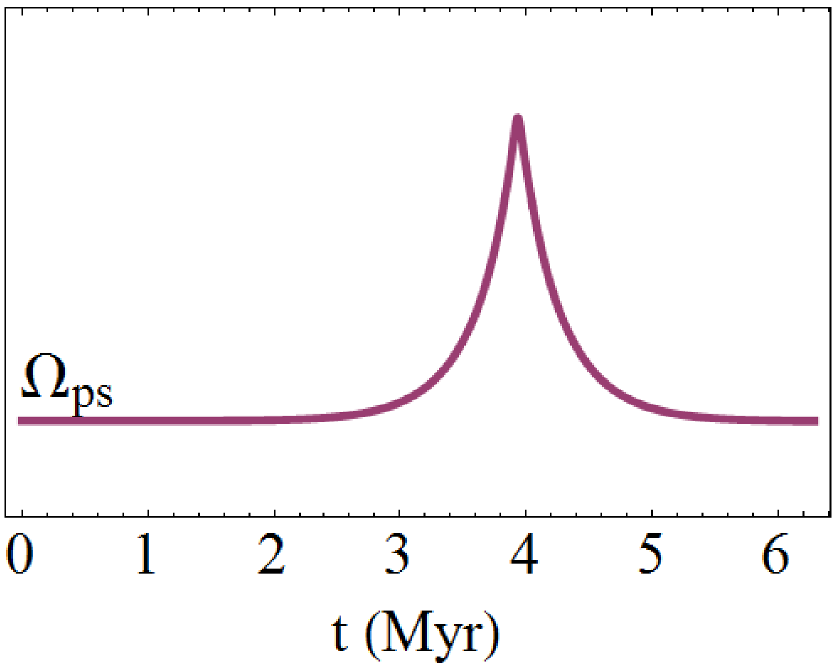
Throw them together → what happens?

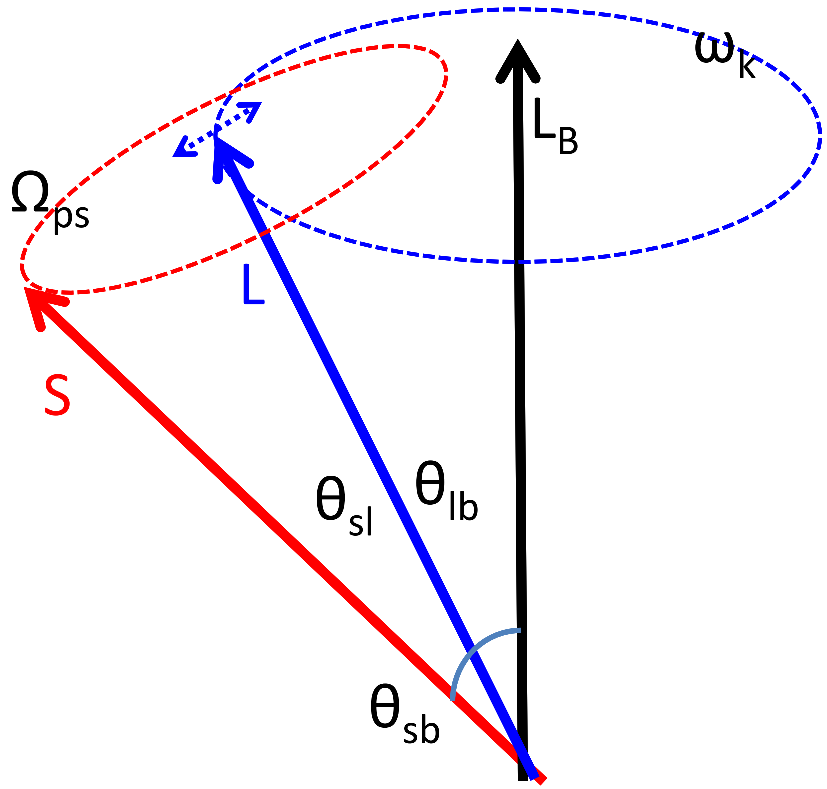


During a Kozai cycle…

The precession frequency changes:

$$\Omega_{\rm ps} \propto \frac{\Omega_{\rm s} M_{\rm p}}{a^3(1-e^2)^{3/2}}$$





This leads to three qualitatively different regimes…

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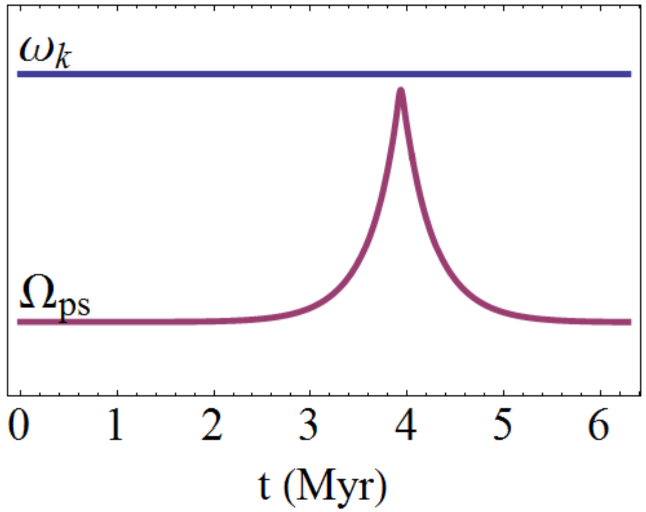
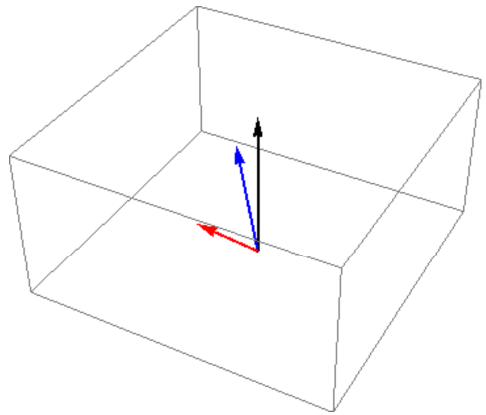


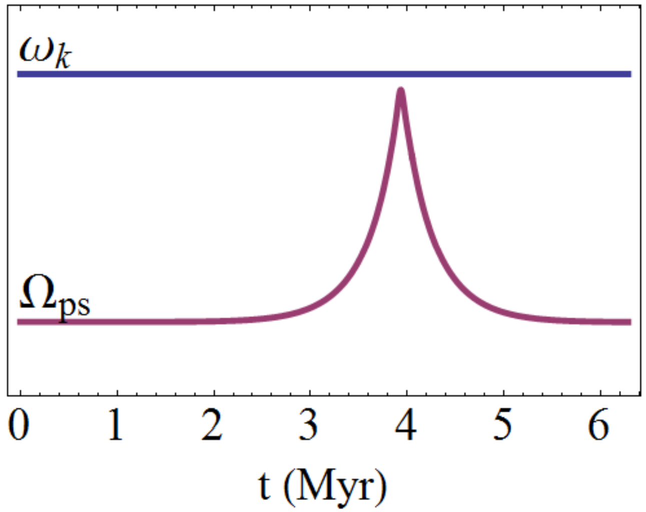


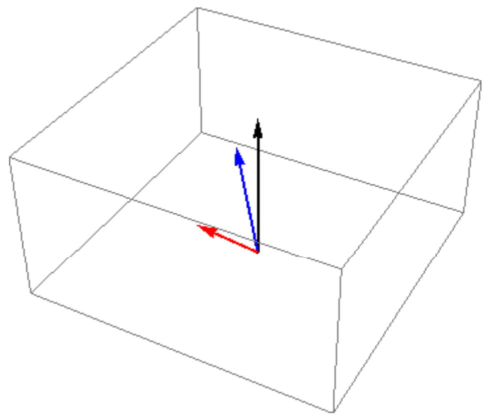
“<u>Non-adiabatic</u>”

— L_B
— L
— S

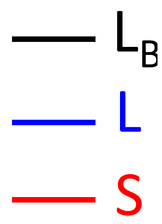
This leads to three qualitatively different regimes…

I





"<u>Non-adiabatic</u>"
Prediction:
θ_{sb} constant



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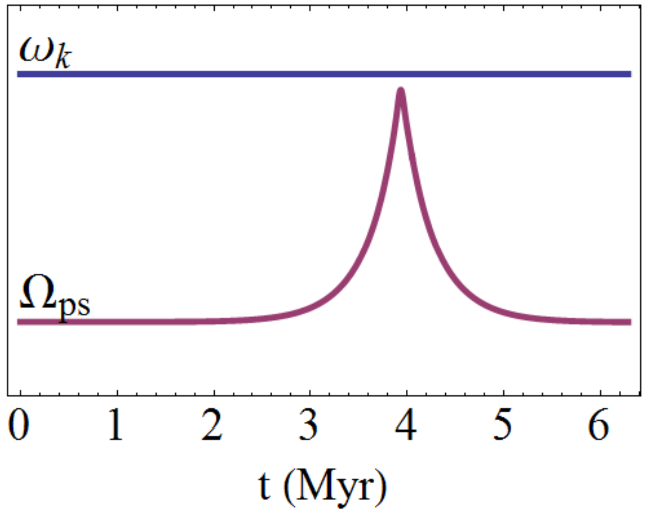

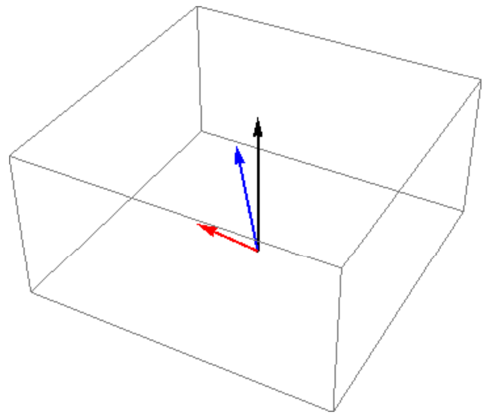

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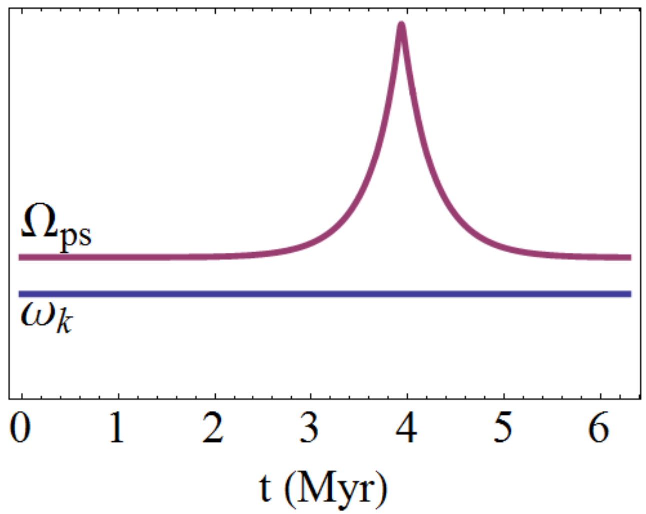
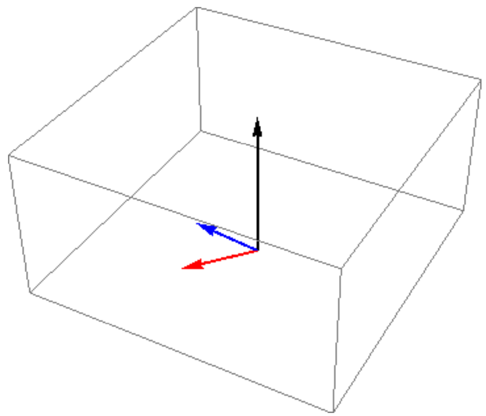
Prediction:

θ_{sb} constant

III





“<u>Adiabatic</u>”

L_B

L

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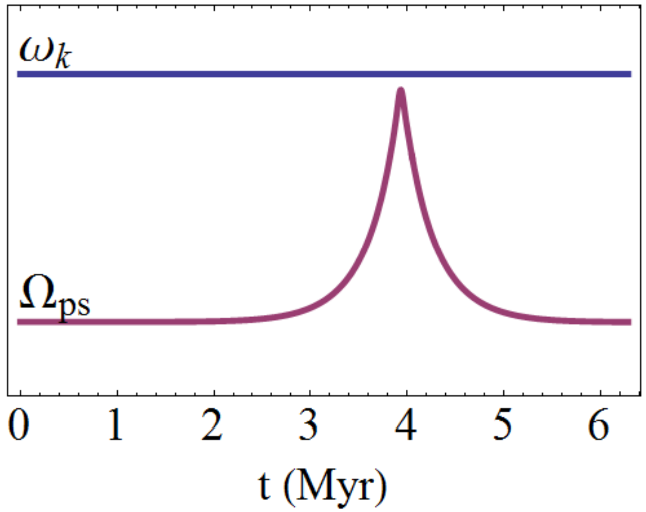

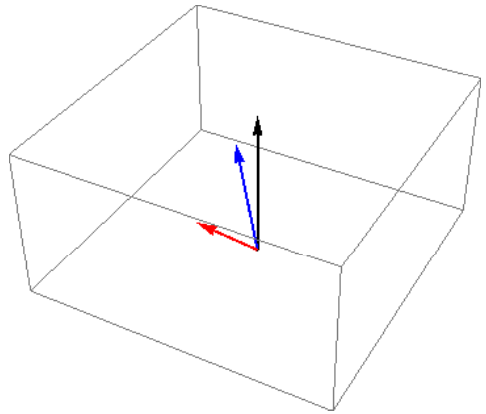

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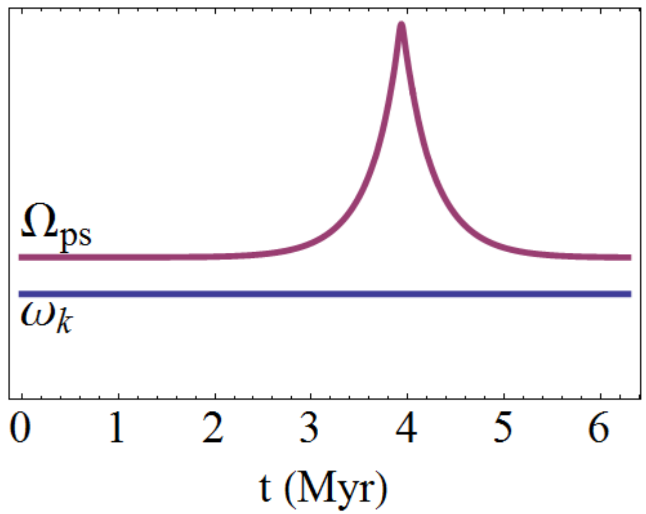
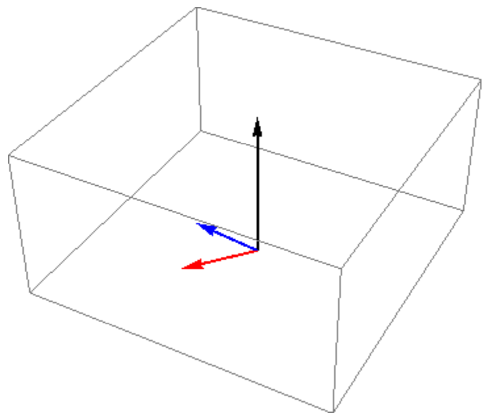
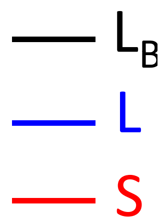




“<u>Adiabatic</u>”
Prediction:
θ_{sl} constant

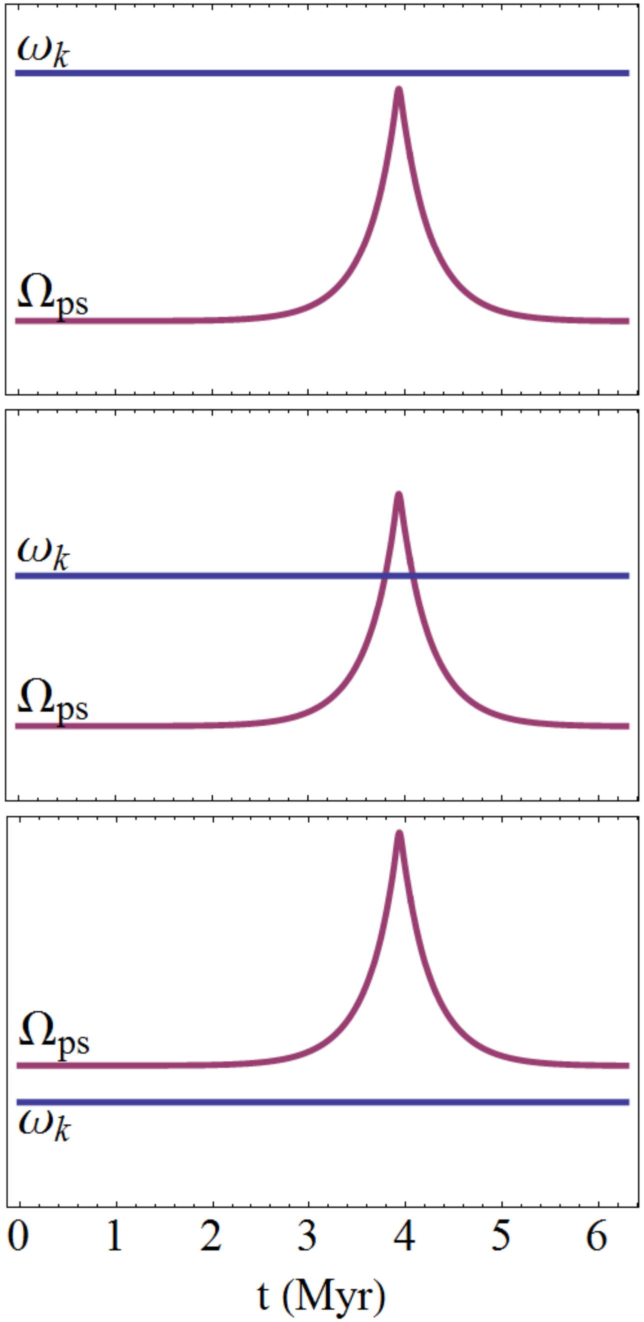


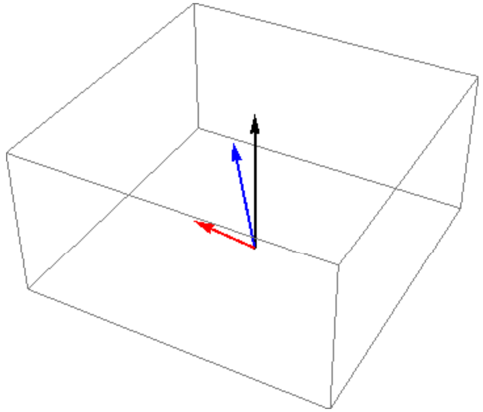
This leads to three qualitatively different regimes…

I

II

III



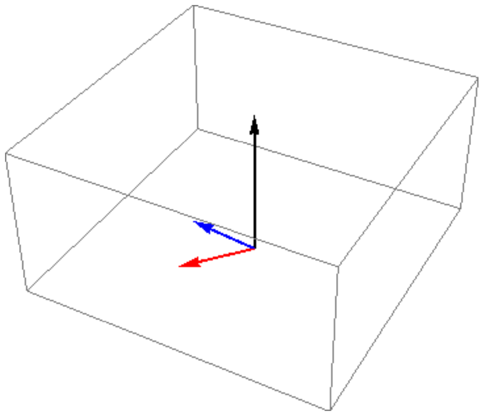


"<u>Non-adiabatic</u>"

Prediction:

θ_{sb} constant

"<u>Trans-adiabatic</u>"



"<u>Adiabatic</u>"

Prediction:

θ_{sl} constant

L_B

L

S

This leads to three qualitatively different regimes…

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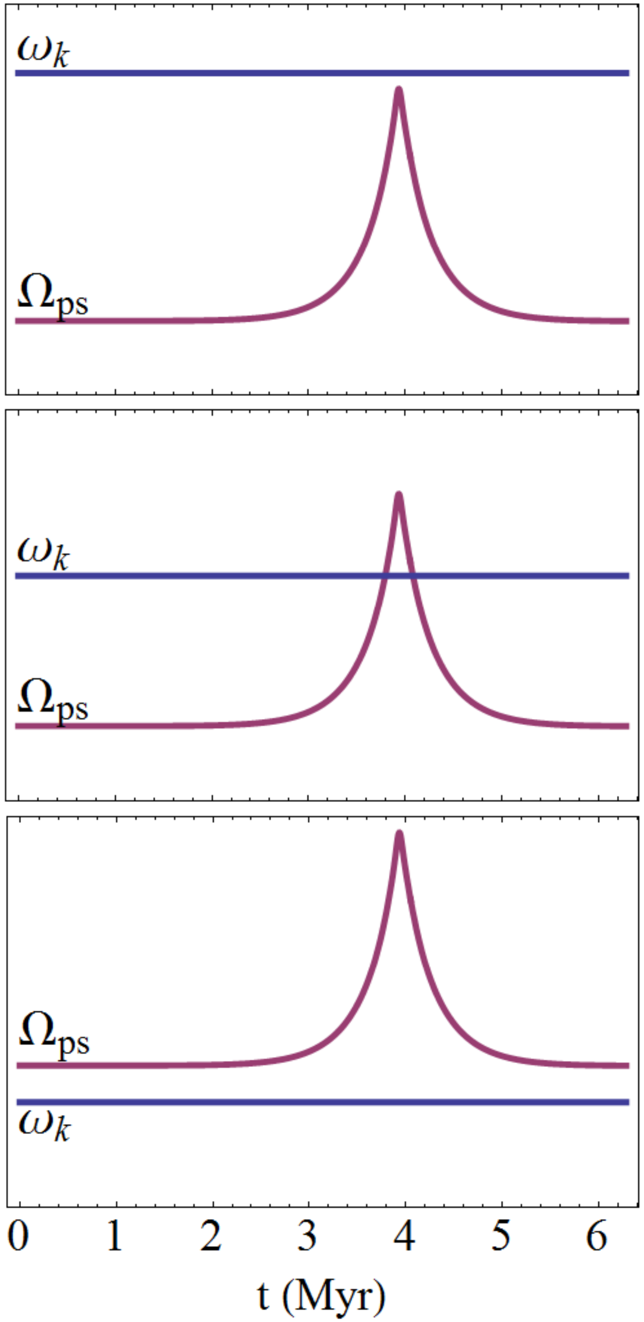
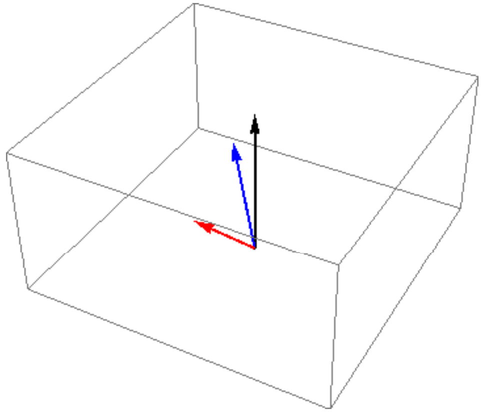




"<u>Non-adiabatic</u>"

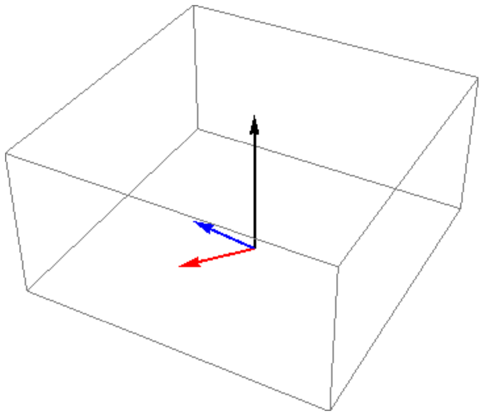
Prediction:

θ_{sb} constant

"<u>Trans-adiabatic</u>"

~~Prediction~~: Speculation:

Interesting behavior due to secular resonance!



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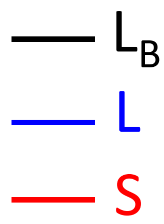
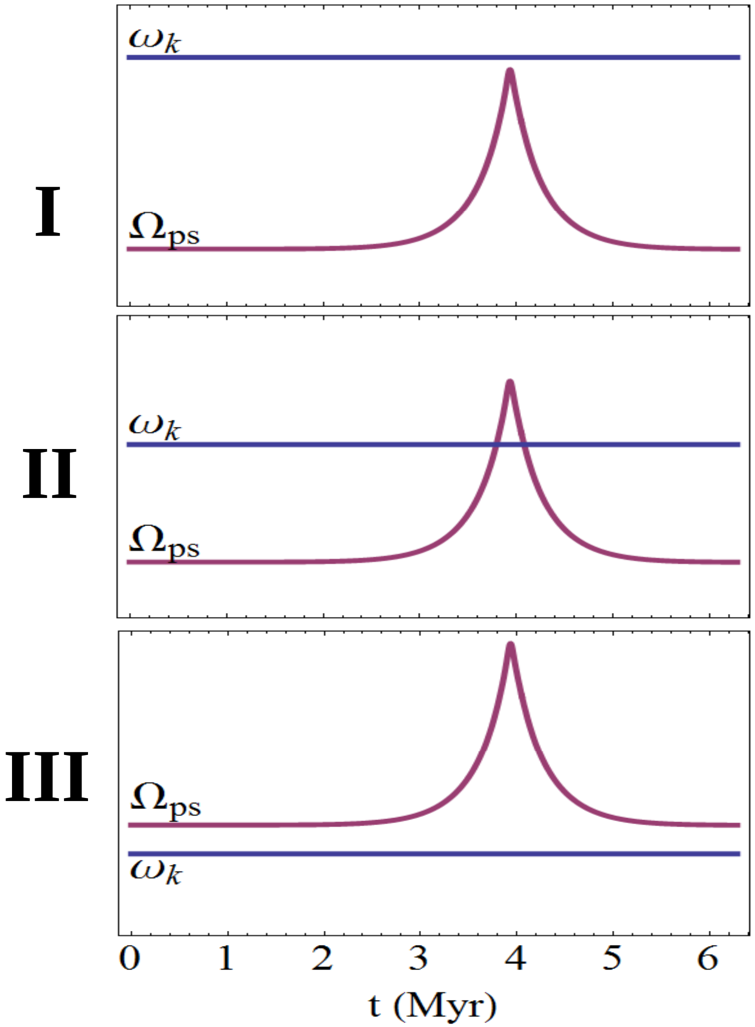
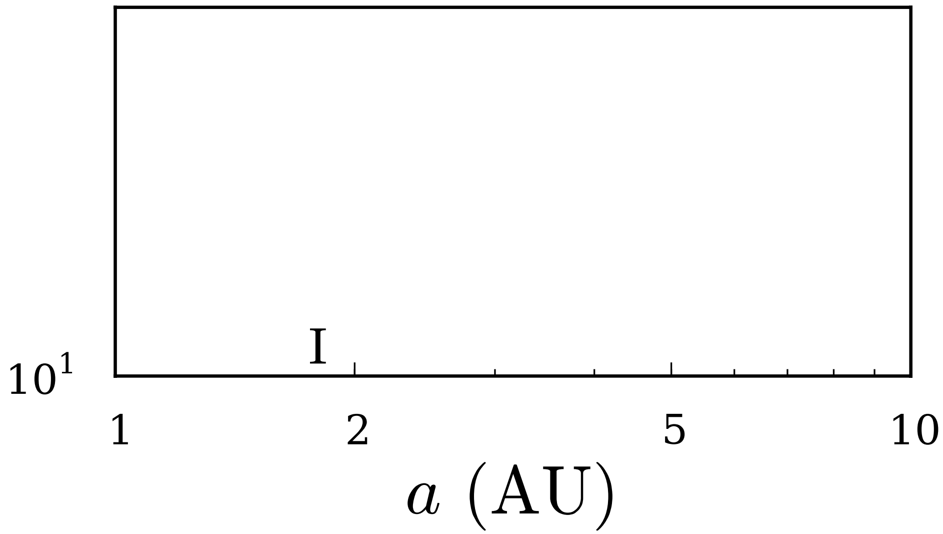


How to find these regimes:

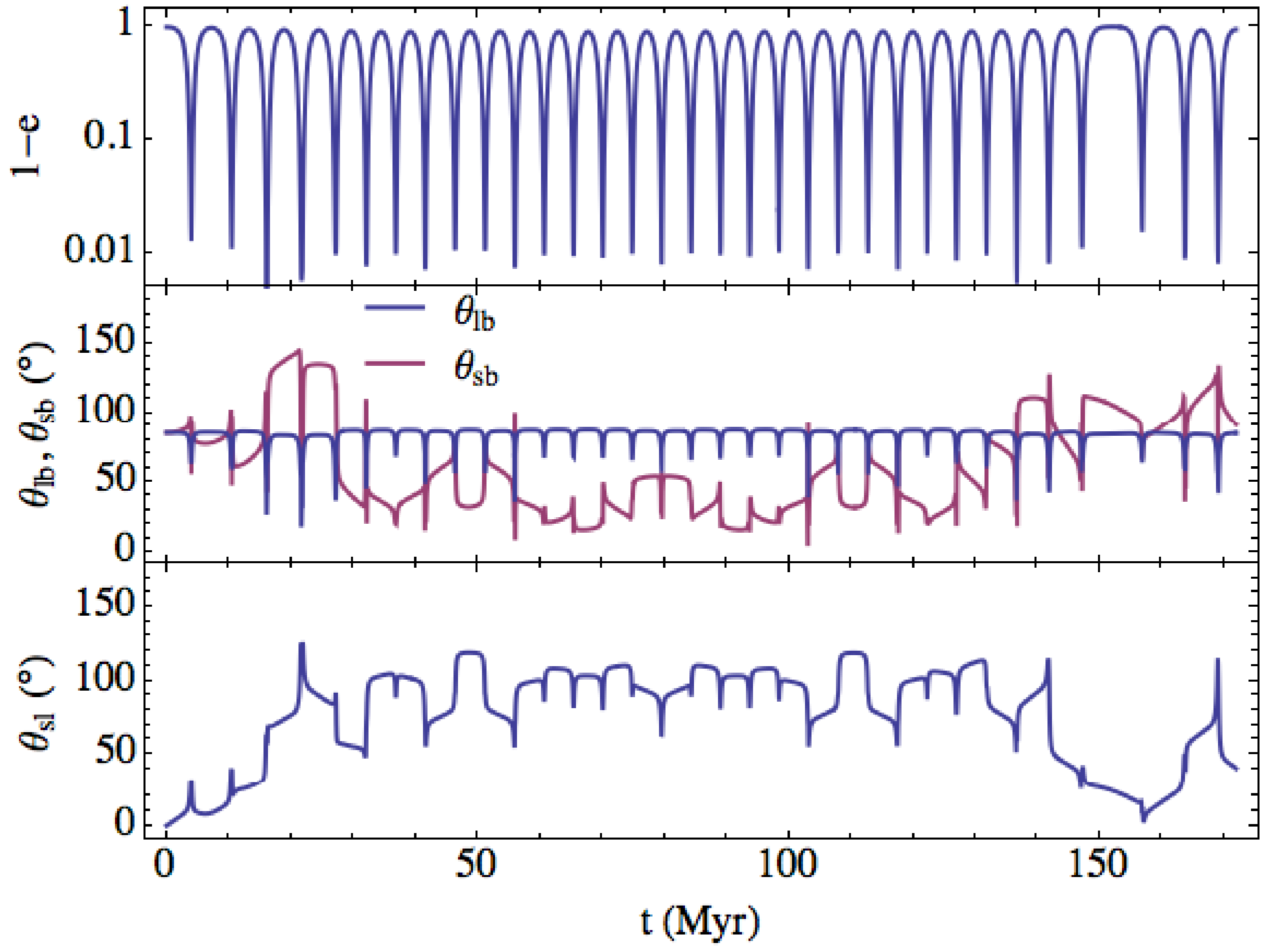


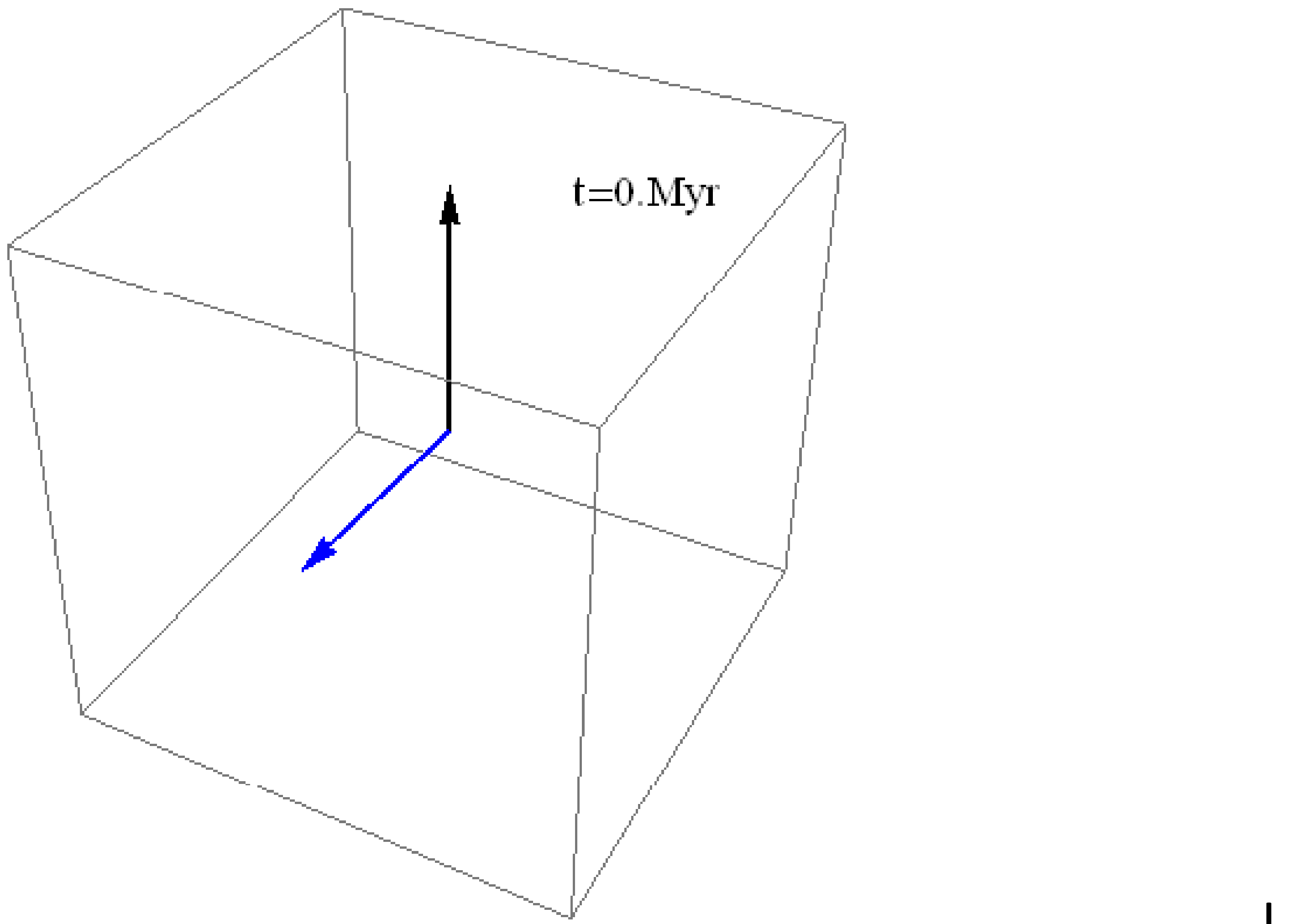
$$\hat{M}_{\rm p} \propto \frac{\Omega_{\rm s}}{2\pi/(3\ \rm days)}\frac{M_{\rm p}}{M_{\rm J}}$$



Finally, numerics!

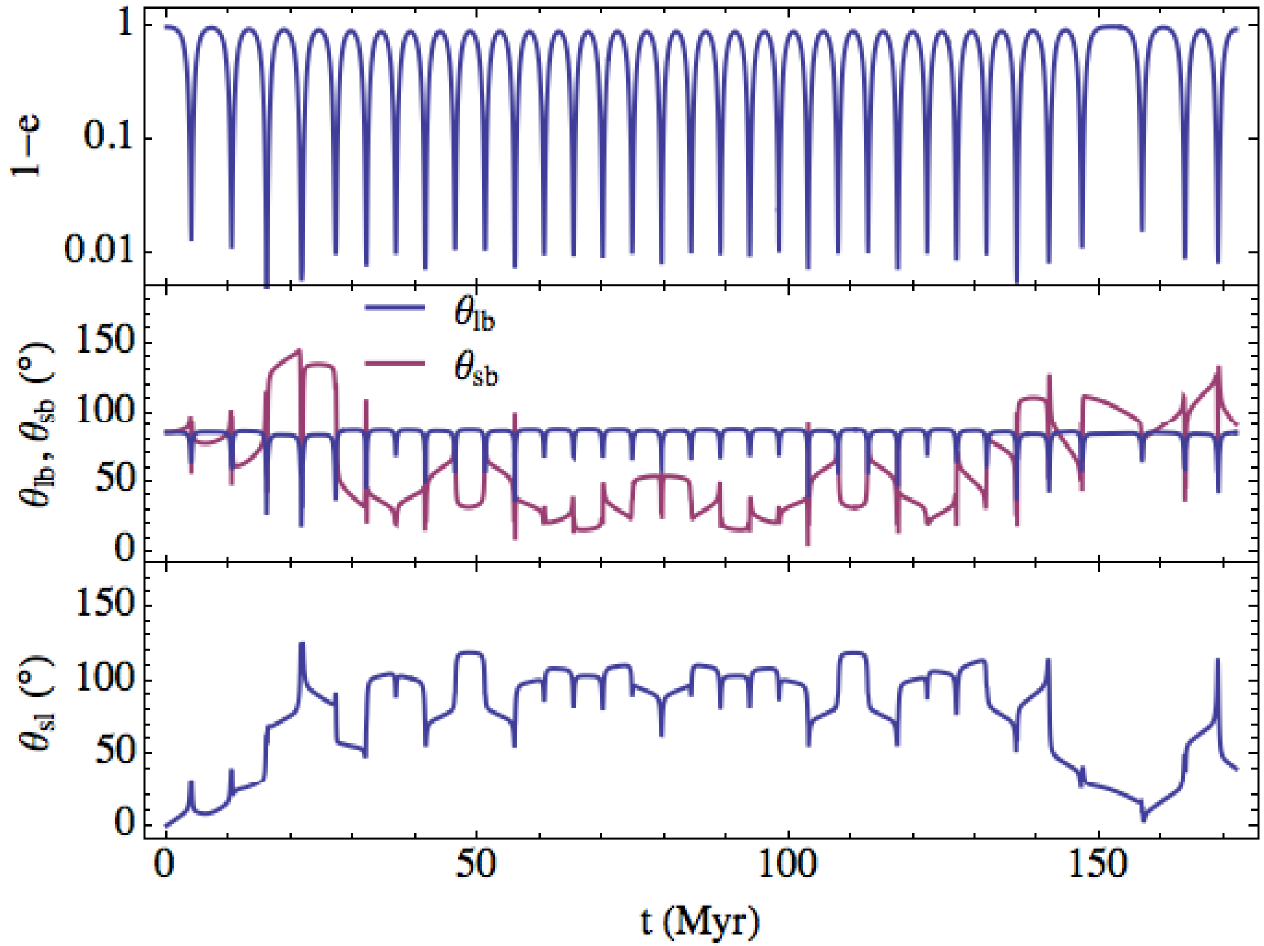
Armed with a qualitative understanding of what's going on, we can now actually take the Kozai+spin precession ODEs and see what happens!




t=0.Myr
L_B
L
S

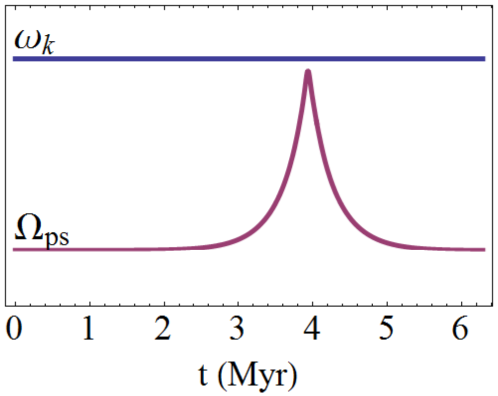
Visualizing the spin behavior

To check whether our predictions for different regime behavior were correct, we construct surfaces of section: scatter plots of the variables evaluated at each eccentricity maximum.



Numerical results for each of the three regimes

I



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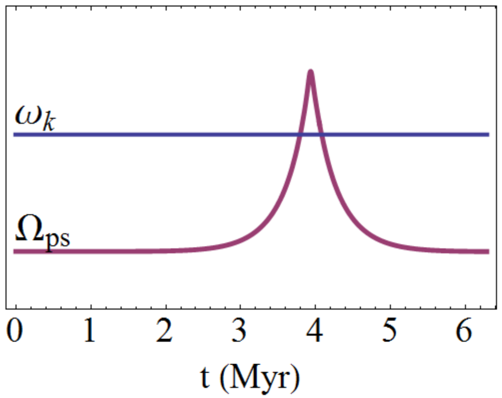

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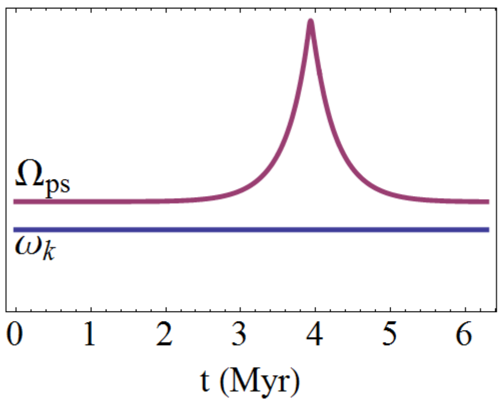

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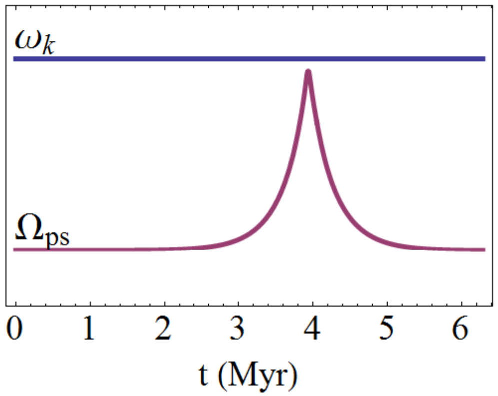

"<u>Non-adiabatic</u>"

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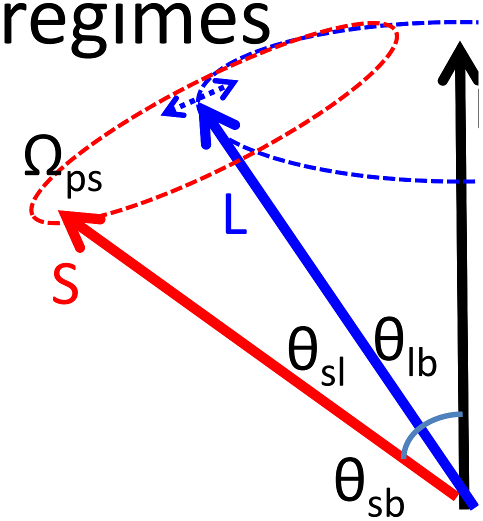

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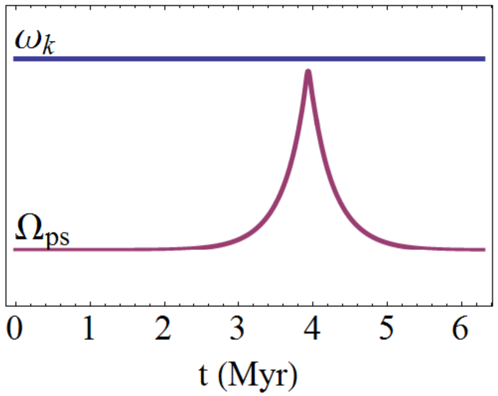

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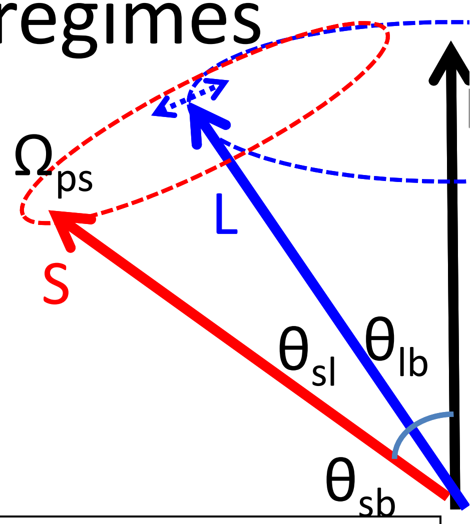

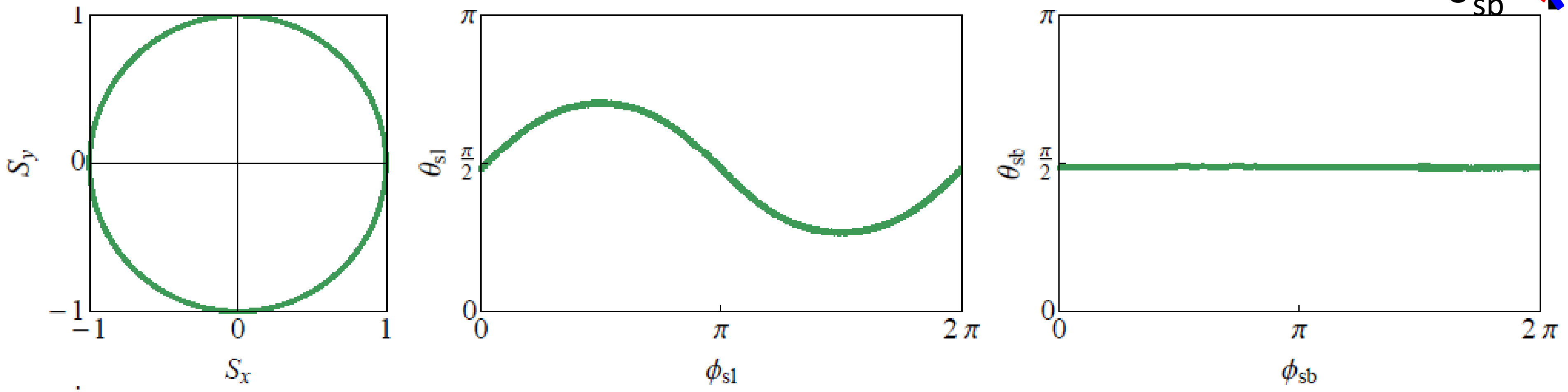

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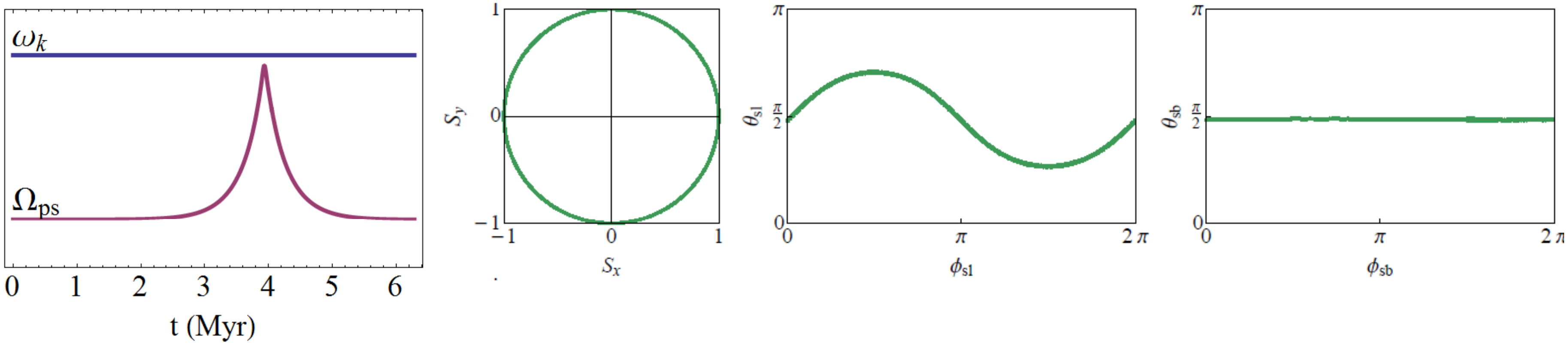

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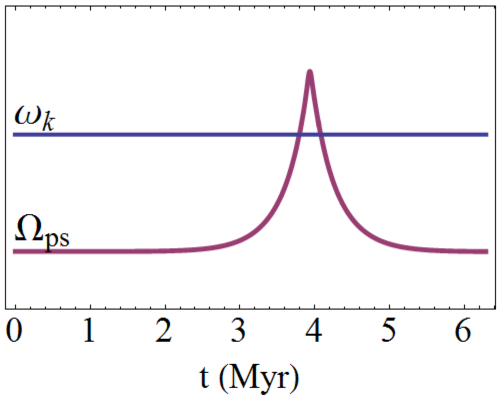

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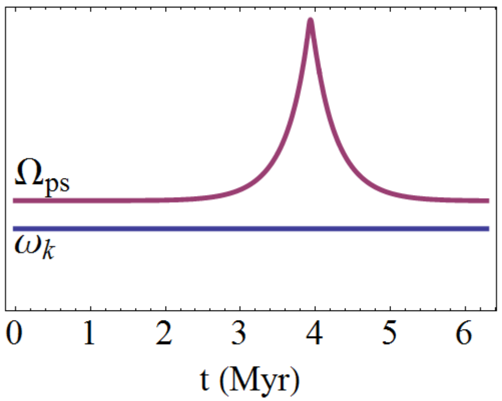

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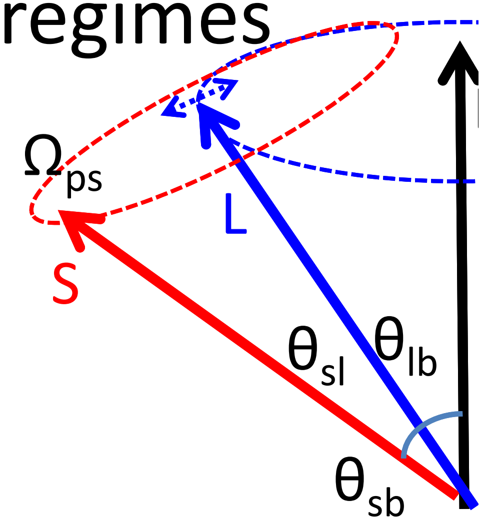

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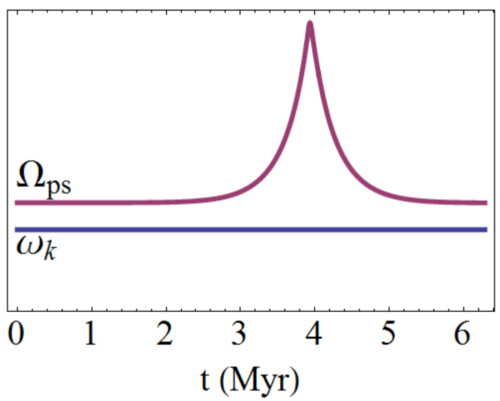

“<u>Adiabatic</u>”

Prediction:

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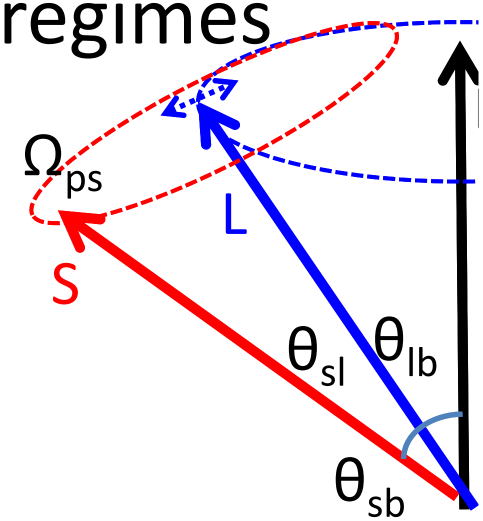
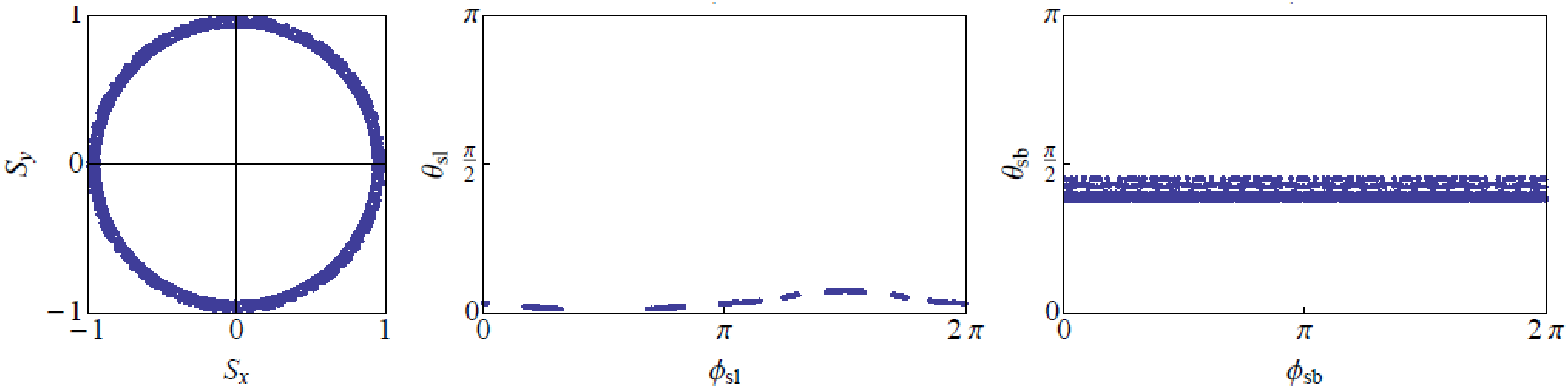
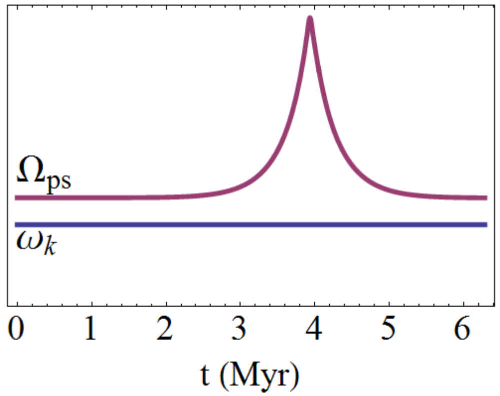
Numerical results for each of the three regimes





III



"<u>Adiabatic</u>"

Prediction:

θ_{sl} constant

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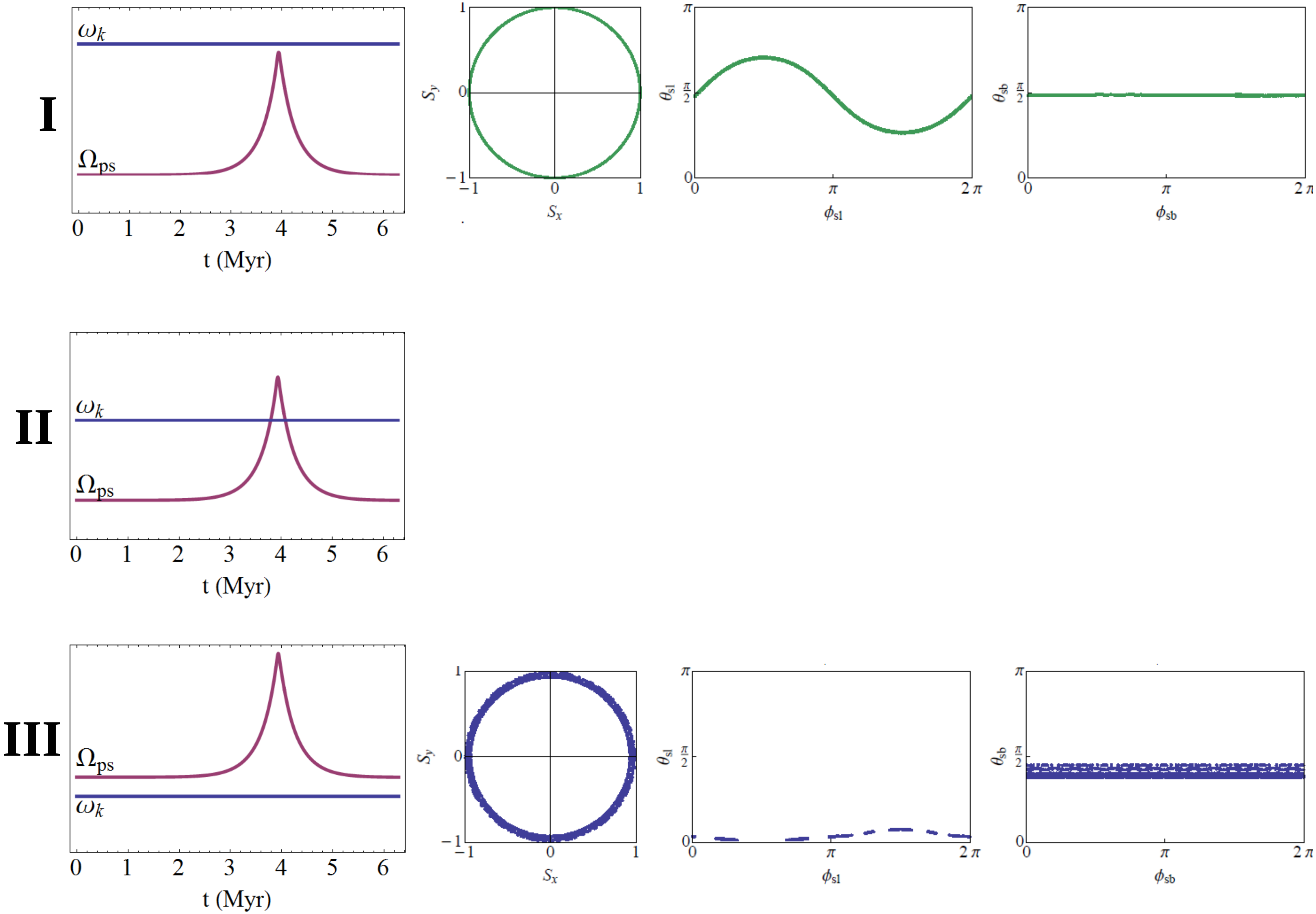
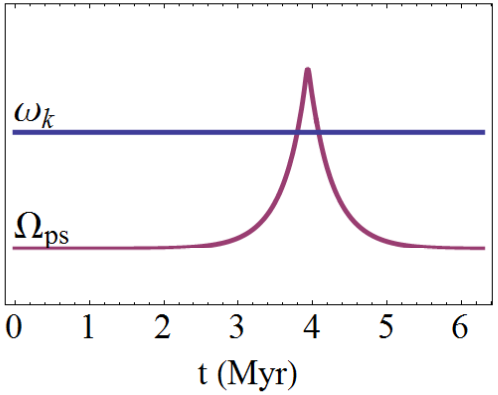


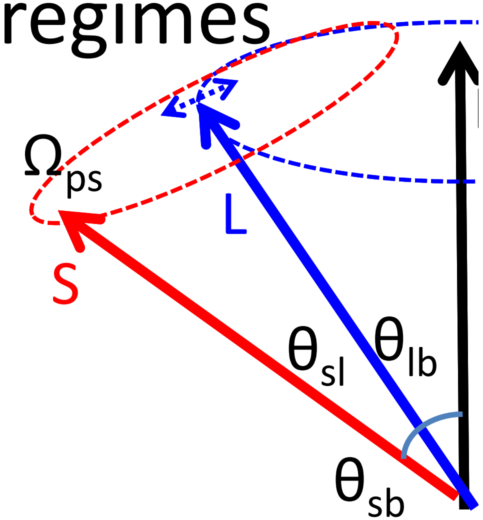
Numerical results for each of the three regimes

II



“Trans-adiabatic”

~~Prediction~~: Speculation: Interesting behavior due to secular resonance!



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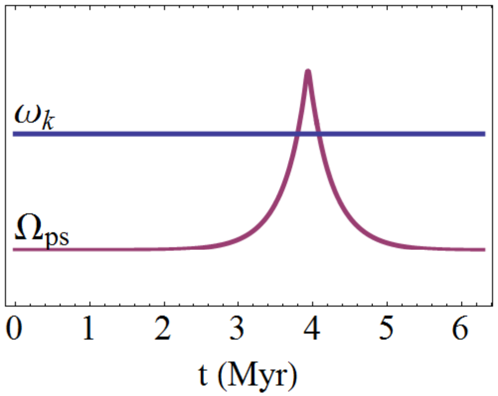
II



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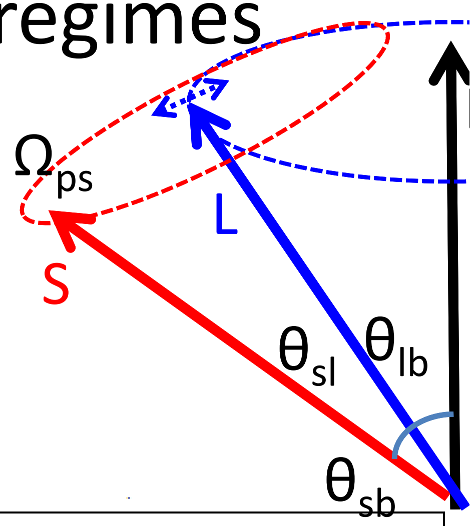

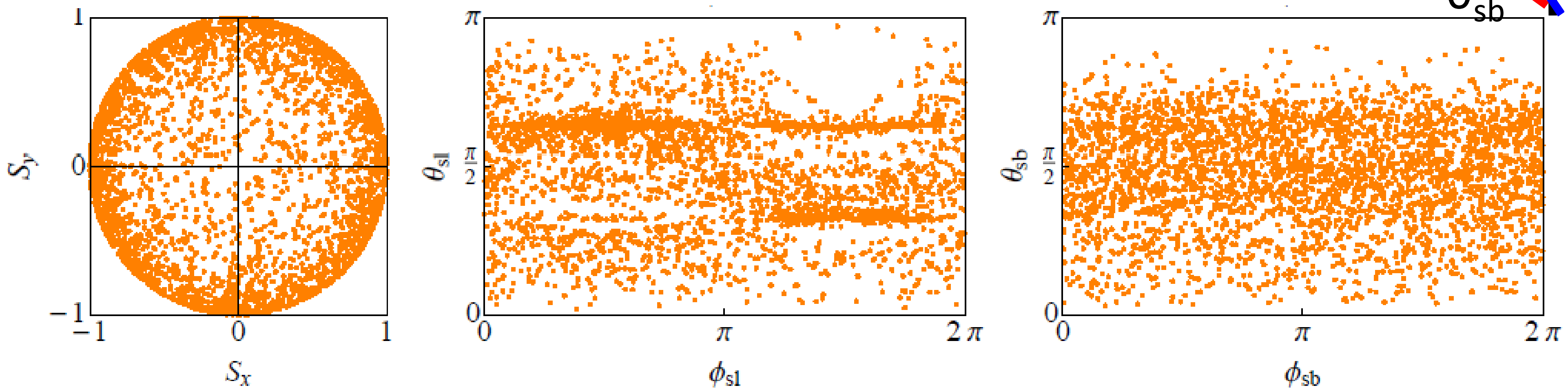

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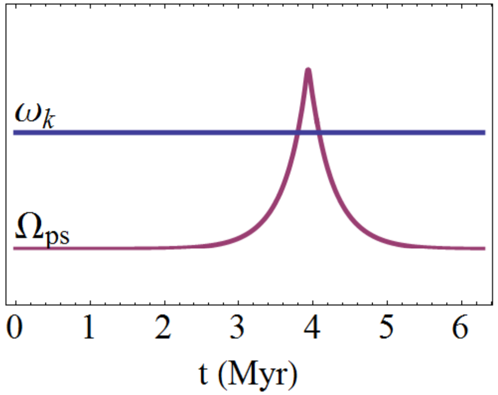

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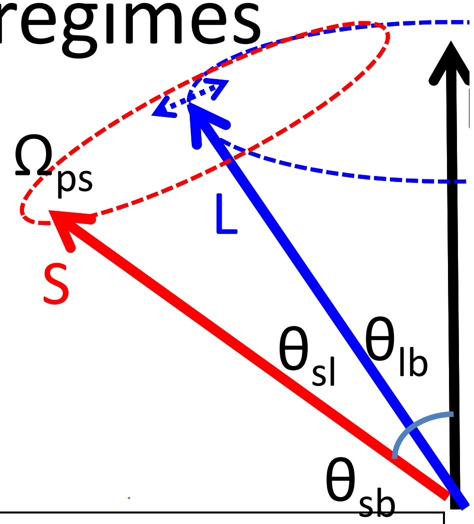

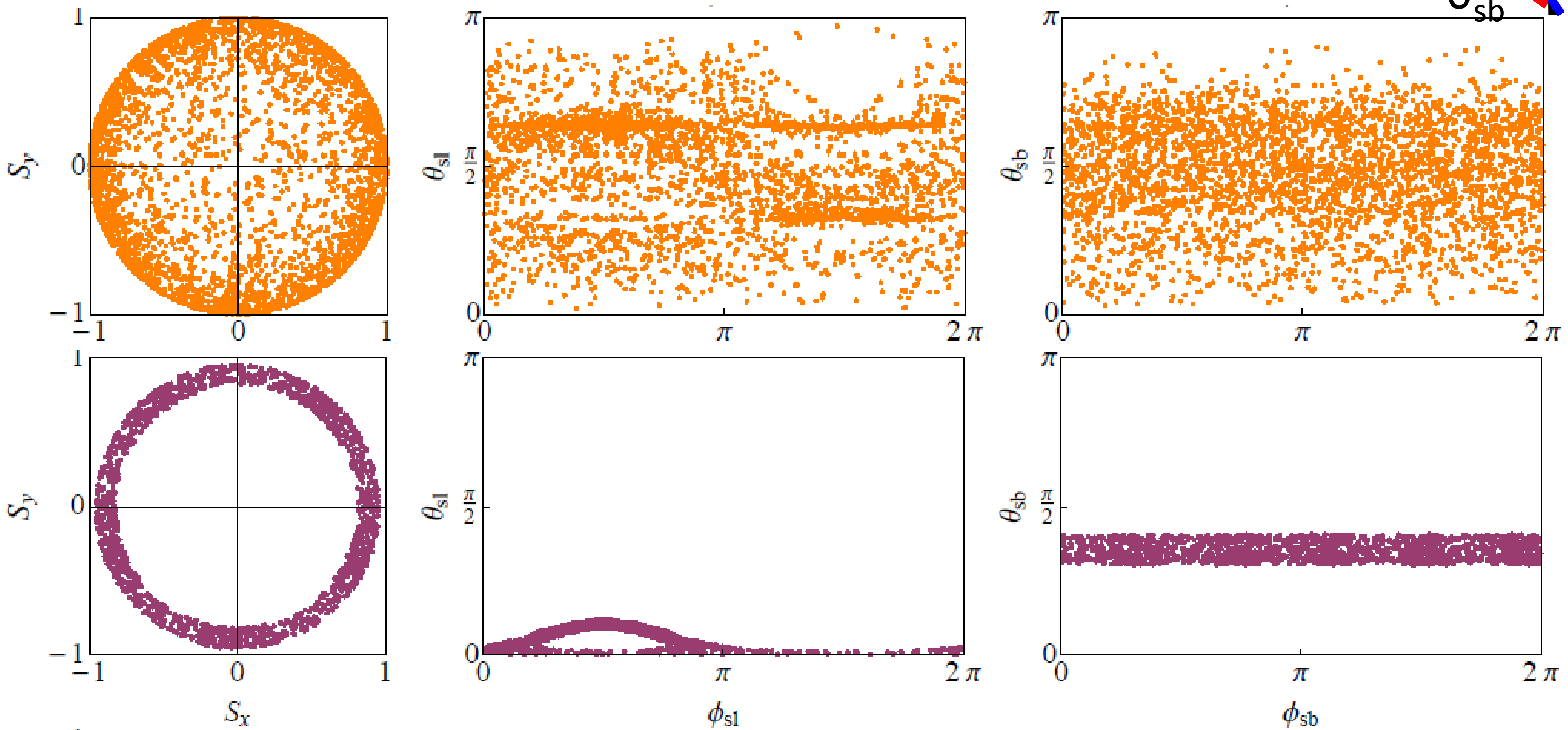

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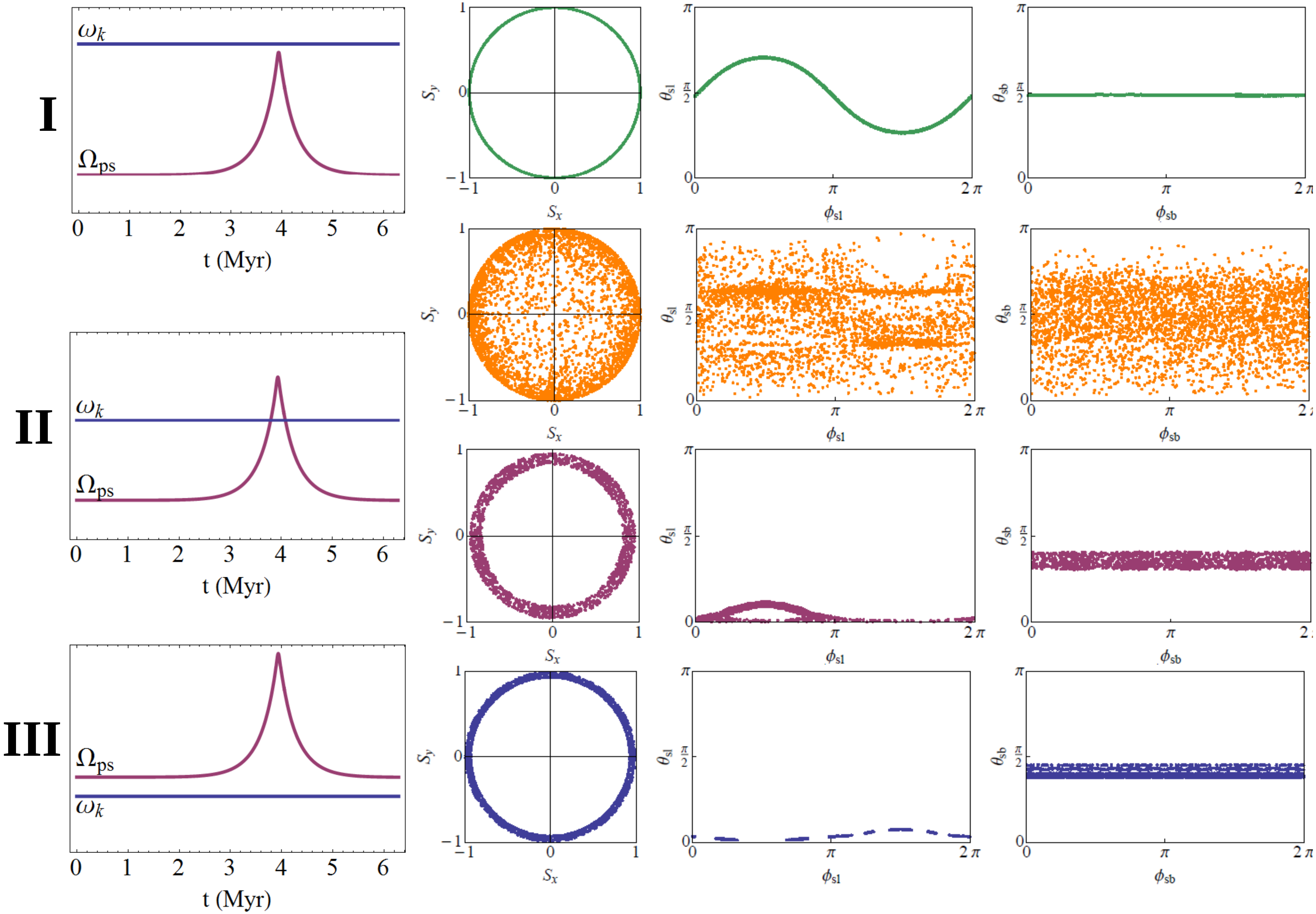

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Quasiperiodic
(not chaotic)

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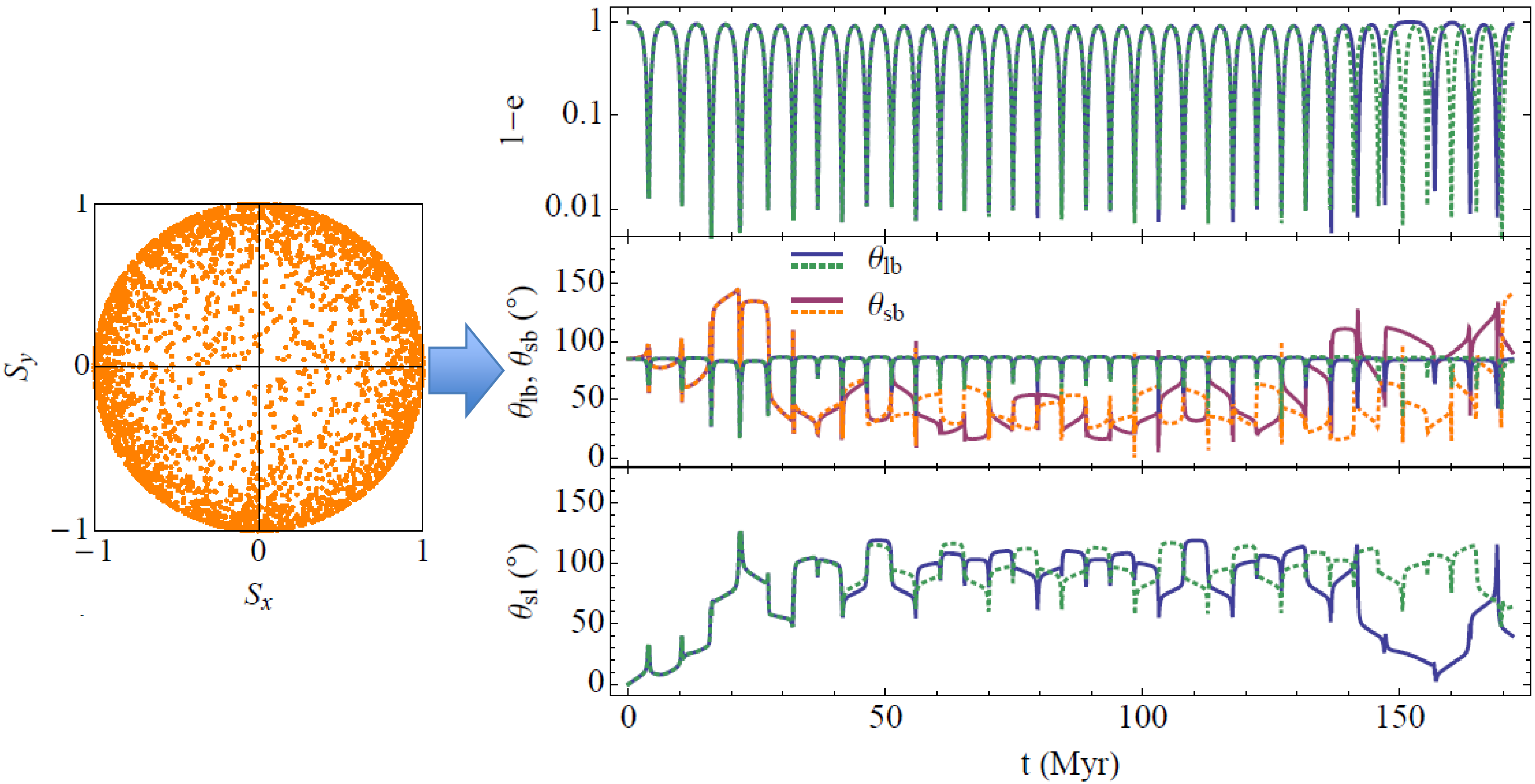
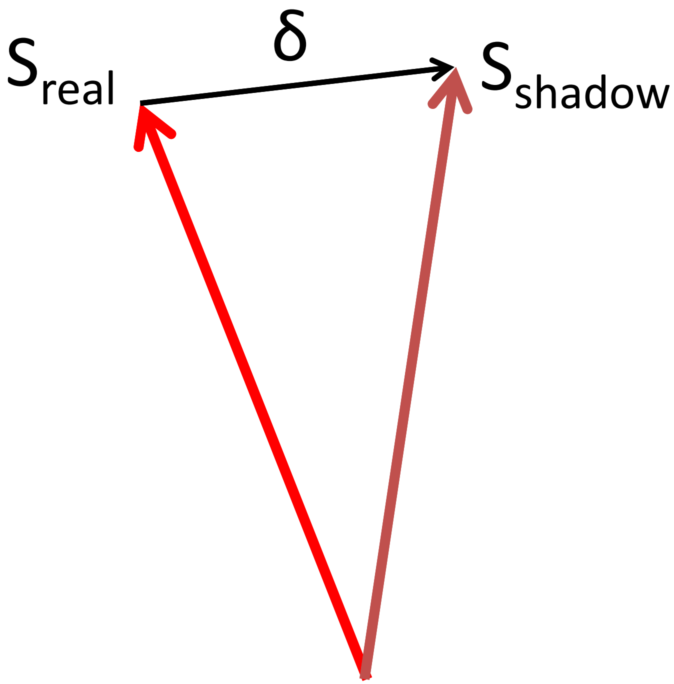
Chaotic

By eye…



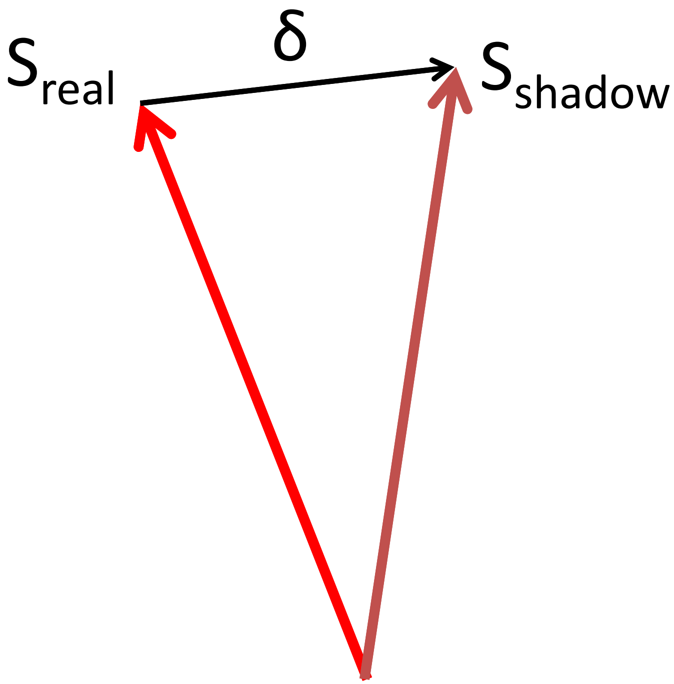
More precisely…

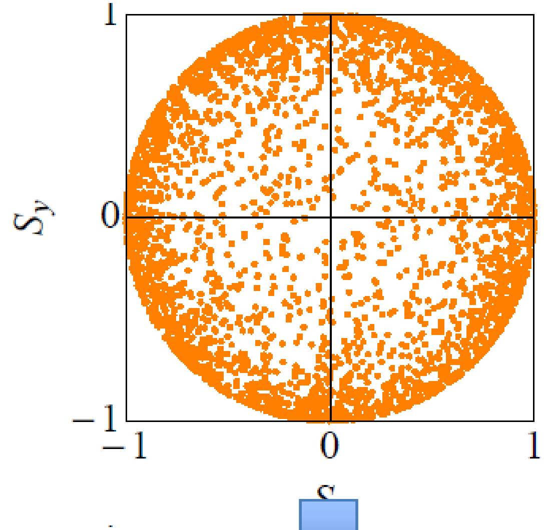


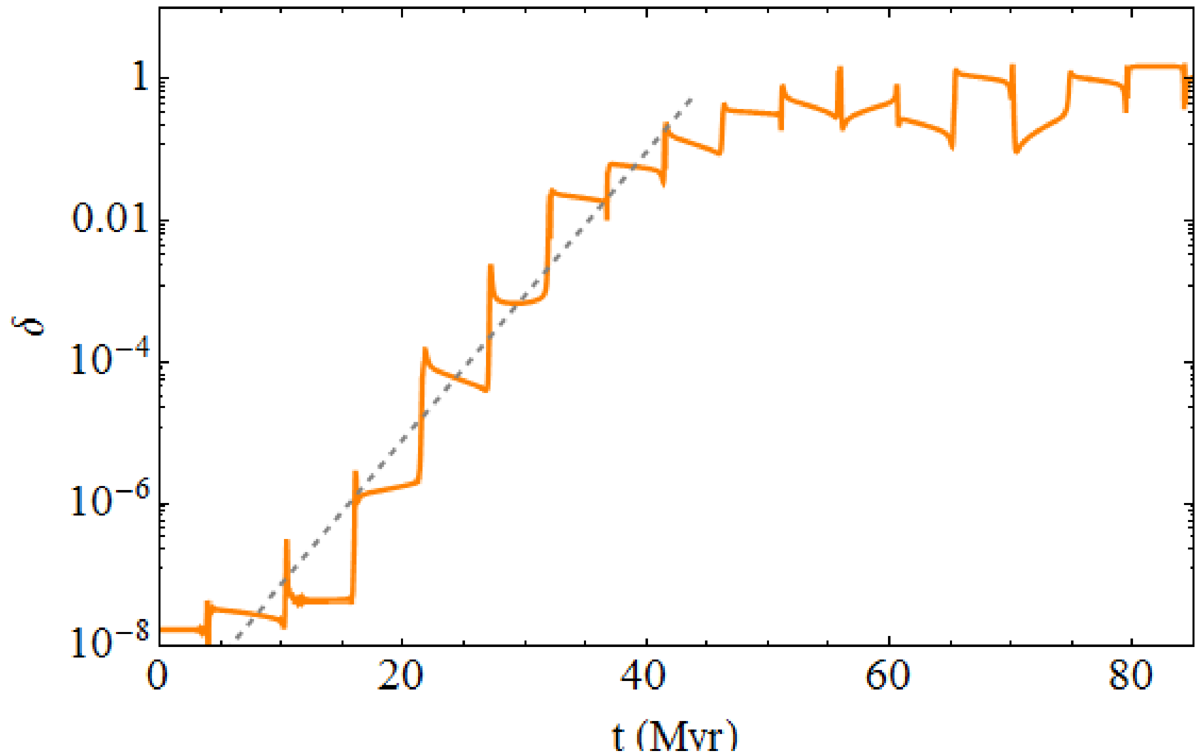
$\delta_0 = 10^{-8}$

$\max(\delta) = 2$ (unit vectors)

More precisely…







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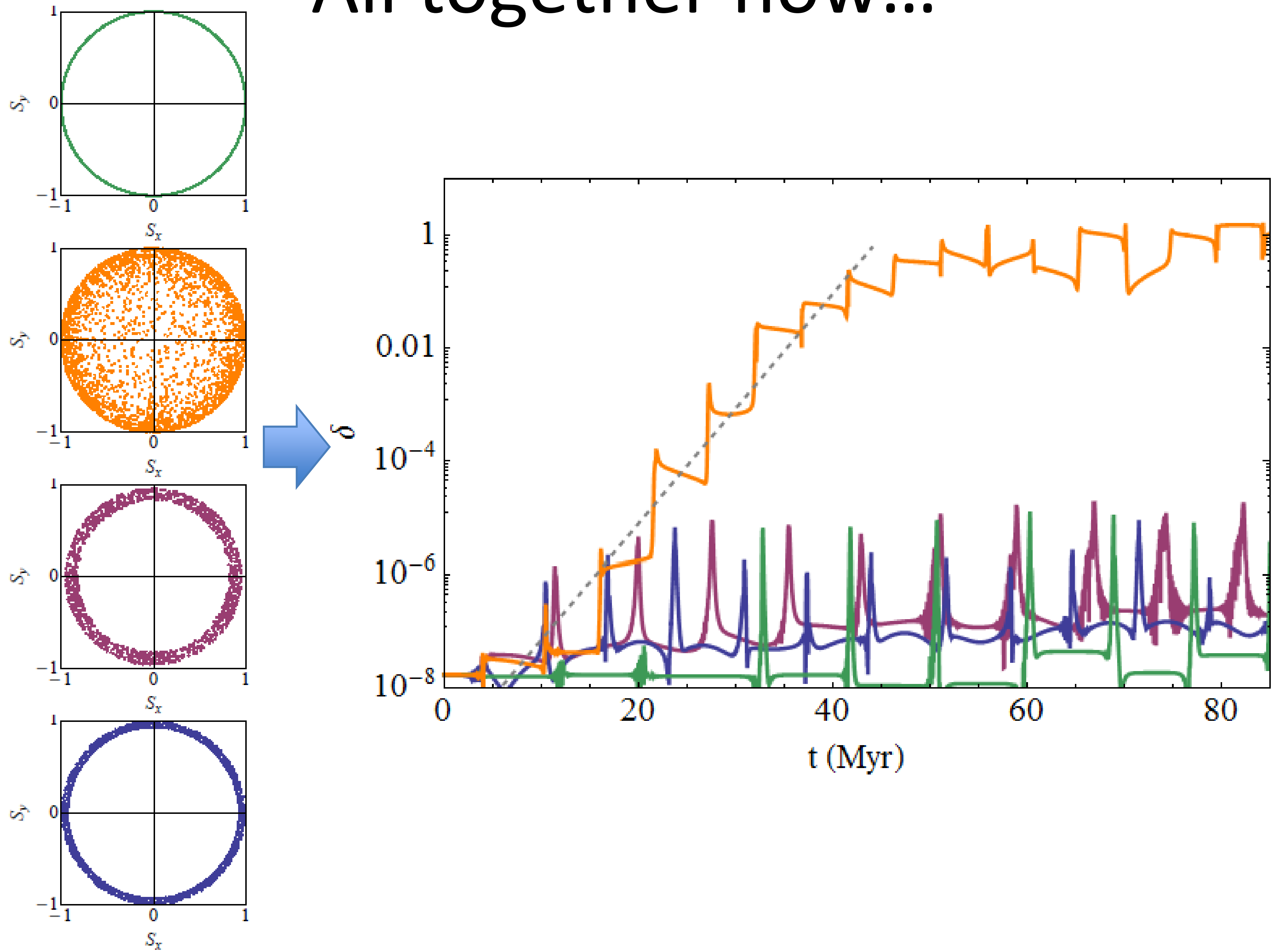
All together now…



Toward realism…

We now add in extra orbital precession terms due to:

- GR
- Static tides in planet
- Planet oblateness (due to spin)

Does this change the qualitative picture?

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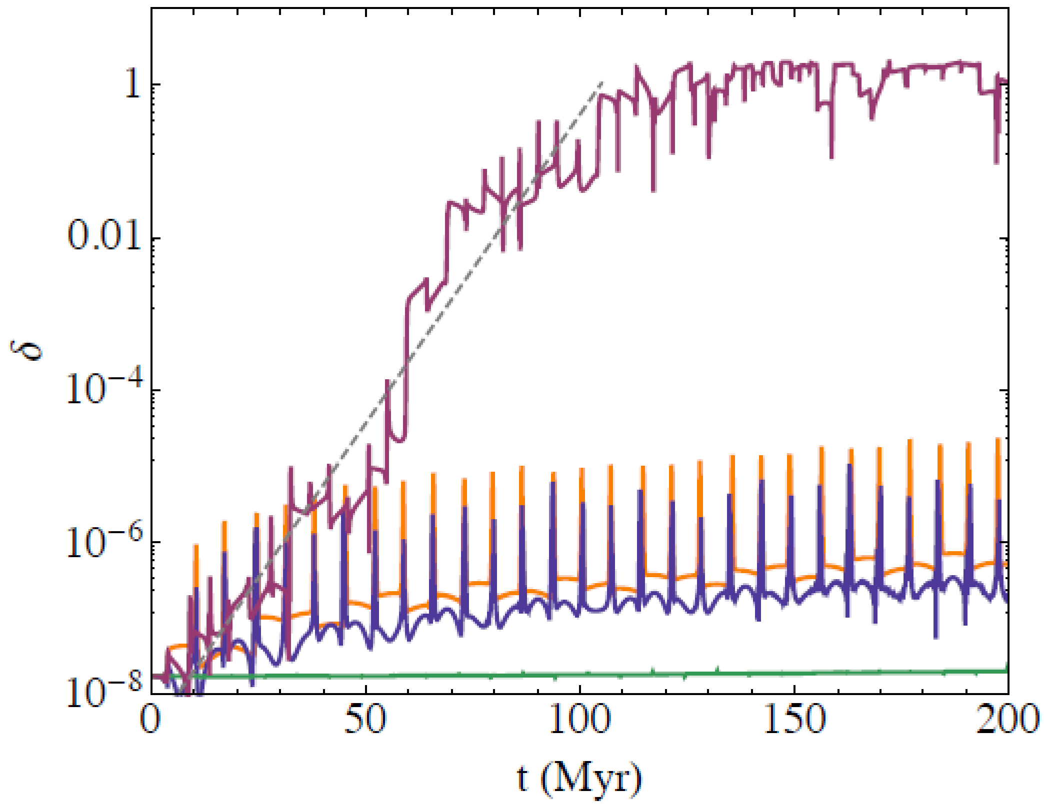
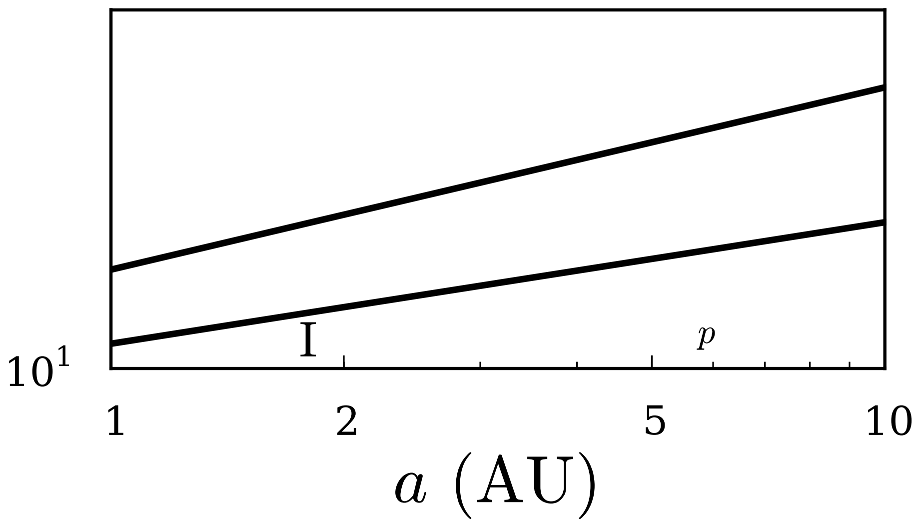


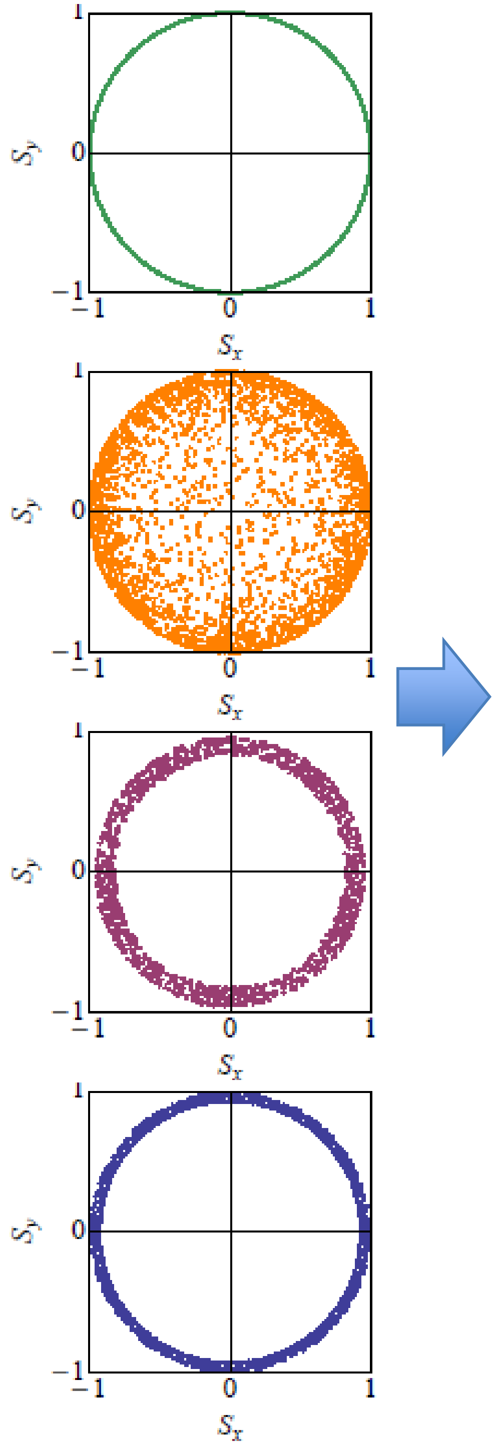
Not really

But it does limit the interesting parameter space…

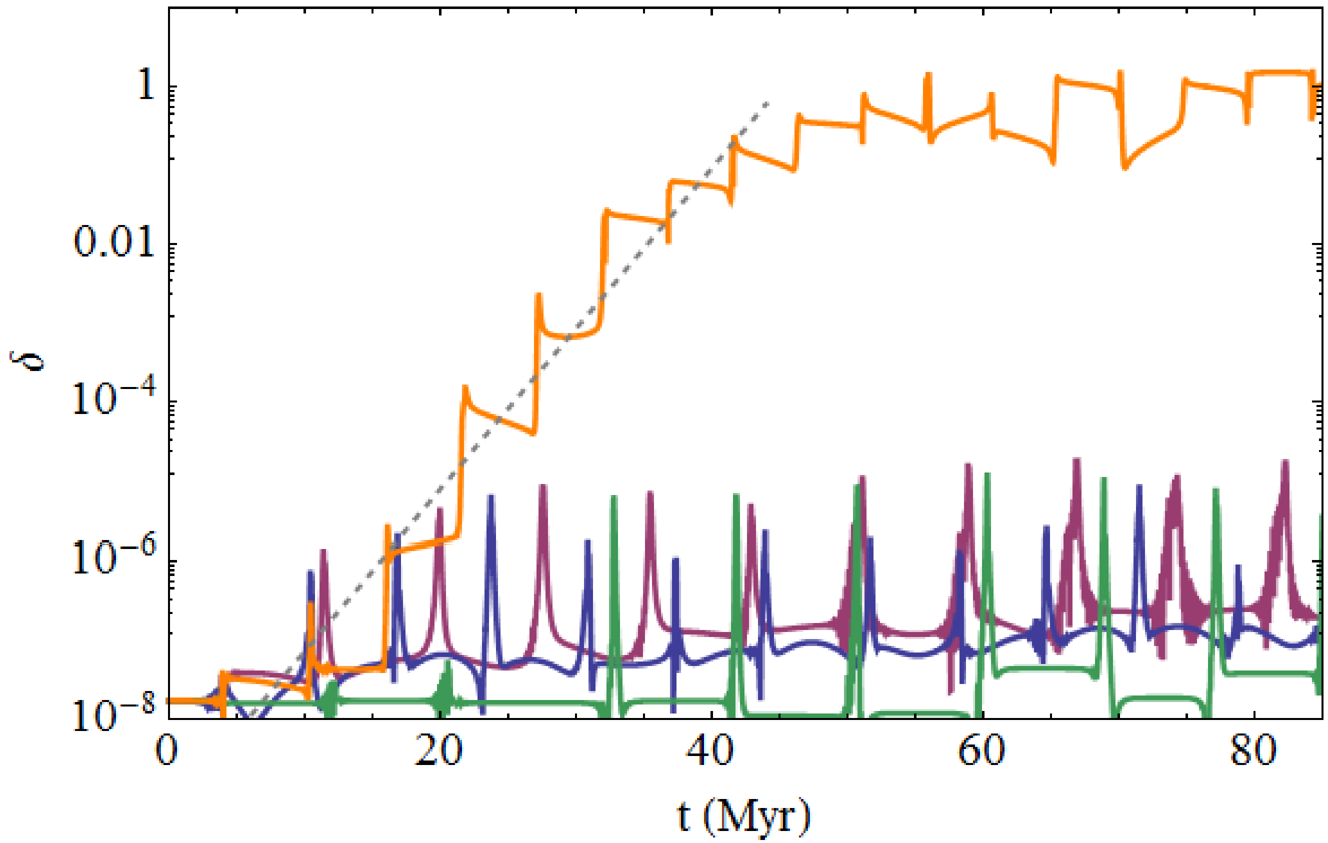


Now we’d like to explore this parameter space

- We have seen that the “trans-adiabatic” regime has both regions of chaos and quasiperiodicity. We would like to explore this further!



Presence of scatter is indicative of chaos



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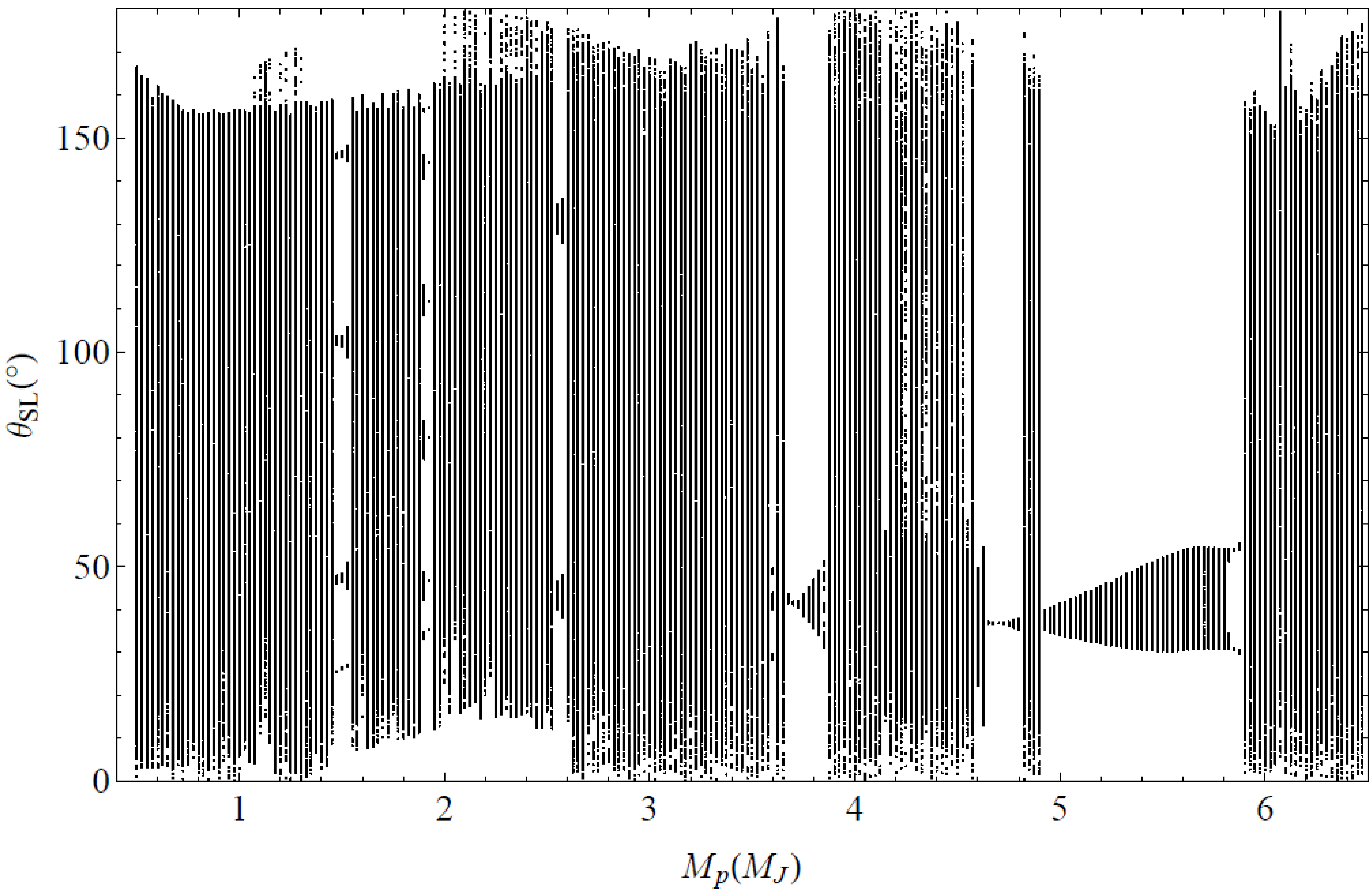
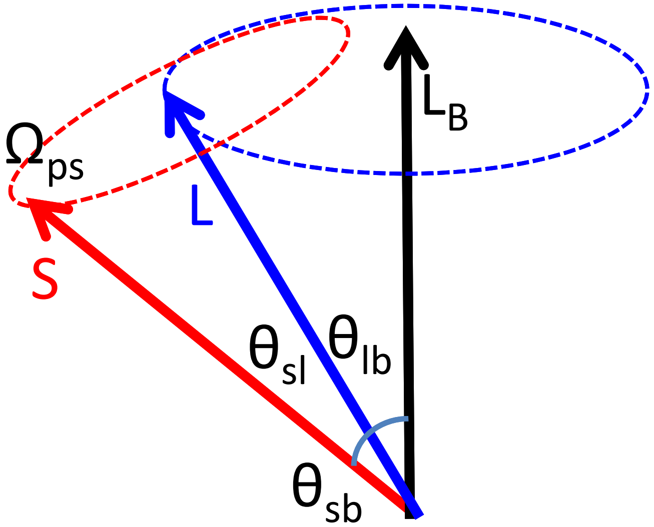
- So scatter on the y-axis will be indicative of how much phase space the stellar spin has explored

“Bifurcation” diagram (stellar spin period: 5 days)



A Toy Model



$$\frac{d\hat{\mathbf{S}}}{dt} = \Omega_{\rm ps}\hat{\mathbf{L}} \times \hat{\mathbf{S}}$$

$$\frac{d\hat{\mathbf{L}}}{dt} = \Omega_{\rm pl}\hat{\mathbf{L}}_{\mathbf{b}} \times \hat{\mathbf{L}} + \frac{S}{L}\Omega_{\rm ps}\hat{\mathbf{S}} \times \hat{\mathbf{L}}$$

$$\Omega_{\rm ps} \propto (1 - e^2)^{-3/2}$$

Adopt the ansatz

$$\Omega_{\rm ps} = \Omega_{\rm ps0} f(t) \cos\theta_{\rm sl}$$

$$f(t) \equiv \frac{1 + \varepsilon}{1 + \varepsilon \cos \Omega_{\rm pl} t}$$

$\Omega_{\rm ps0}$ a free parameter,

where $\Omega_{\rm ps0} \propto M_p \Omega_\star$

$\varepsilon =$ 'eccentricity' $= 0.99$

A Toy Model

Periodic

Chaotic

0 20 40 60 80 100 120 140 16

t

0 20 40 60 80 100 120 140 160

t

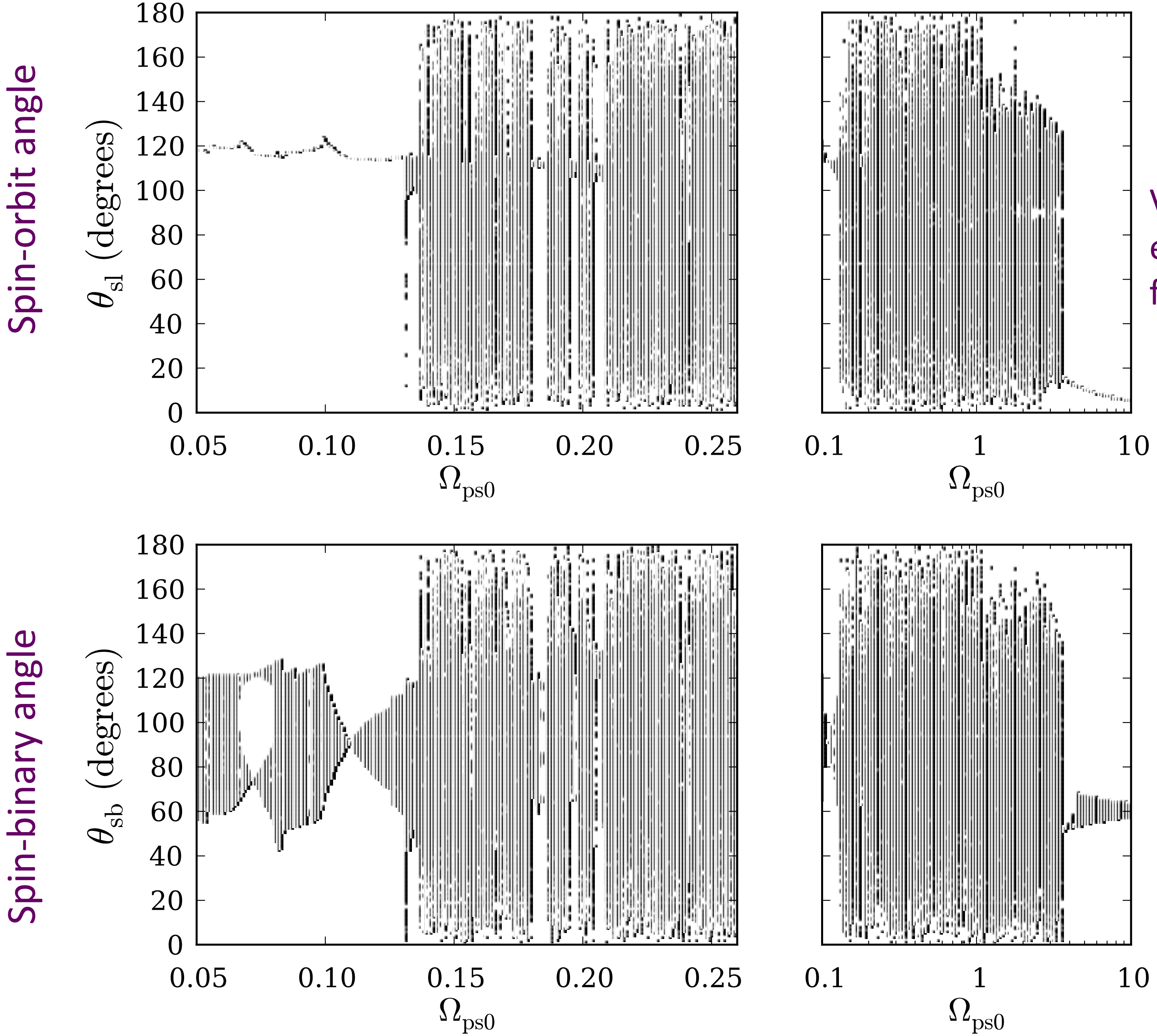
Qualitatively similar to "real" system

“Bifurcation” Diagram

- Specify Ω_{ps0} and integrate toy-model equations for 1000 “Kozai cycles”

- Record the spin-orbit and spin-binary angles at each eccentricity maximum

- Repeat for different values of Ω_{ps0}

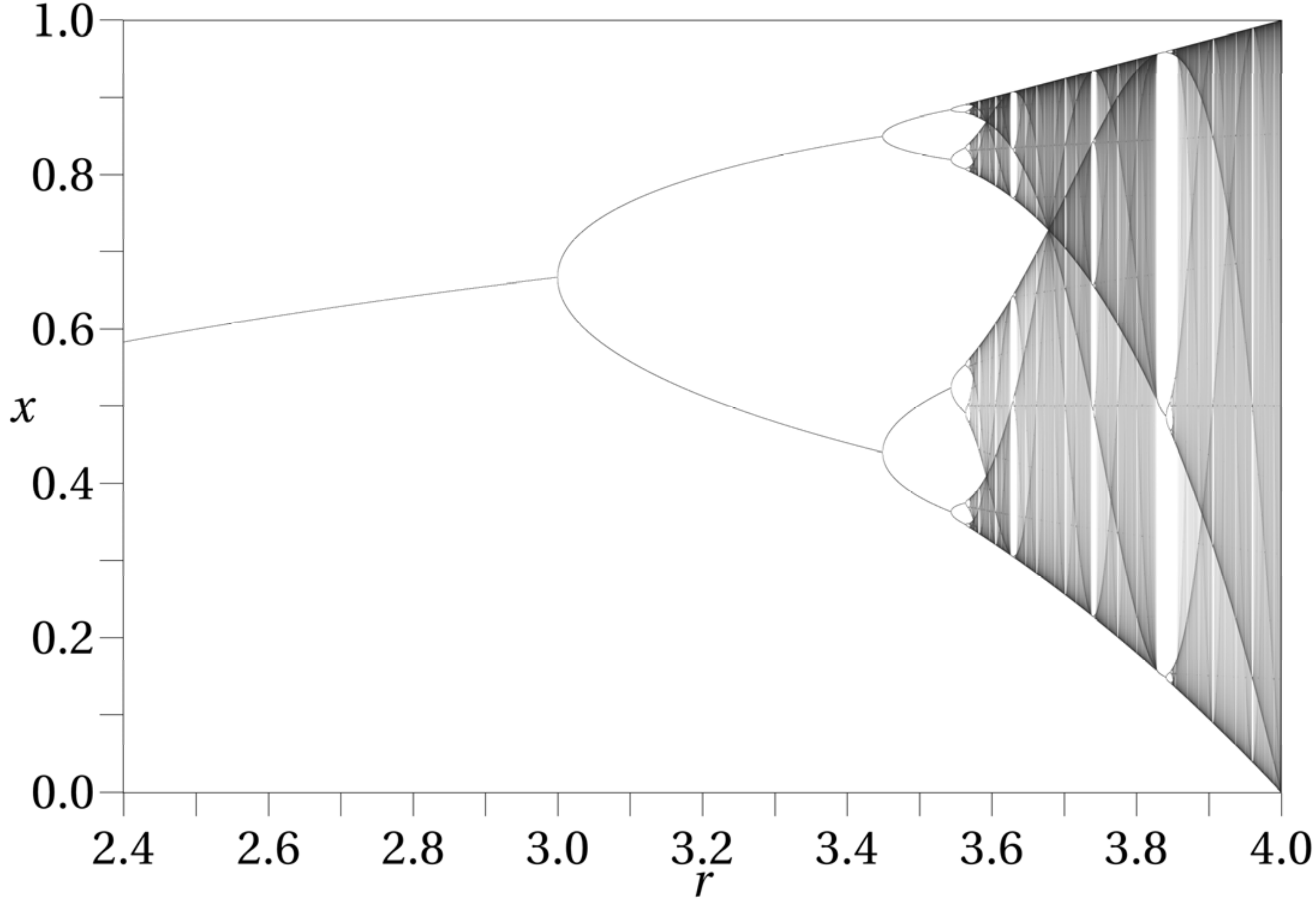


$$\Omega_{\rm ps0} \propto M_p \Omega_\star$$

Values recorded at eccentricity maxima, for 1000 cycles

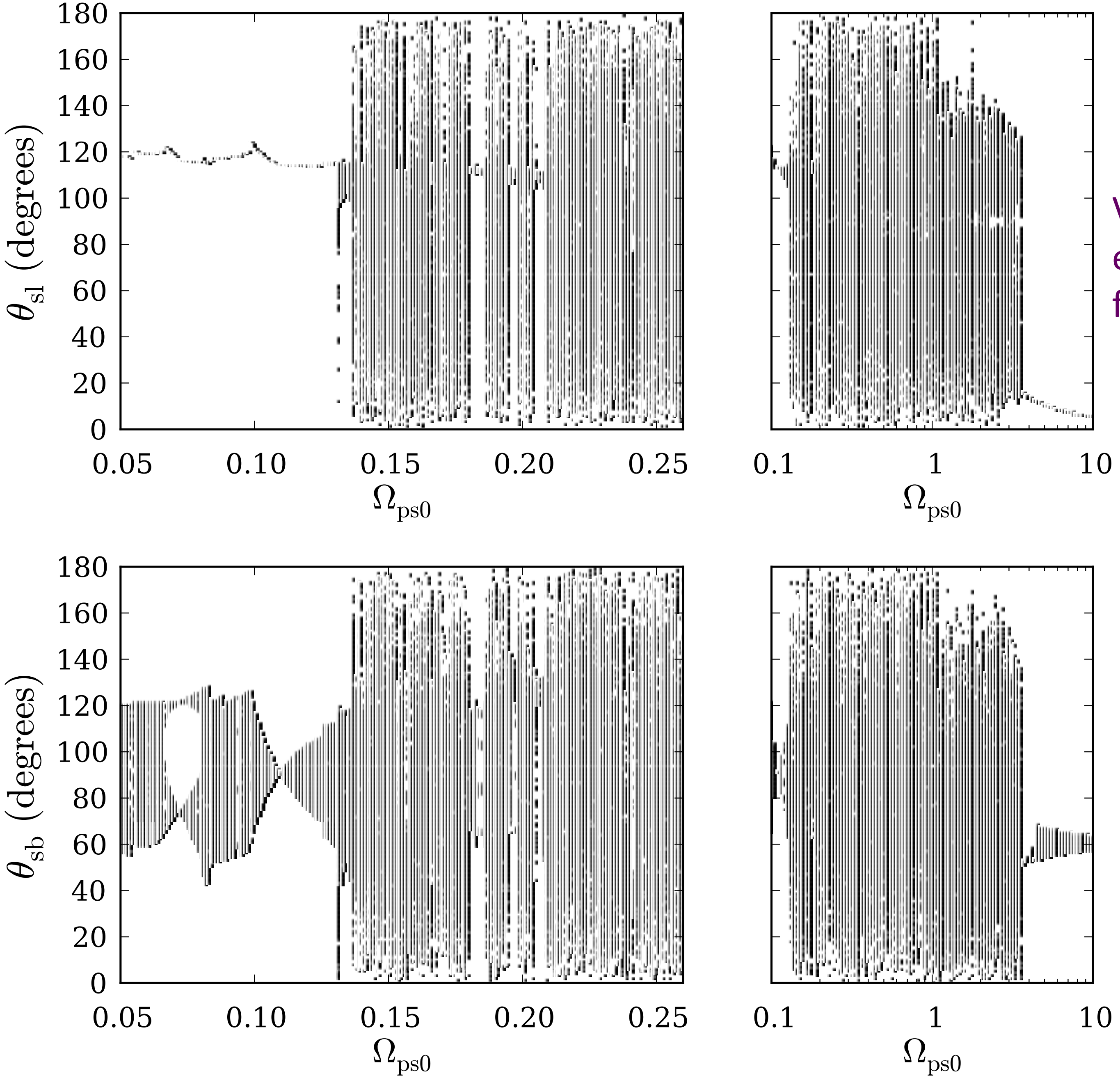
Logistic Map

$$x_{n+1} = r x_n (1 - x_n)$$



Spin-orbit angle

Spin-binary angle



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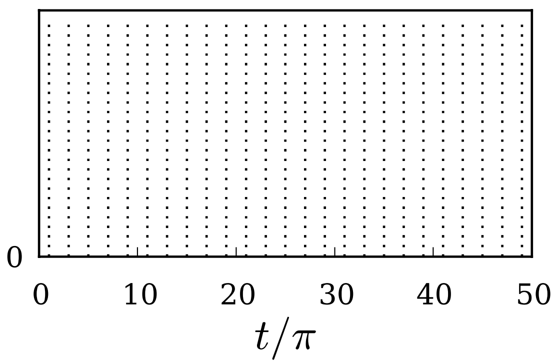
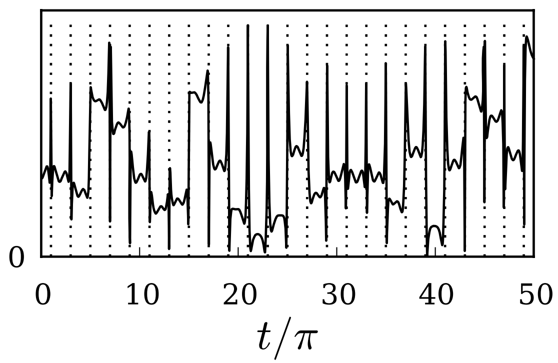
Pe ean



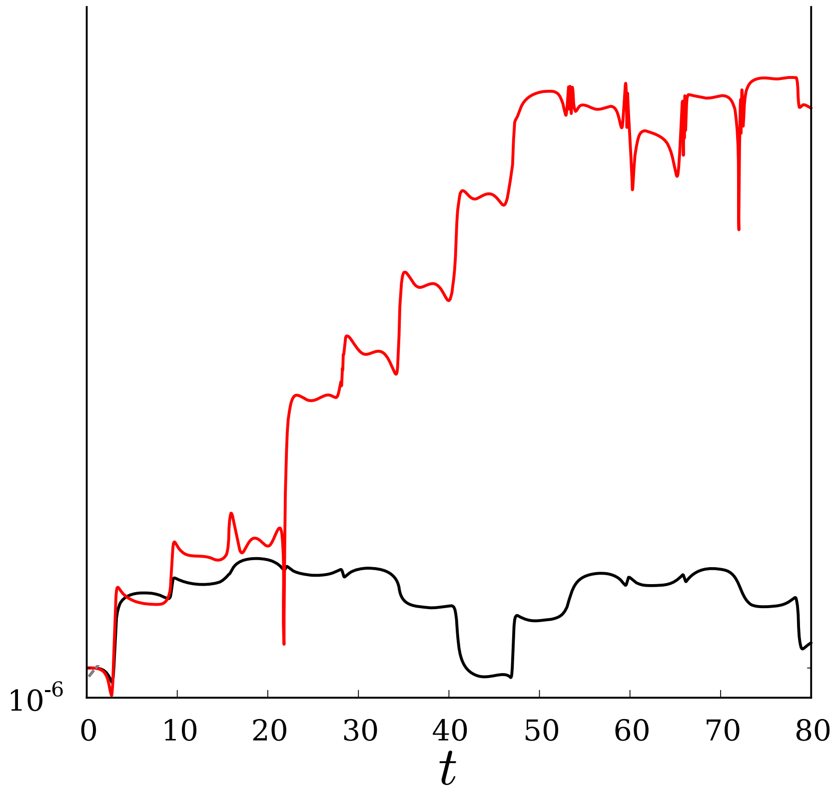


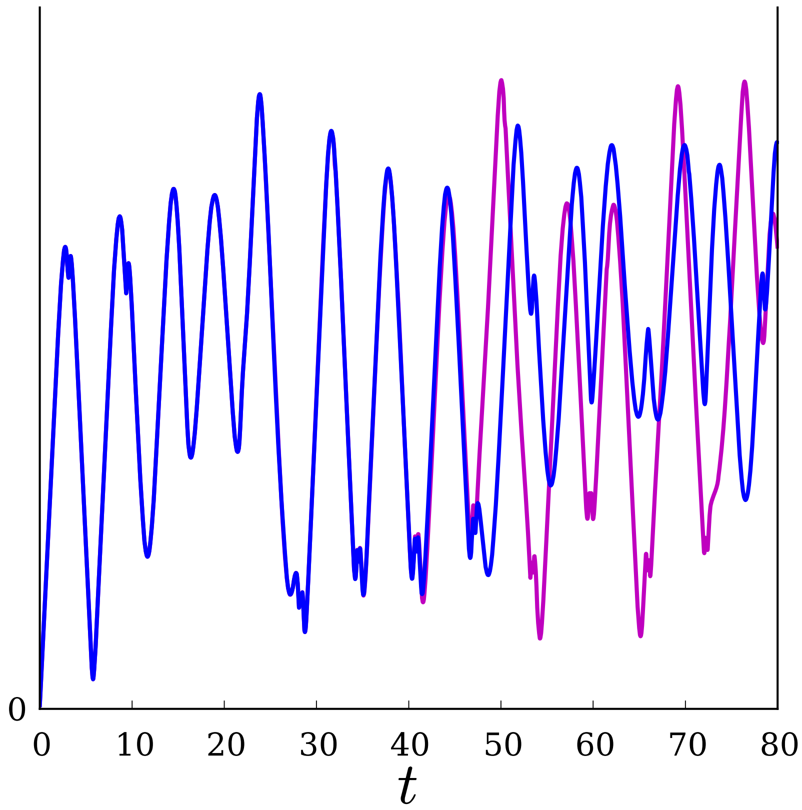
Measuring Chaos

Define a “real” system with set of initial conditions, and “shadow” system with initial conditions differing by a small amount

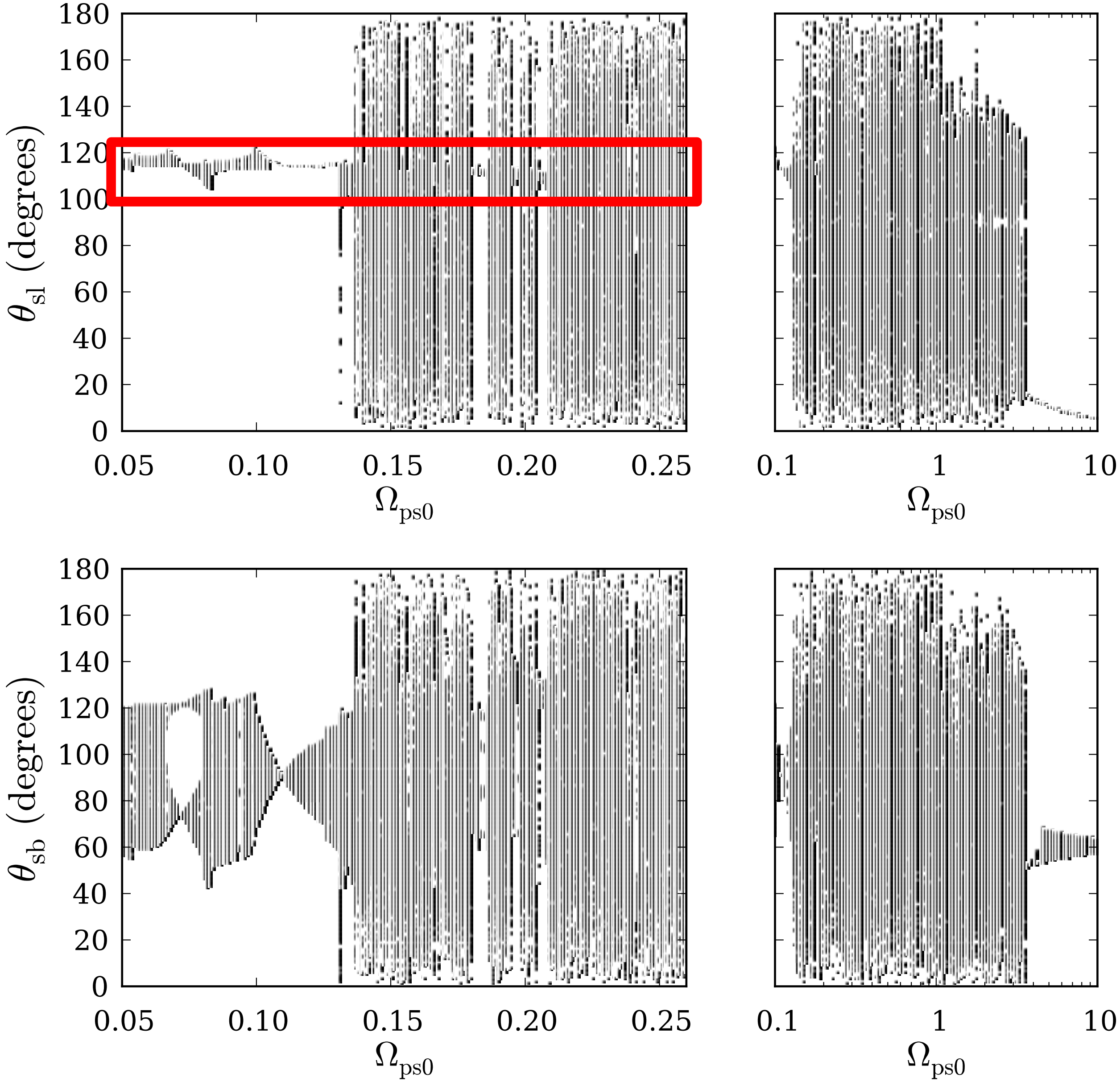
$$\delta \equiv \left|\hat{\mathbf{S}}_{\text{real}} - \hat{\mathbf{S}}_{\text{shadow}}\right|$$

$$\delta(t) = \delta_0 e^{\gamma t}$$





A Curiosity

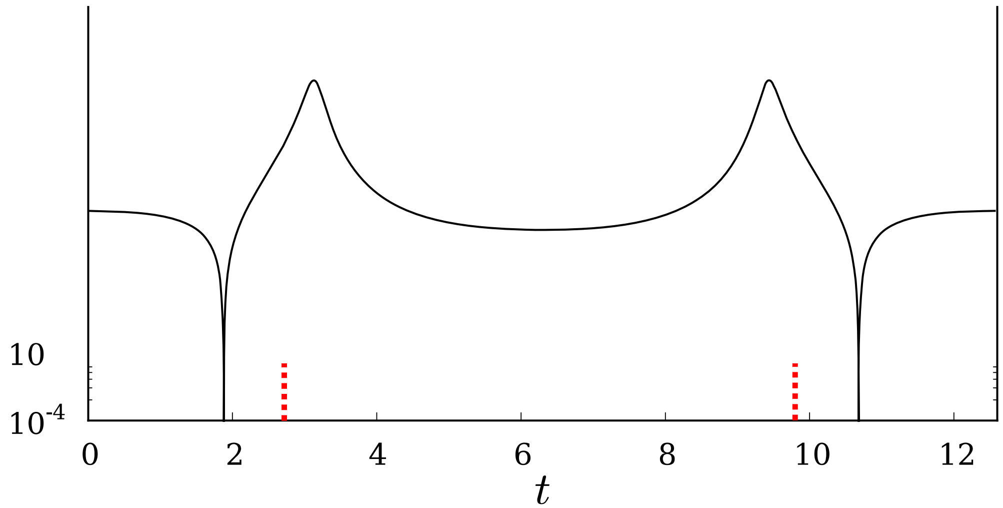


180
160
140
120
100
80
60
40
20
0
θ_{sl} (degrees)
0.05
0.10
0.15
0.20
0.25
Ω_{ps0}
0.1
1
10
Ω_{ps0}
θ_{sb} (degrees)

Estimate of spin-orbit angle in periodic islands

During a trans-adiabatic cycle,
ɔst of the change in the spin
bit angle occurs during the
n-adiabatic portion of the cycle

$$\frac{\text{ɔs}\,\theta_{\rm sl}}{dt} \approx \Omega_{\rm pl} \sin^2(\theta_{\rm lb}) \sin(\phi_{\rm sb} - \Omega_{\rm pl} t)$$

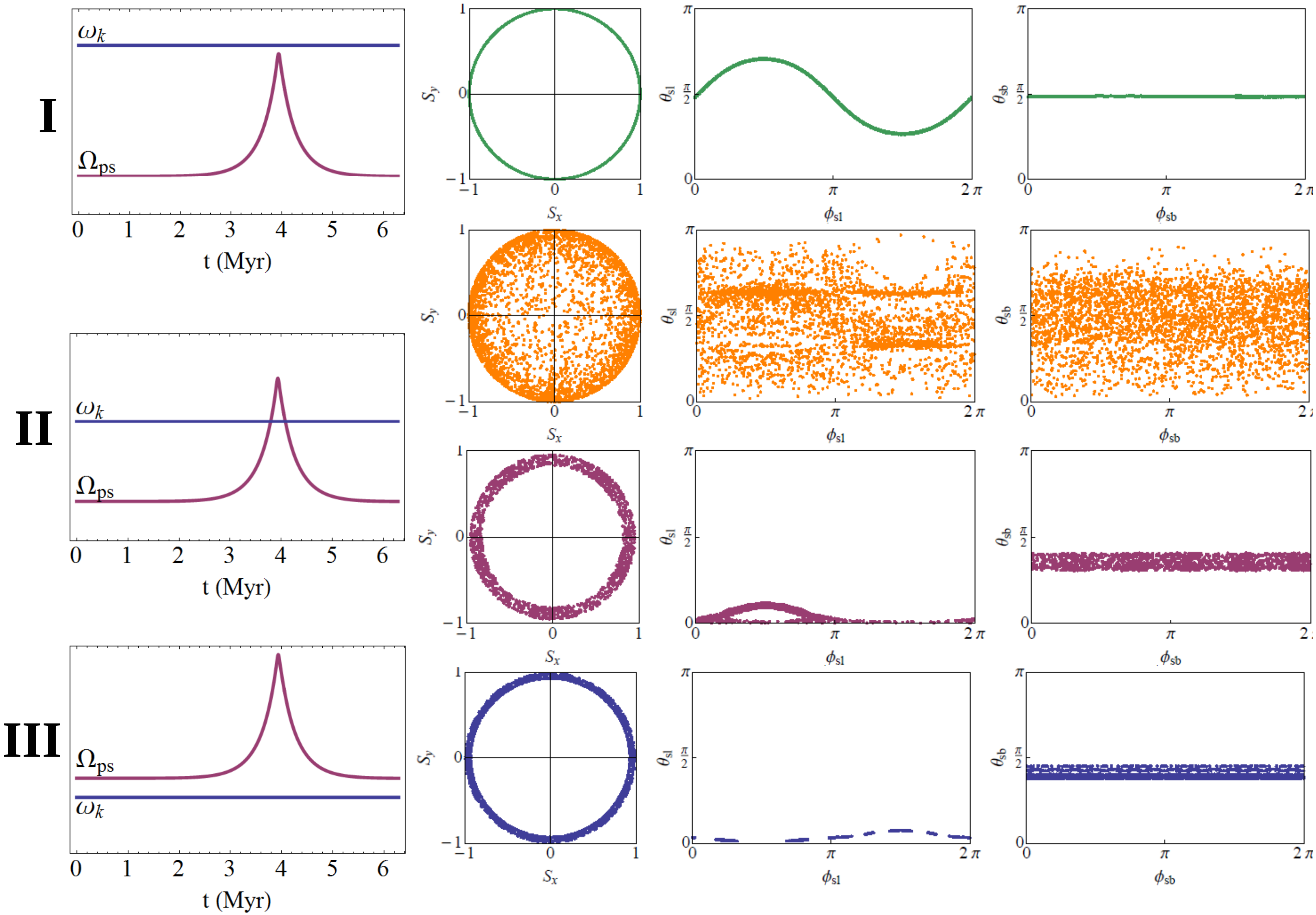


ɡrate up to the point where
ːm becomes adiabatic, beyond
h, spin-orbit angle is ~ constant

icted spin-orbit angles in good
ement with values in "islands"

Recap

Three regimes



Recap

- We have found that the stellar spin exhibits a lot of interesting behavior during Kozai cycles, and we have a qualitative understanding of why that is

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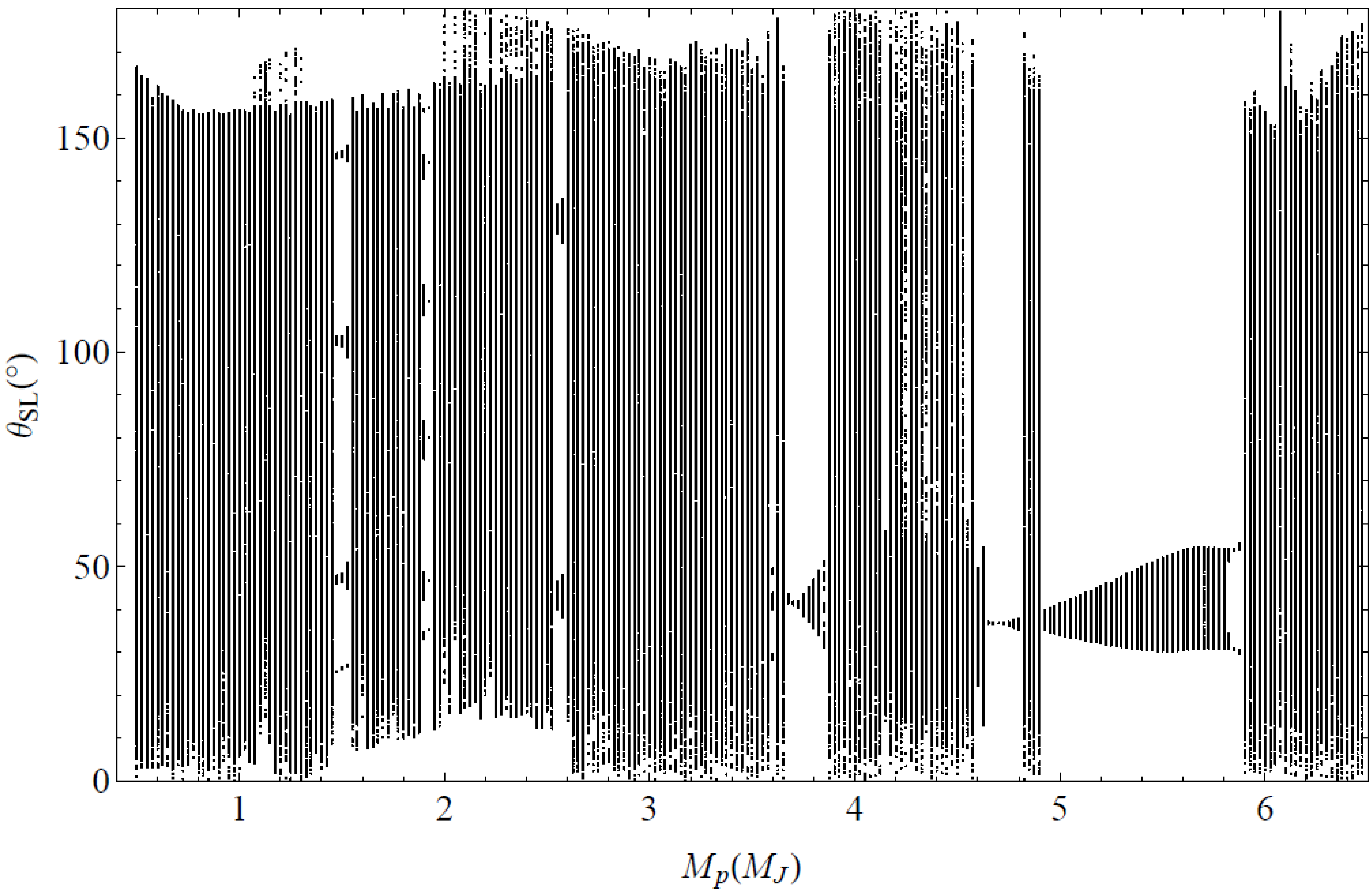

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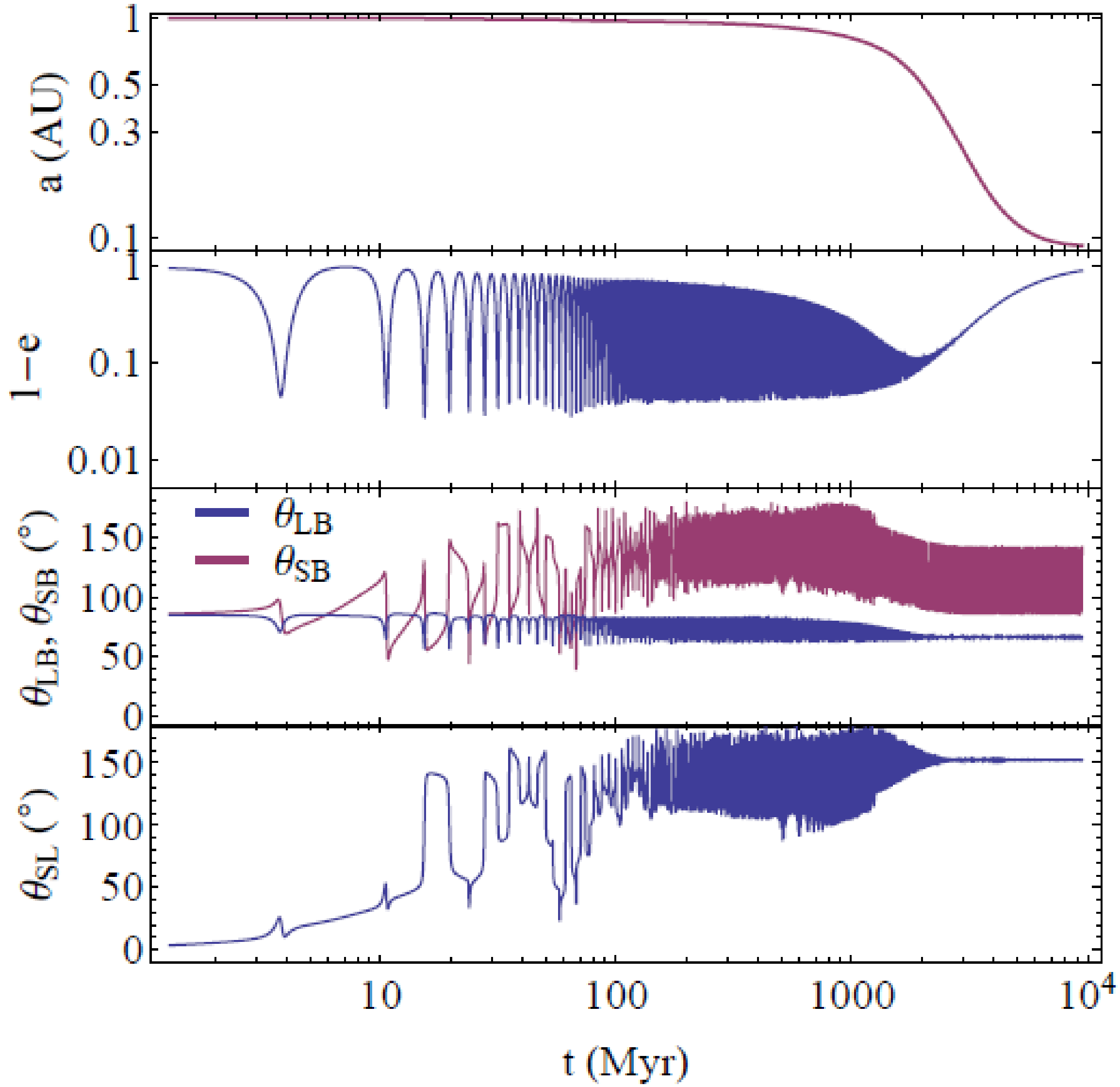
- The "trans-adiabatic" regime definitely shows complex, including chaotic, behavior

- Now we'd like to see what, if any, effect this has on the spin-orbit misalignment of Hot Jupiters
- → so we add in tidal dissipation

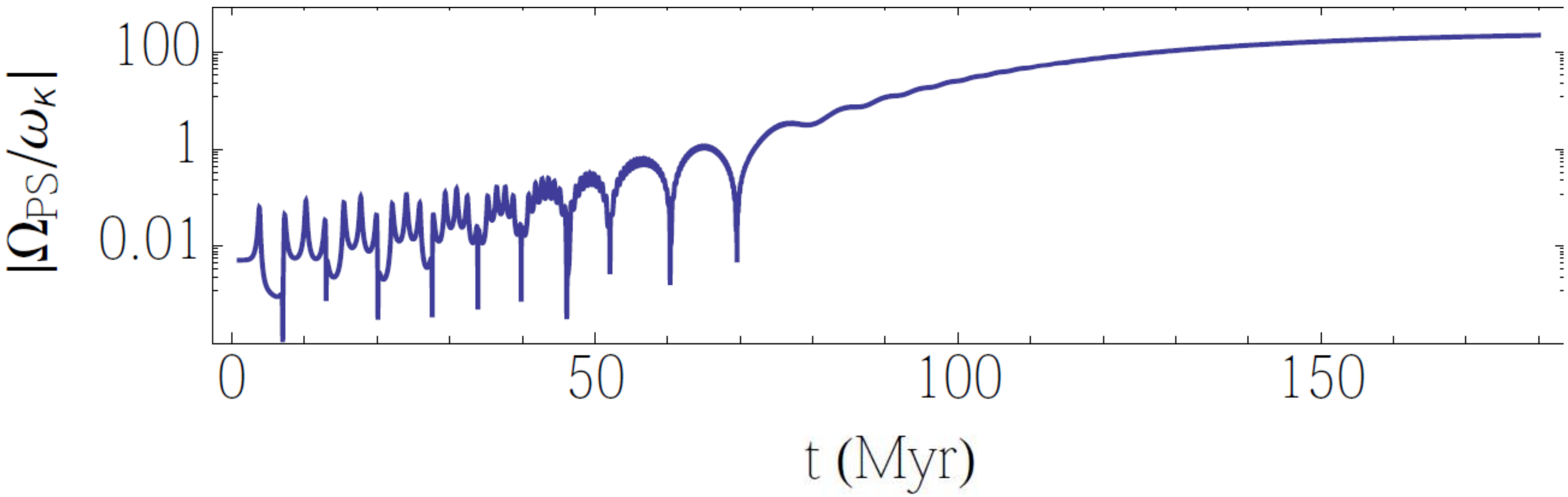
Effect of tidal dissipation



Blurring of regimes

$$\frac{\Omega_{\rm ps}}{\omega_{\rm k}} \propto a^{-9/2}$$

- As semi-major axis decays, even if the initial conditions were in "non-adiabatic" regime, they will always end up adiabatic



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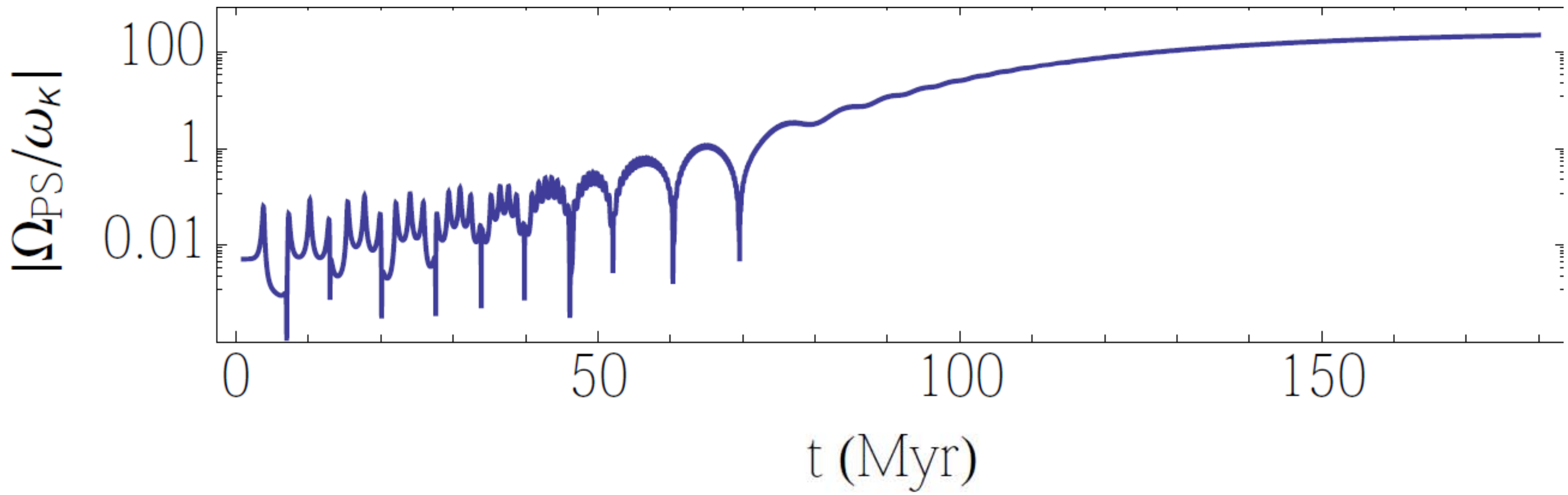

→ Stellar spin always gets a chance to affect things

Variation in final spin-orbit misalignment

- Recall: in a chaotic system, the phase space distance between two initially neighboring trajectories increases exponentially
  - →Things that start out similar end up different

Variation in final spin-orbit misalignment

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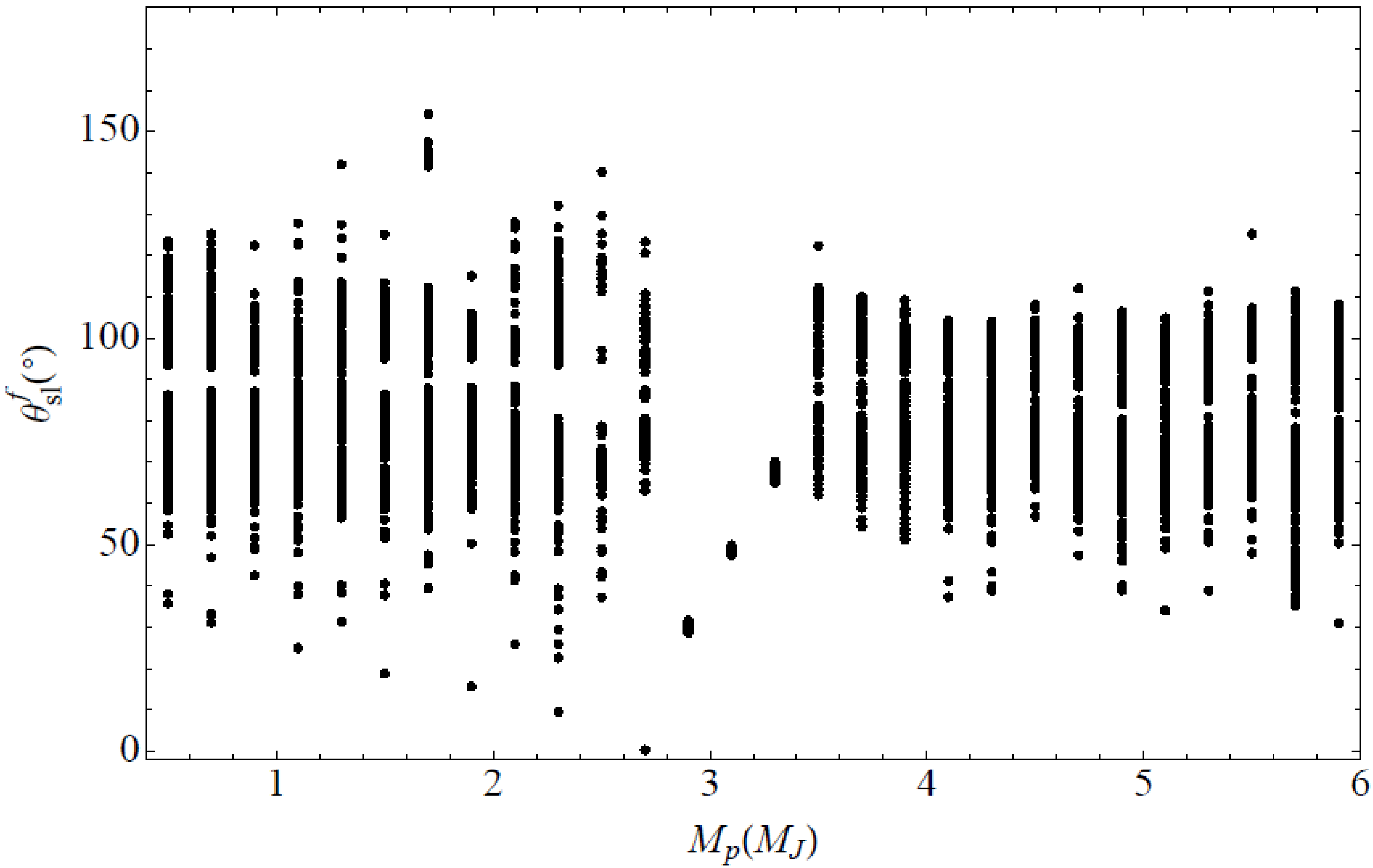
  - → Things that start out similar end up different

- So we do the following:
  - Vary initial orbital inclination by ± 0.05°
  - S and L always start out parallel
  - Record final misalignment angle
  - Repeat for different planet masses

Variation in final spin-orbit misalignment



Here stellar spin period is 3 days, initial orbital inclination θ_{lb}=85± 0.05°

Now let the star spin down!

$$\dot{\Omega}_{\rm s} \propto -\Omega_{\rm s}^3$$

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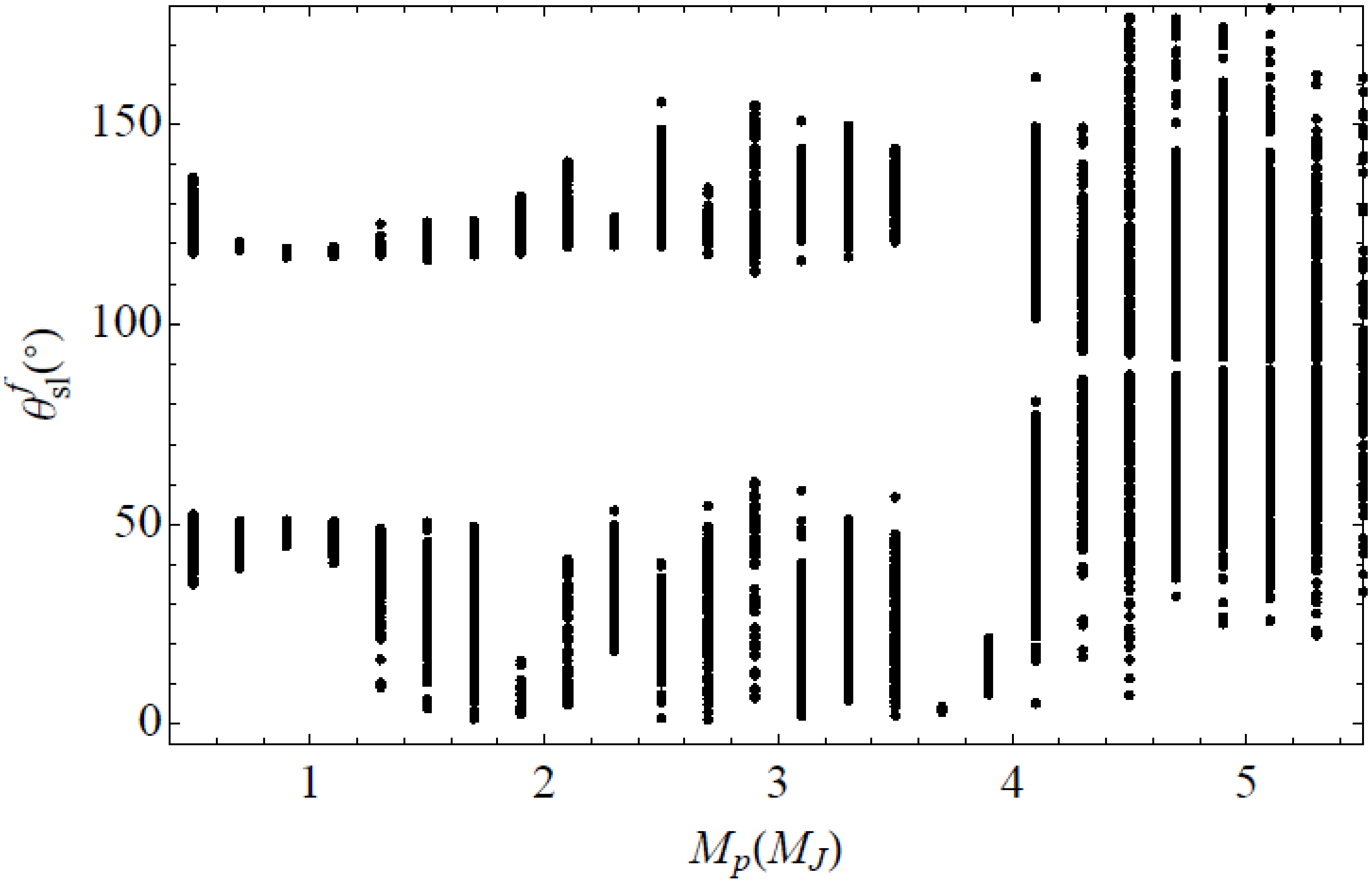

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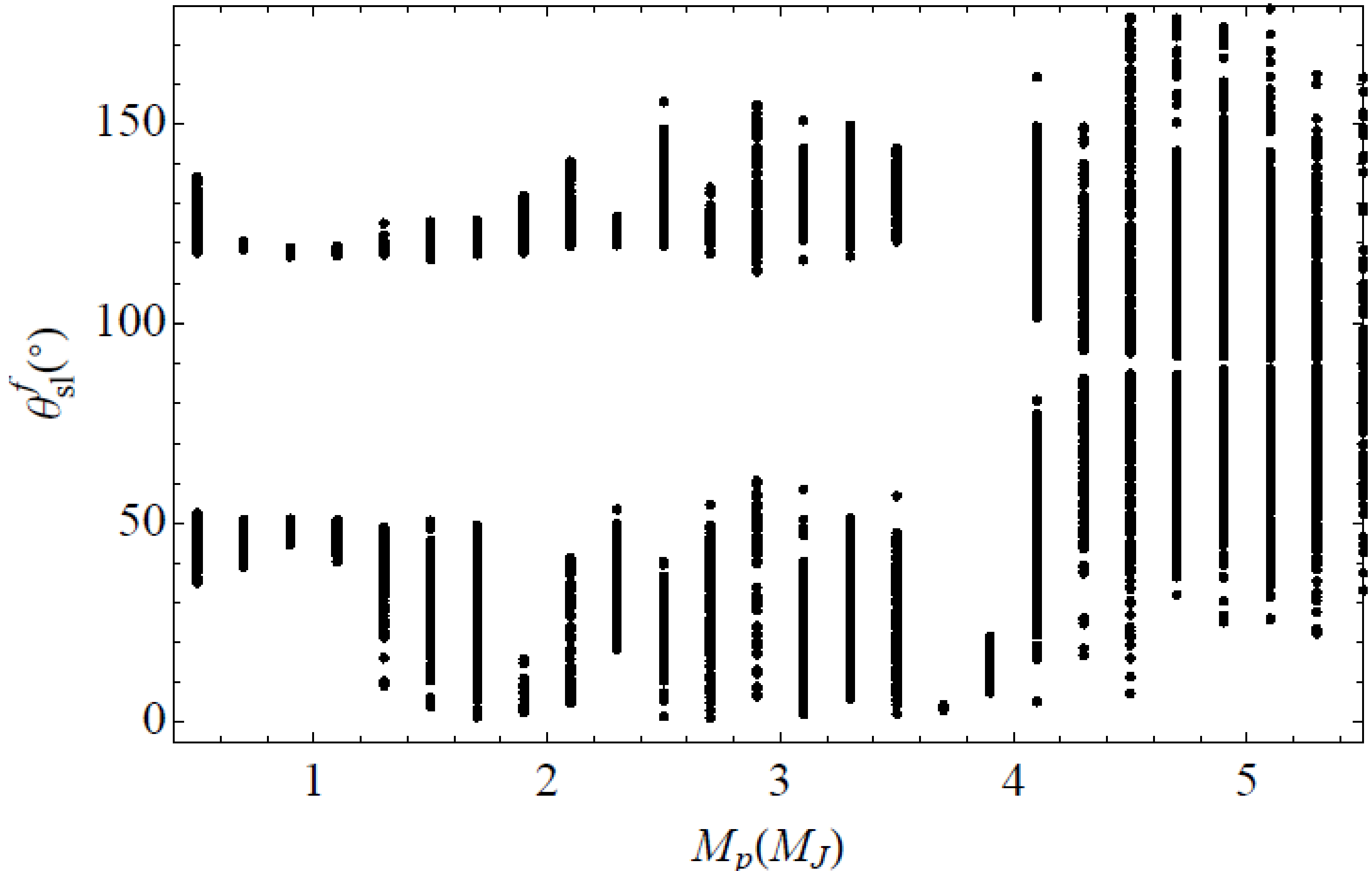

Conclusion: a tiny initial spread in inclination leads to a very large spread in spin-orbit misalignment

Take-away messages

- Stellar spin often does a crazy dance during Kozai!
- We can understand qualitatively under what circumstances this happens (“trans-adiabatic” regime)
- This certainly impacts the final distribution of spin-orbit misalignments in hot Jupiters

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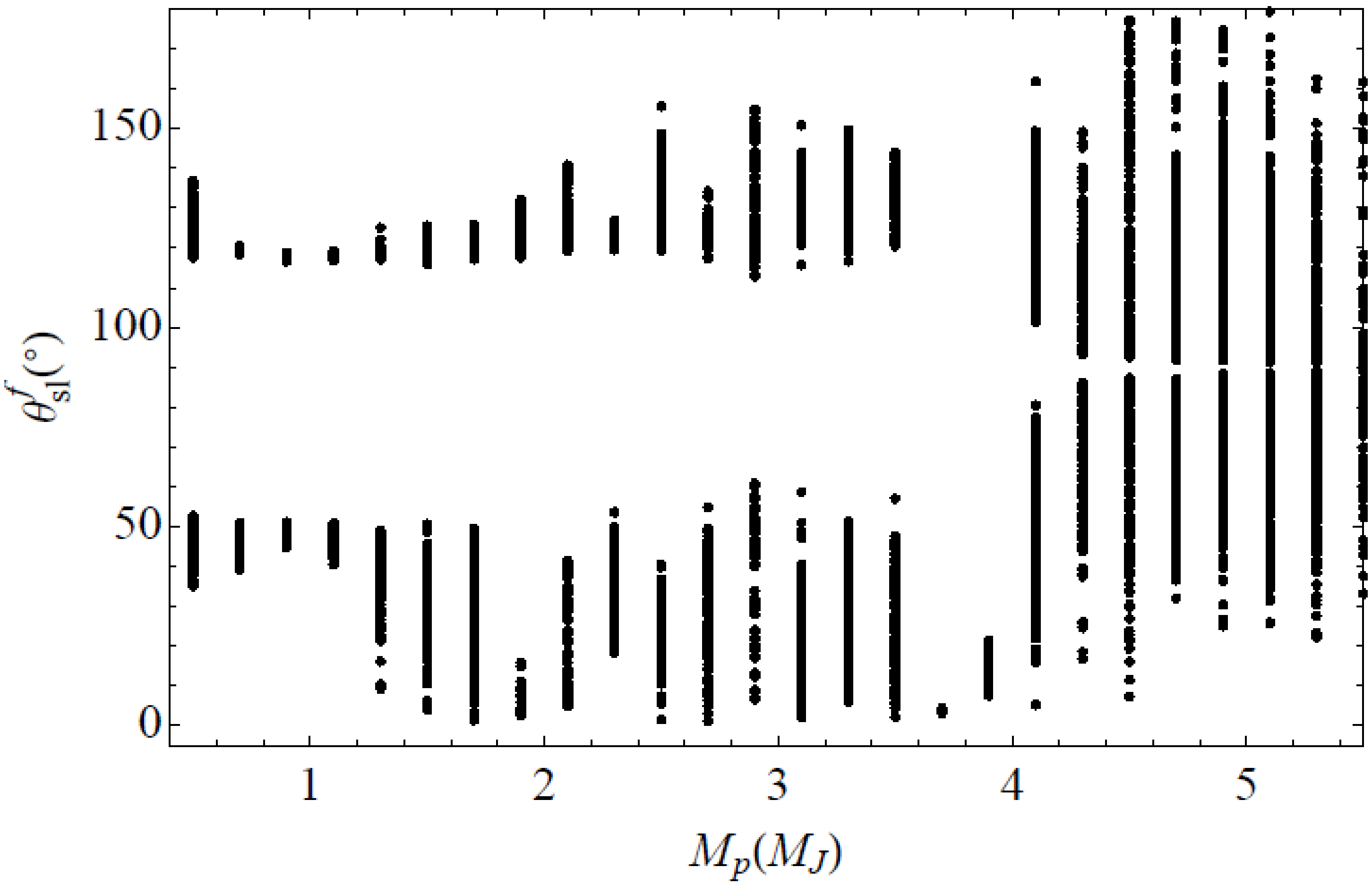
Bimodality



Bimodality: has been numerically found before…

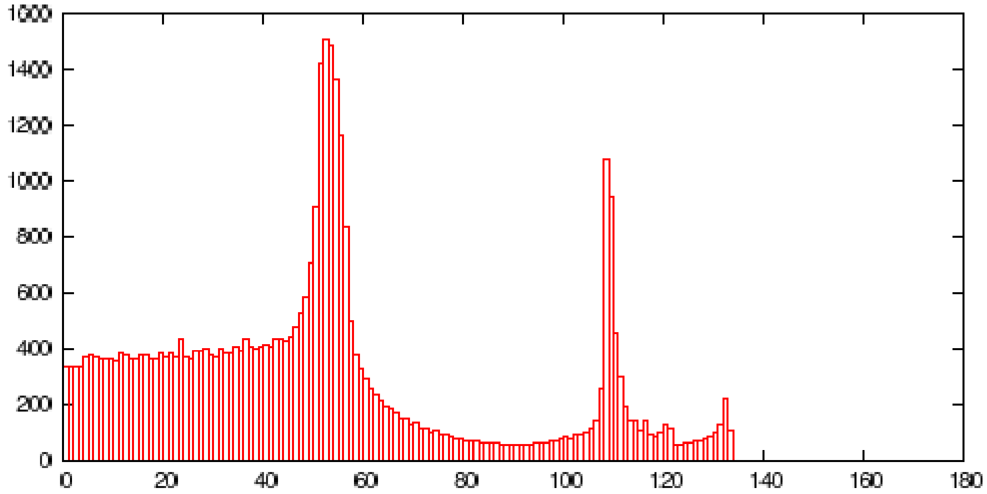
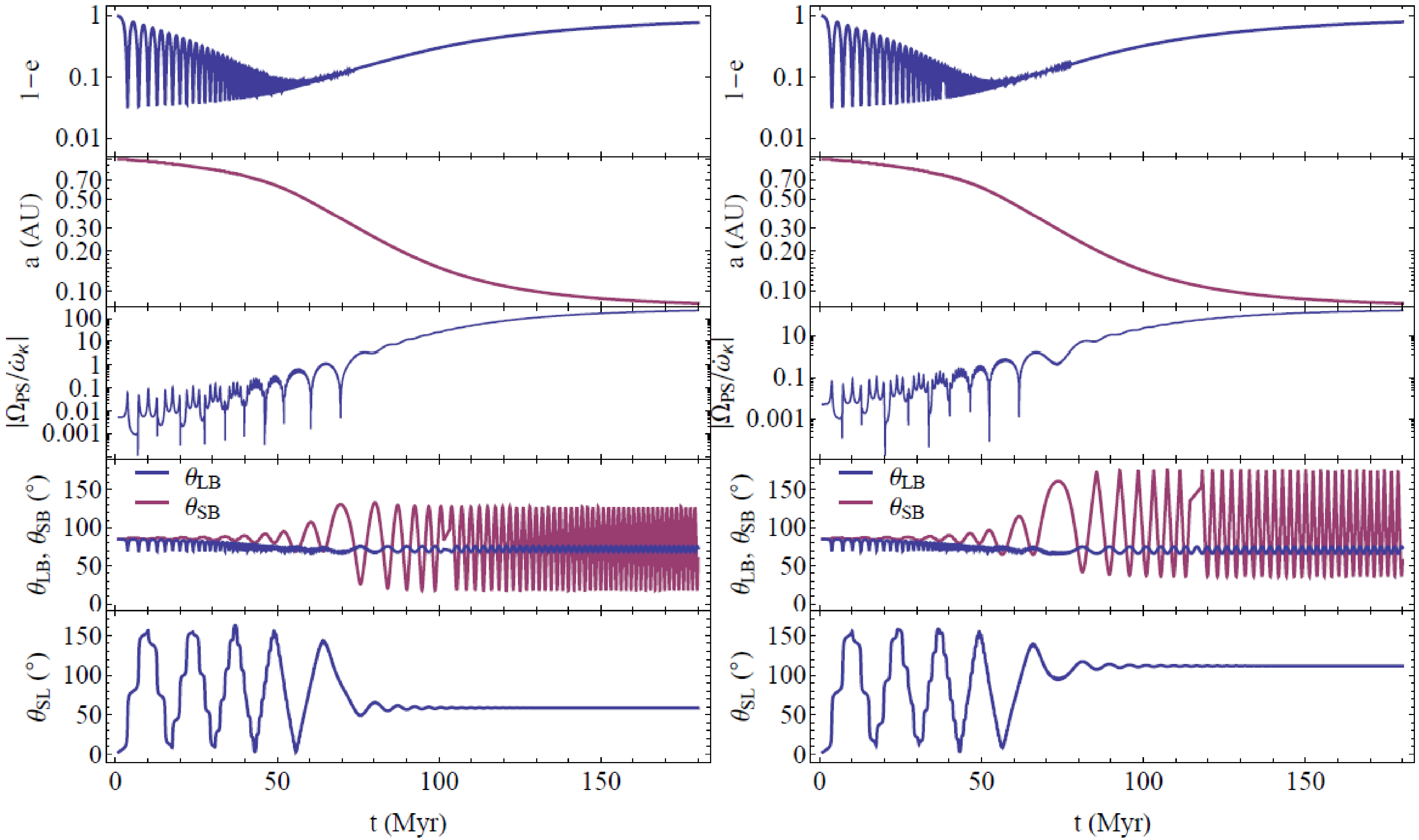


Fig. 2 Histogram of the final distribution of the misalignment angle θ_0. We have integrated a series of 40 000 systems with the same initial conditions from Table 1 except for the obliquity $\theta_0 = 0°$, and inclination I, which range from $\pm 84.3°$ to $90°$. We observe two pronounced peaks of higher probability around $\theta_0 \approx 53°$ and $\theta_0 \approx 109°$, which is consistent with the observations of the Rossiter-McLaughlin anomaly for the HD 80606 system (Pont et al, 2009).

Correia, Laskar, Farago, & Boué (2011)

Bimodality: definitely has to do with spin…



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