Self-bound droplets of a dipolar Bose-Einstein condensate:

stabilized by the Lee-Huang-Yang corrections

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work with Danny Baillie, Russell Bisset, Ryan Wilson

UNIVERSITY of OTAGO

SAPERE AUDE

Te Whare Wānanga o Otāgo

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Outline

- Introduction to Dipolar Bose-Einstein Condensates (BECs)
- Stability and motivating experiments
- LHY fluctuations stabilising a dipolar BEC - Trapped system
- Self-bound droplets in free-space.
- Collective excitations of self-bound droplets

Relevant preprints/papers from my group:

Phys. Rev. A **94**, 021602(R) (2016)	self-bound droplets
Phys. Rev. A **94**, 033619 (2016)	trapped droplets
arxiv.org/abs/1703.07927	Collective excitations of self-bound droplets

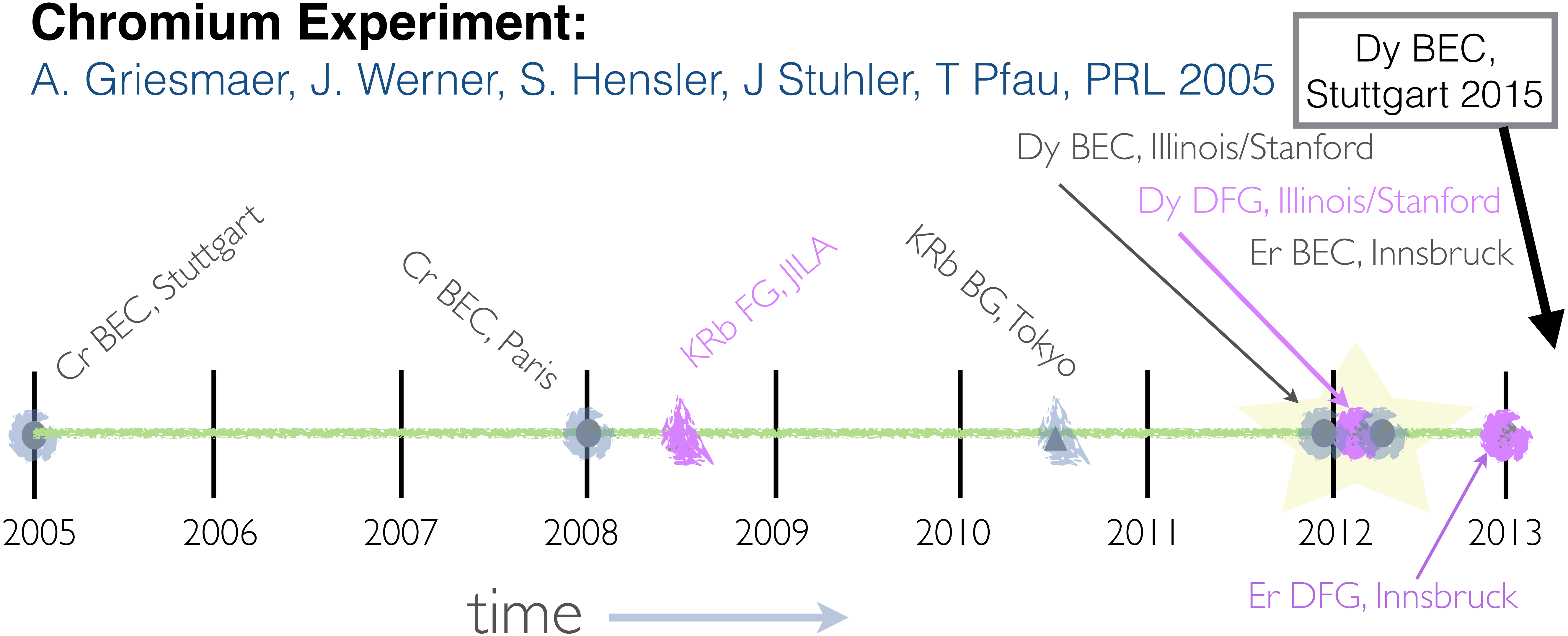
A brief history of dipolar BECs

Early theory proposals:

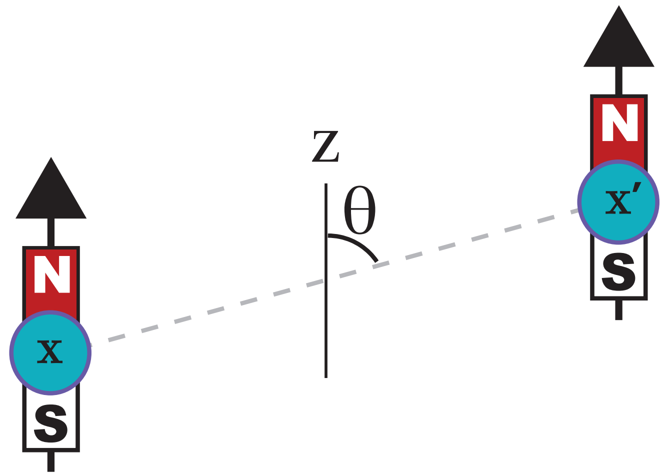
K. Goral, K. Rzazewski, T. Pfau, PRA 2000
L. Santos, G. Shlyapnikov, P Zoller, M. Lewenstien, PRL 2000
S.Yi, L. You PRA 2001

Chromium Experiment:

A. Griesmaer, J. Werner, S. Hensler, J Stuhler, T Pfau, PRL 2005



Dipole-Dipole interaction



$$V_{\rm dd}(\mathbf{r}) = \frac{3g_{\rm dd}}{4\pi}\left(\frac{1-3\cos^2\theta}{r^3}\right)$$

Atoms:

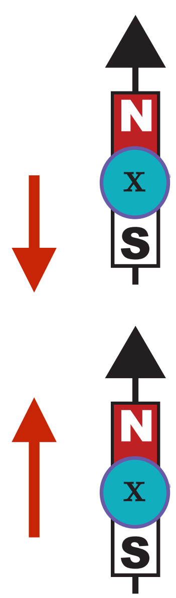
$$g_{\rm dd} = \frac{1}{3}\mu_0\mu_m^2,$$

magnetic dipole moment

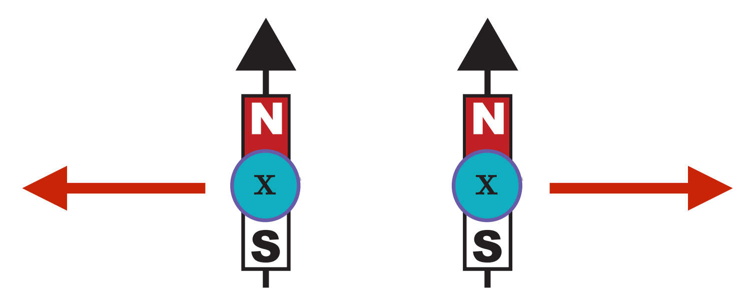
Molecules:

$$g_{\rm dd} = \frac{1}{3}\frac{d^2}{\epsilon_0}$$

electric dipole moment



attractive



repulsive

Long-ranged + anisotropic

Dipolar interaction strengths

Species	Dipole moment	$a_{\rm dd}$	$\epsilon_{\rm dd}$
^{87}Rb	$1.0\,\mu_{\rm B}$	$0.7\,a_0$	0.007
^{52}Cr	$6.0\,\mu_{\rm B}$	$16\,a_0$	0.16
^{168}Er	$7.0\,\mu_{\rm B}$	$67\,a_0$	0.38
^{164}Dy	$10\,\mu_{\rm B}$	$130\,a_0$	1.3
KRb	0.6 D	$2.0 \times 10^3 a_0$	

$$a_{\rm dd} = \frac{m\mu_0\mu^2}{12\pi\hbar^2}$$

contact interaction $g_{\rm s} = \dfrac{4\pi a_{\rm s}\hbar^2}{m}$

s-wave scattering length

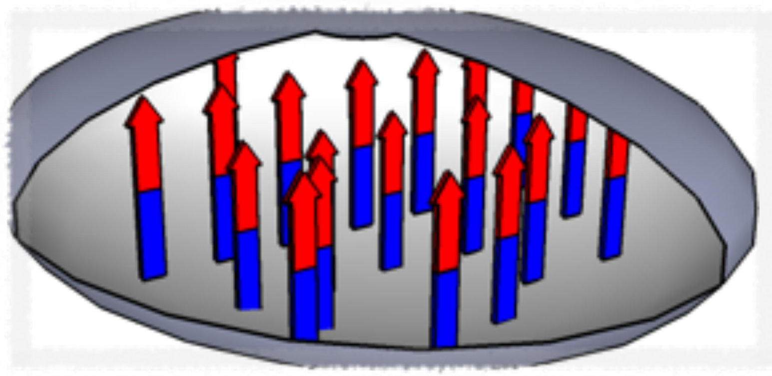
Dipolar interaction $g_{\rm dd} = \dfrac{4\pi a_{\rm dd}\hbar^2}{m}$

dipole length

Ratio of dipole to contact interactions $\epsilon_{\rm dd} \equiv \dfrac{a_{\rm dd}}{a_{\rm s}} = \dfrac{g_{\rm dd}}{g_{\rm s}}$

$\epsilon_{\rm dd} > 1$ **“dipole dominated”**

“strongly” dipolar condensate



Trapped condensate
Magnetic Dipoles (moment μ) polarised along z

“dipole dominated”

$$a_{\mathrm{dd}} > a_s$$

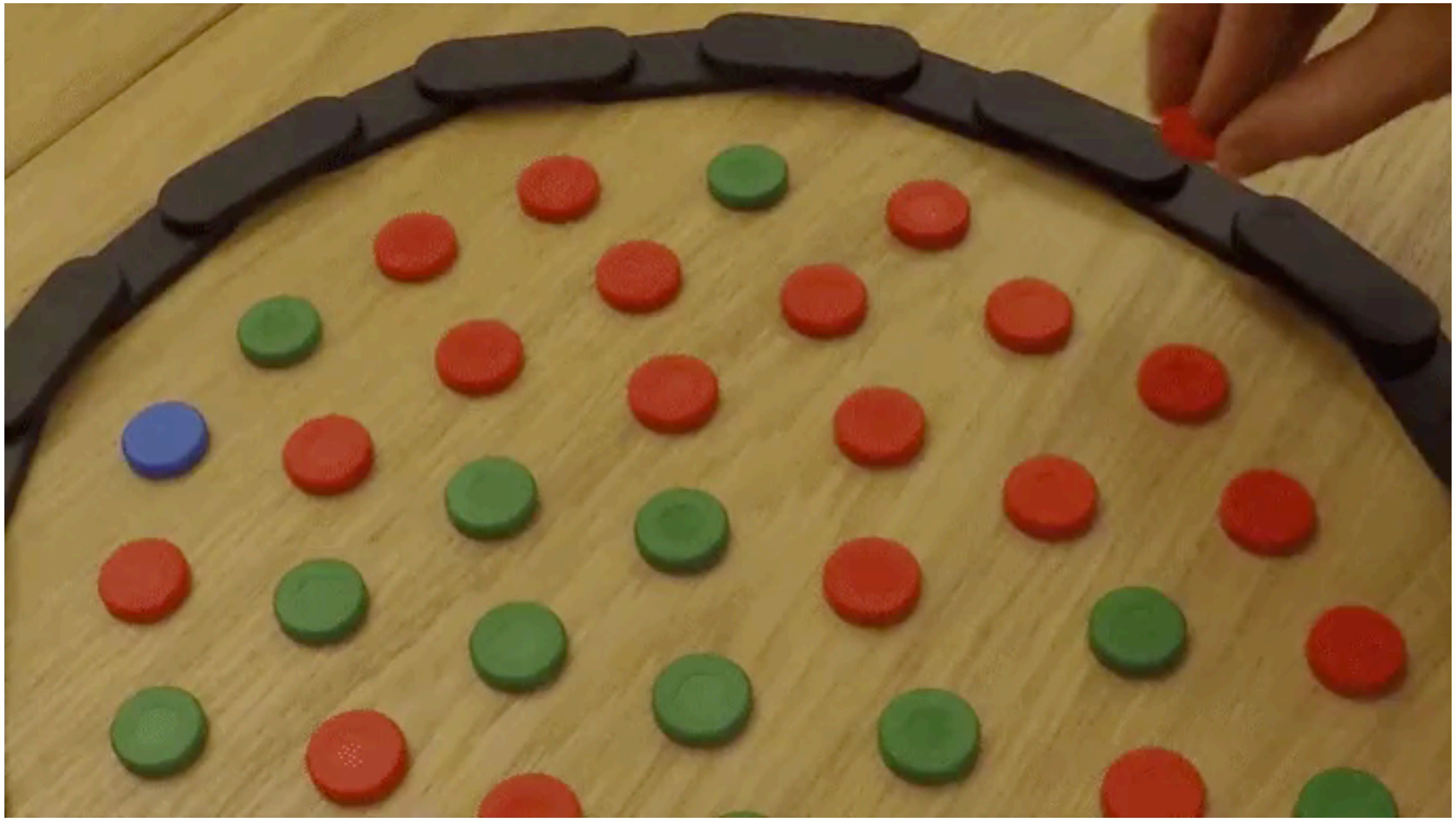
“dipole length” $a_{\mathrm{dd}} = \frac{m\mu_0\mu^2}{12\pi\hbar^2}$

In **dipole dominated regime** new physics *predicted* to emerge: rotons, quasi-2D solitions, structured ground states …. (Ronen *et al.*, Pedri *et al.*,..)

However, condensate is meta-stable to mechanical collapse ➔ **rapid atom loss + heating**

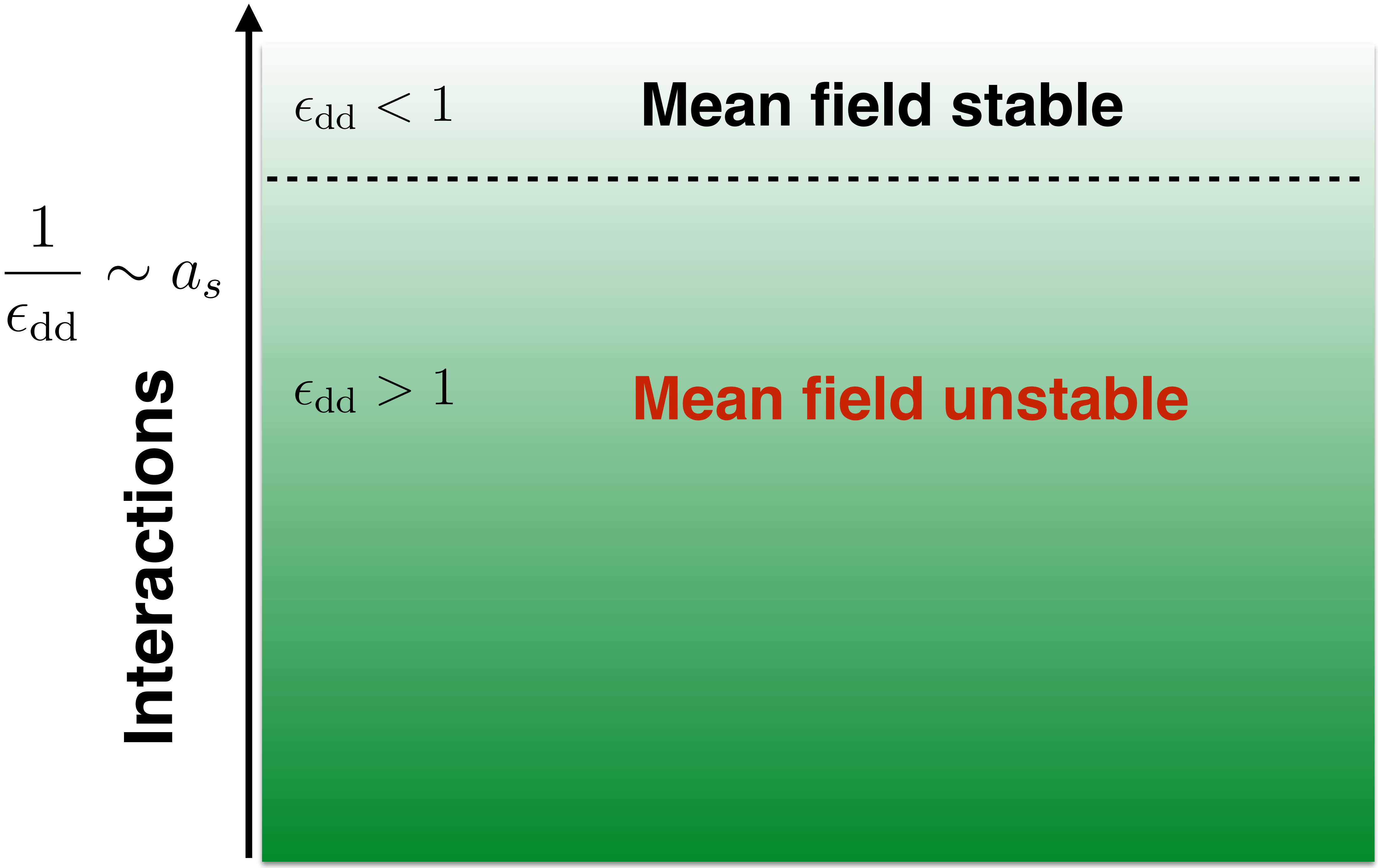
(Koch *et al.*, Wilson *et al.*,Linscott *et al.*)

classical collapse analog

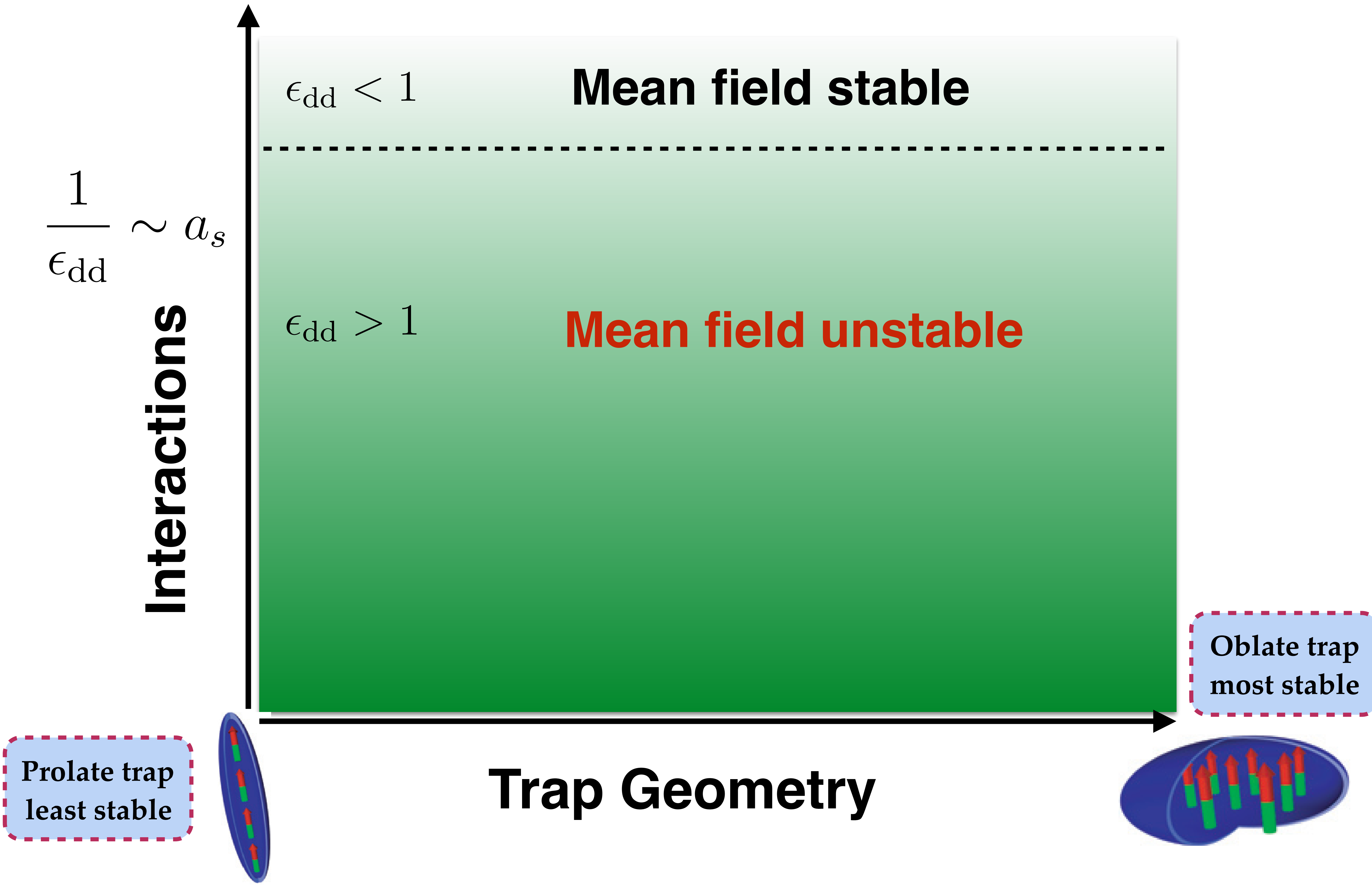


MAGINATION

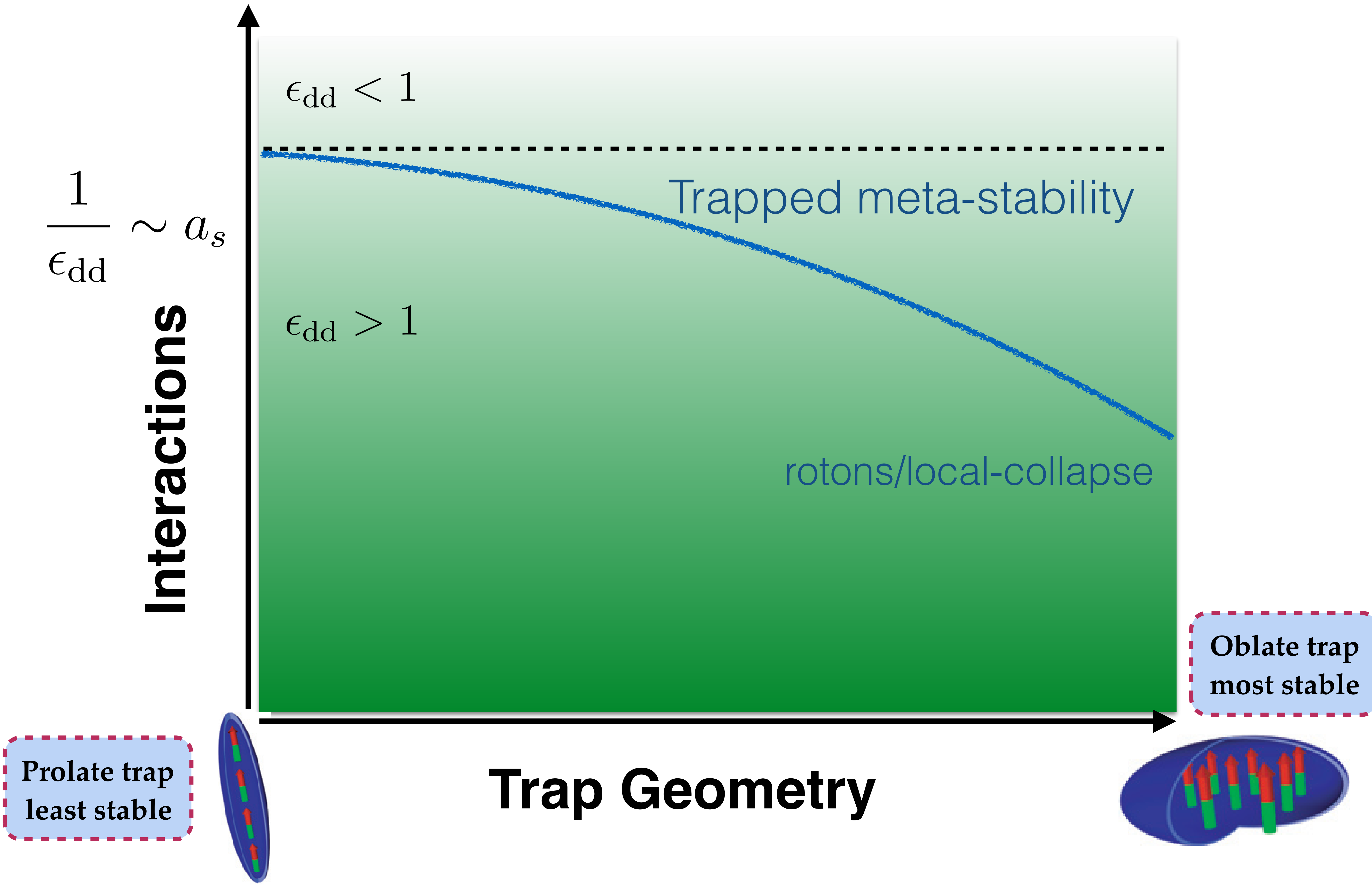
mean field phase diagram



mean field phase diagram

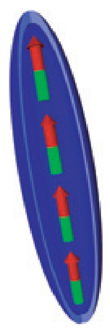


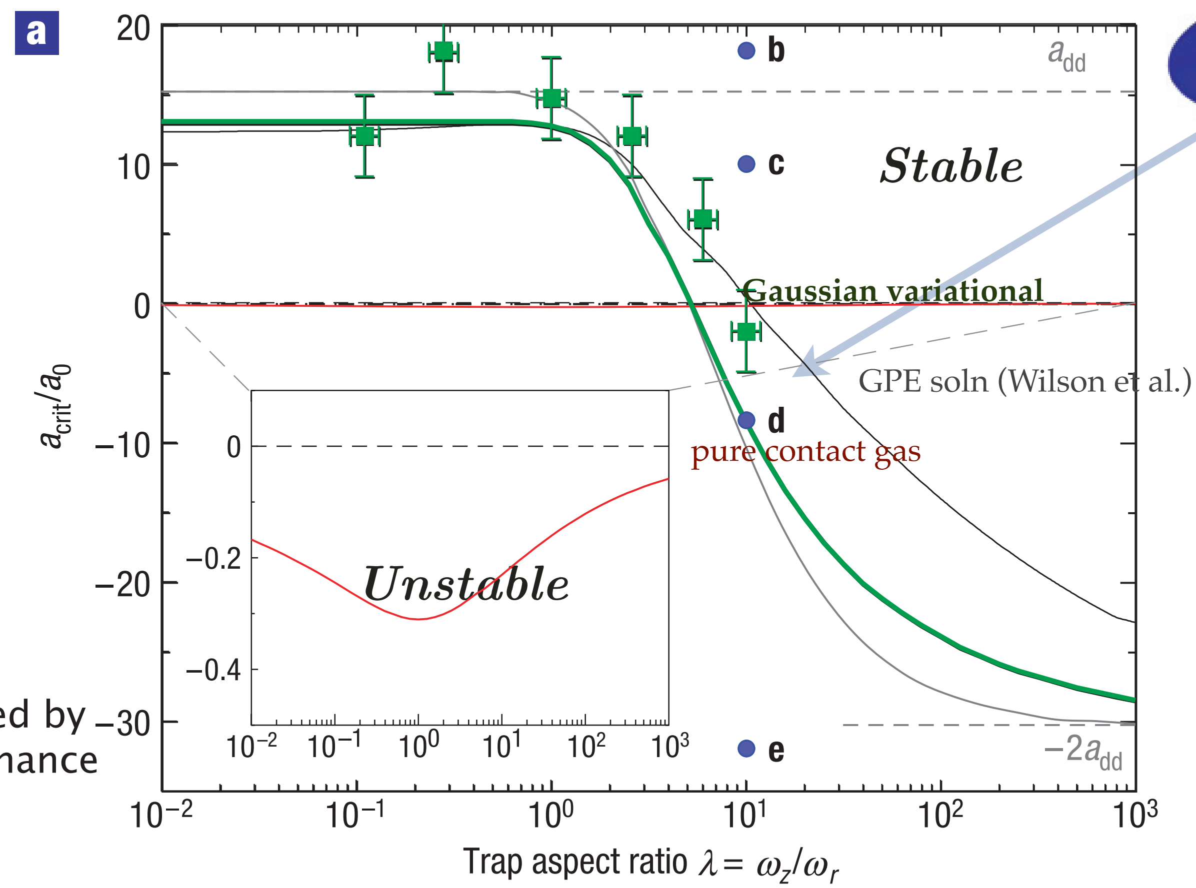
mean field phase diagram

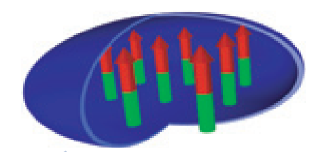


mean field phase diagram

Experiments with ^{52}Cr   T Koch et al., Nat. Phys. **4**, 218 (2008)





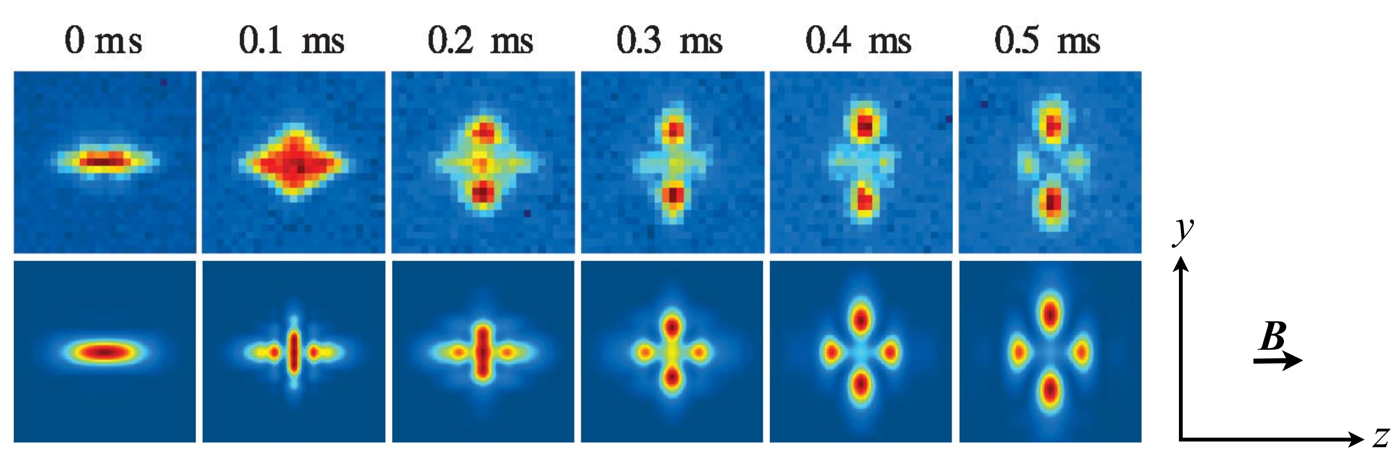


s-wave adjusted by Feshbach resonance

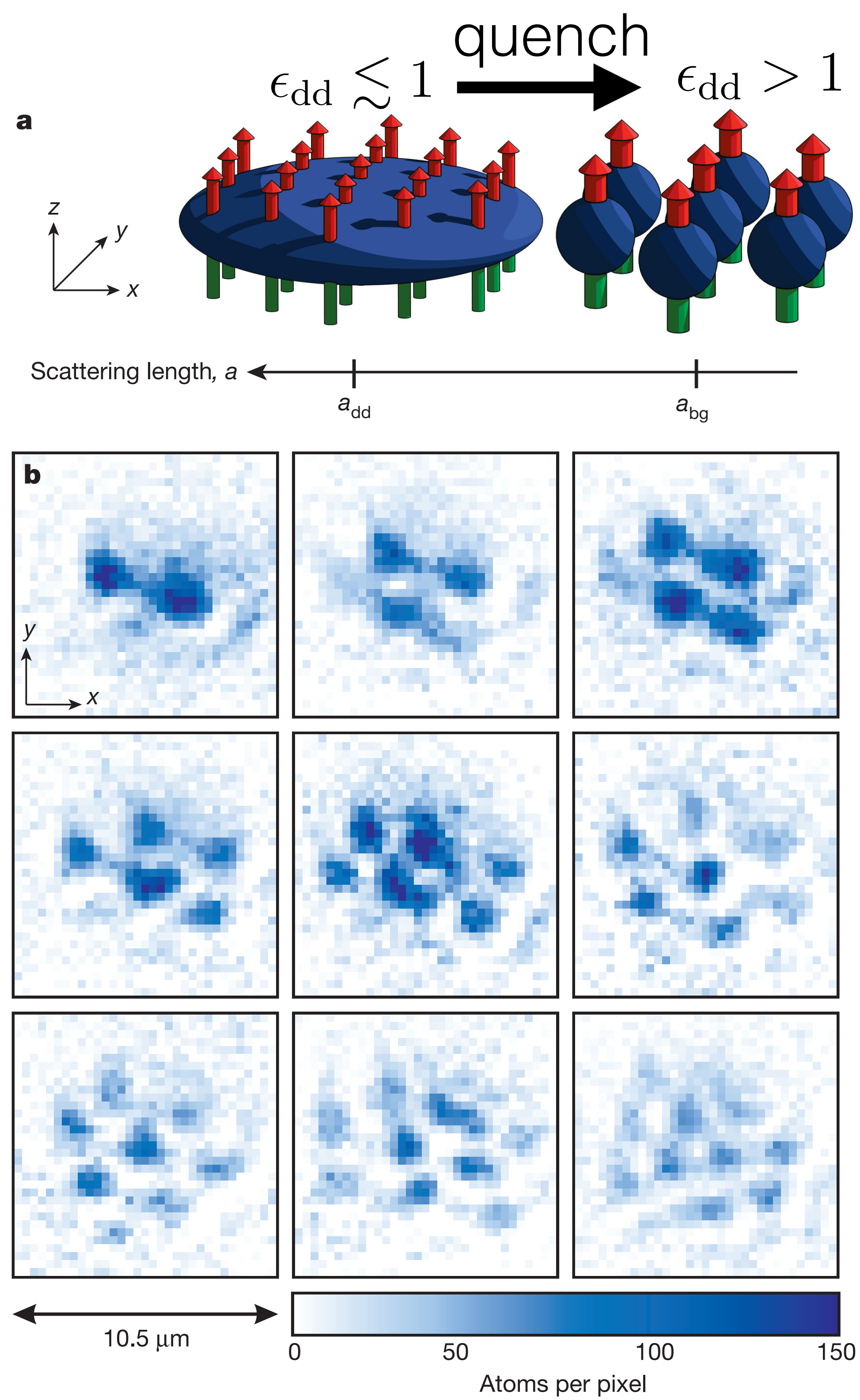
Key theory:

S. Ronen, et al., PRL **98**, 030406 (2007)

Collapse dynamics with ^{52}Cr: PRL **101** 080401 (2008).



Arrested Development



Quenching to dipole dominated regime experiments observe stable long-lived droplet crystals

- Lifetime > 100's ms
- ~1000 atoms per droplet
- ~3 micron spacing
- peak density > 5×10^{20}m^{-3}
- $a_{dd} \gtrsim 1.4 a_s$

Not predicted by standard meanfield theory

(Kadau *et al.*,Nature 2016, also see Ferrier-Barbut *et al.* PRL 2016)

What is this new state?

nature International weekly journal of science

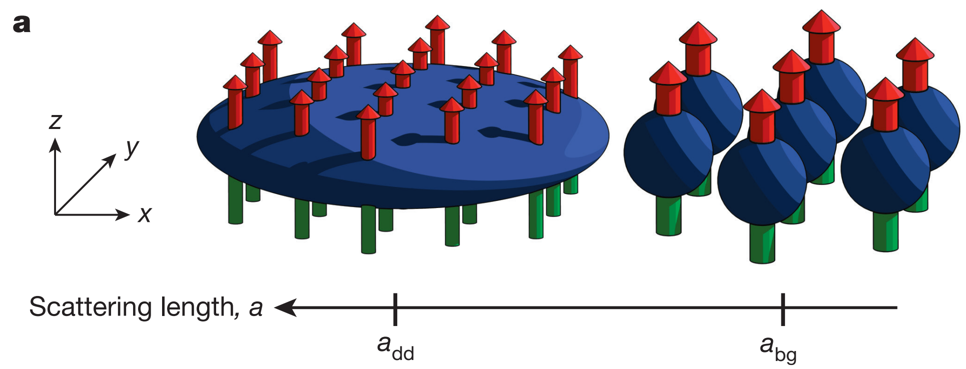
NATURE | LETTER

日本語要約

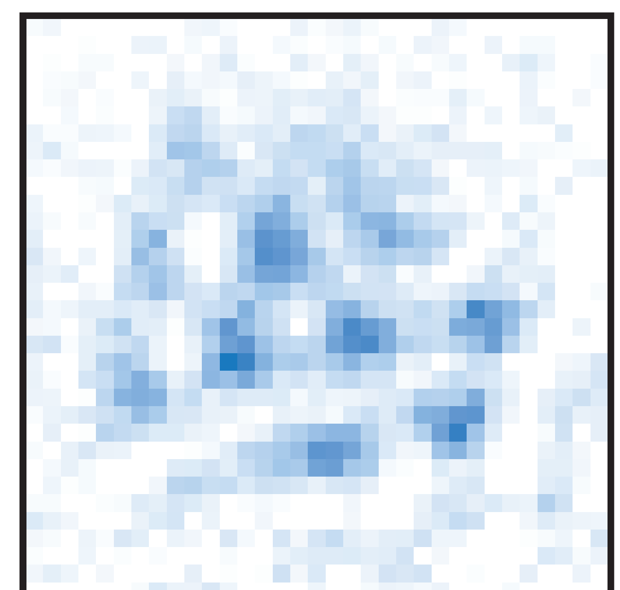
Observing the Rosensweig instability of a quantum ferrofluid

Holger Kadau, Matthias Schmitt, Matthias Wenzel, Clarissa Wink, Thomas Maier, Igor Ferrier-Barbut & Tilman Pfau

Nature **530**, 194–197 (11 February 2016)



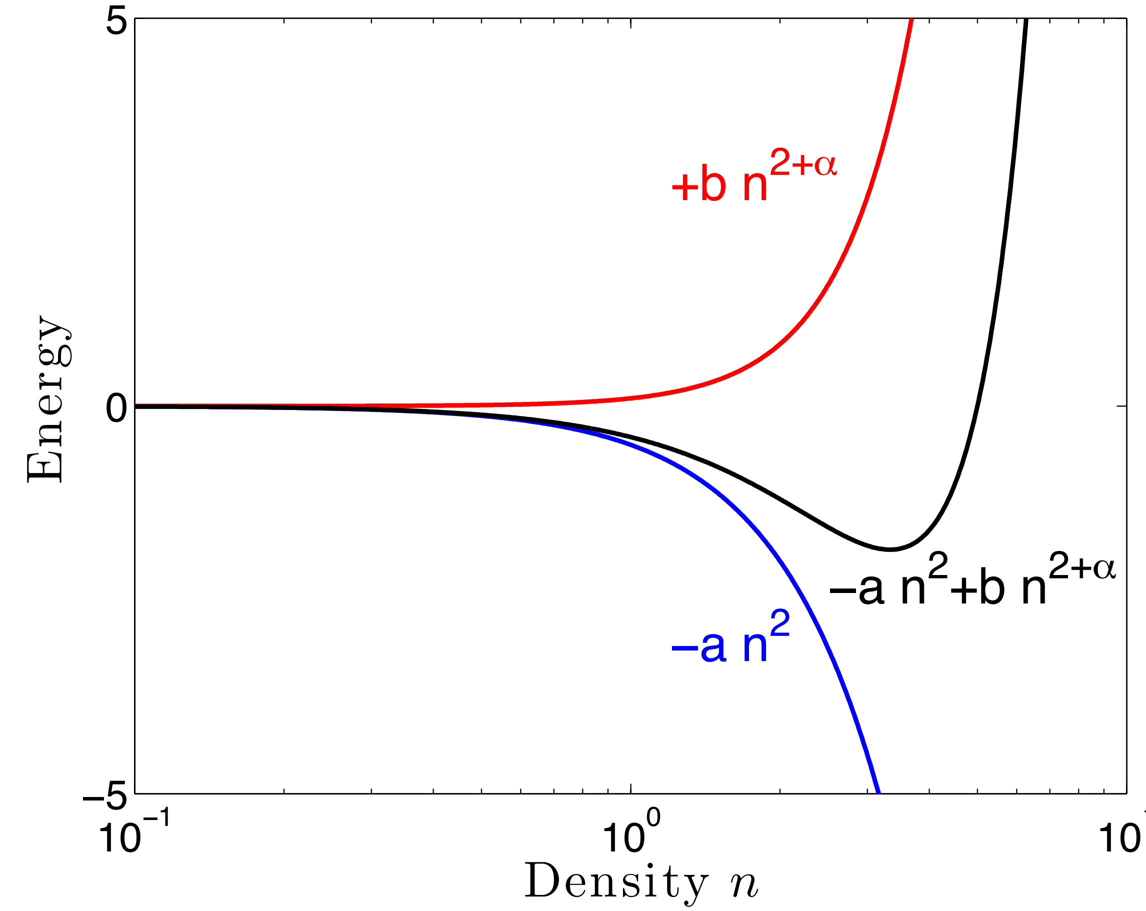
supersolid?



Rosensweig instability of a ferrofluid

Arrested by Higher Order Interaction

Competition between interactions



attractive (collapsing) 2-body interaction n^2 vs. $n^{2+\alpha}$ **repulsive** (stabilizing) "(2+α)-body" interaction

density

(similar ideas discussed in He droplets, nuclear physics…)

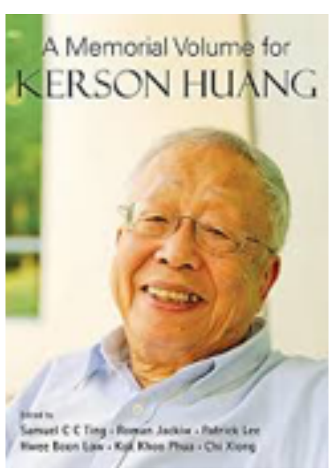
(Petrov, Liu *et* al., Xi *et al.*, Wachtler *et al.*, Bisset *et al.*, Lima *et al.*, Bulgac *et al.*, Ferrier-Barbut *et al.*,….)

Quantum Fluctuations

$$\frac{E}{V} = \overbrace{\frac{1}{2} g_s n^2}^{\text{mean-field energy}} \left[1 + \underbrace{\frac{128}{15\sqrt{\pi}} \sqrt{n a_s^3}}_{\text{leading order quantum corrections}} \right]$$

... the "LHY" corrections

L



H

Y

T. D. Lee and C. N. Yang, Phys. Rev. 105, 1119 (1957).
T. D. Lee, K. Huang, and C. N. Yang, Phys. Rev. 106, 1135 (1957)

LHY Corrections

- Are **small** for a dilute gas: $na_s^3 \ll 1$

typical BEC $na_s^3 \sim 10^{-5} - 10^{-4}$ liquid He $na_s^3 \sim 1$

- First quantified in 2010 with Fermi gas on the BCS-BEC crossover measuring the equation of state

Solomon Group, Science **328,** 729 (2010)
(regime: *weakly bound molecular bosons)*

Generalised mean field theory for dipolar BEC

DDI correction valid for $\epsilon_{\rm dd} \sim 1$

LHY for a dipolar condensate: $\Delta E_{\rm LHY} = V\frac{64}{15} g_s \sqrt{\frac{a_s^3}{\pi}} \left(1 + \frac{3}{2}\epsilon_{\rm dd}^2\right) n^{5/2}$

Chemical potential correction: $\Delta\mu_{\rm LHY} = \frac{\partial \Delta E_{\rm LHY}}{\partial N} = \gamma_{\rm QF} n^{3/2}$

$\gamma_{\rm QF} = \frac{32}{3} g_s \sqrt{\frac{a_s^3}{\pi}} \left(1 + \frac{3}{2}\epsilon_{\rm dd}^2\right)$

local density approximation

$n^{3/2} \rightarrow |\psi|^3$

$$i\hbar\frac{\partial\psi}{\partial t} = \left[H_{\rm sp} + \int d\mathbf{r}'\, U(\mathbf{r}-\mathbf{r}')|\psi(\mathbf{r}')|^2 + \gamma_{\rm QF}|\psi|^3\right]\psi,$$

single particle physics

$$H_{\rm sp} = -\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m(\omega_\rho^2\rho^2 + \omega_z^2 z^2),$$

two-body interactions

$$U(\mathbf{r}) = g\delta(\mathbf{r}) + \frac{\mu_0\mu^2}{4\pi}\frac{1-3\cos^2\theta}{r^3},$$

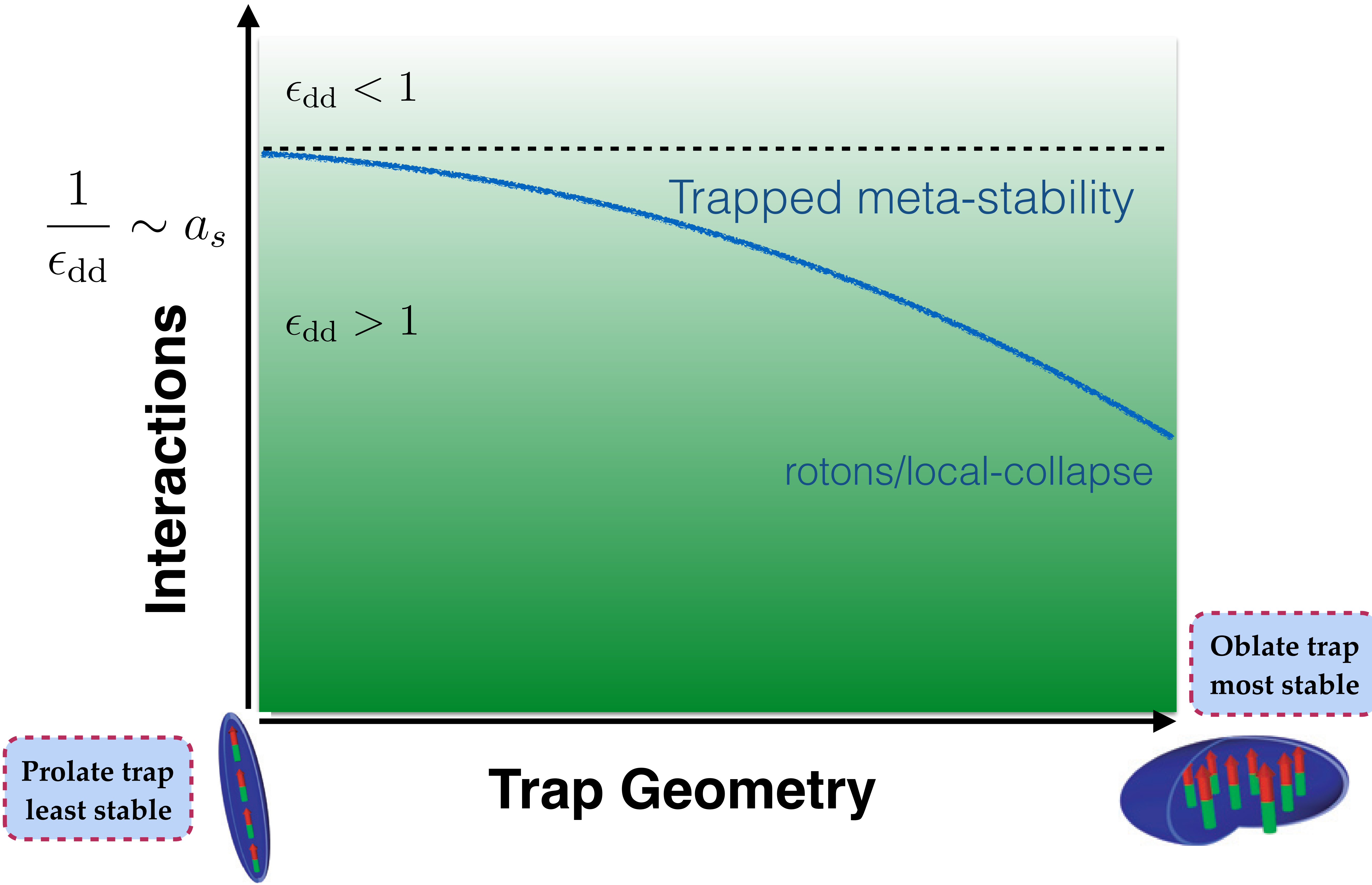
Dipolar LHY theory:

Lima and Pelster, PRA **84**, 041604 (2011).

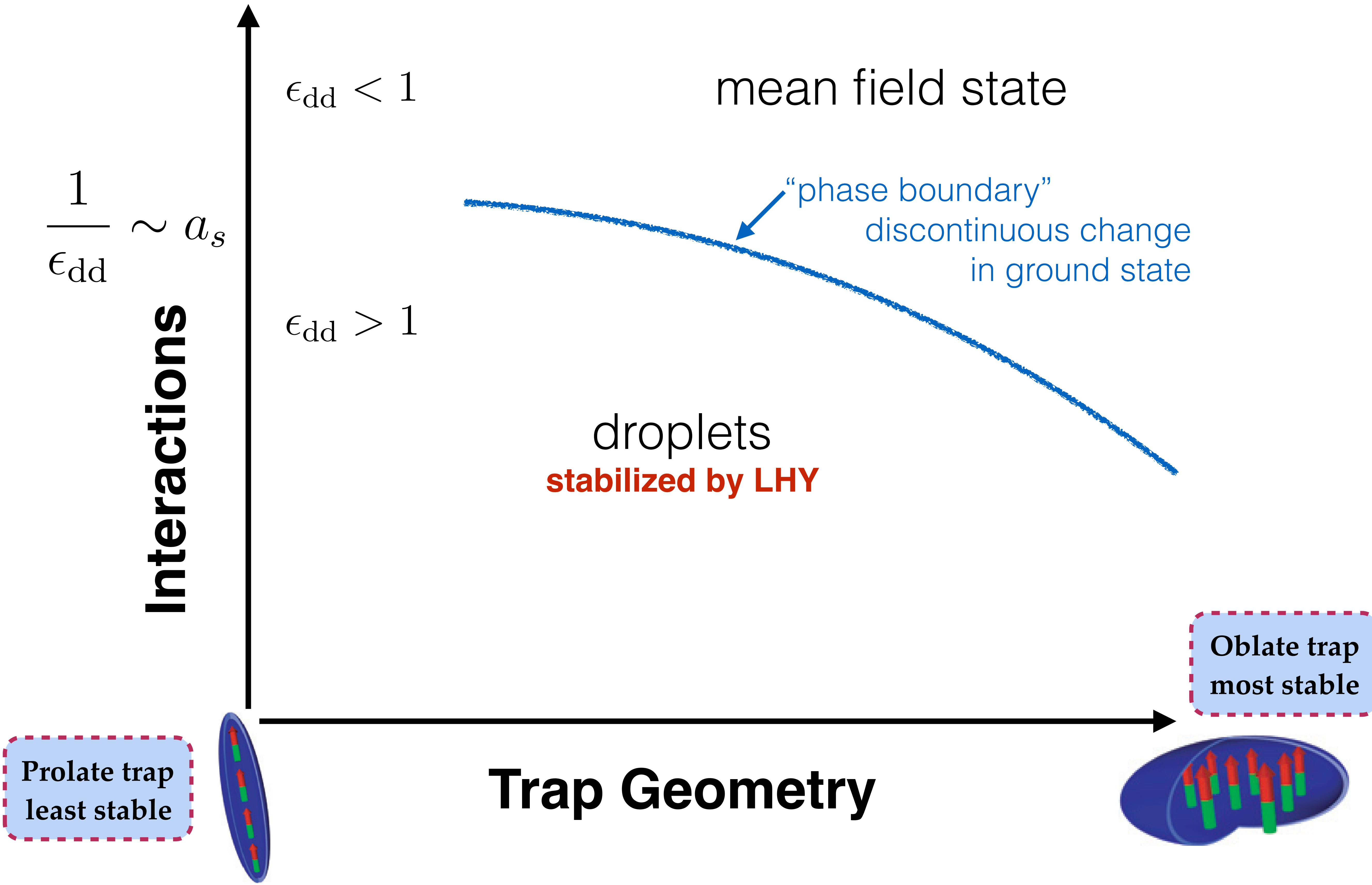
Lima and Pelster, PRA **86**, 063609 (2012).

Path-integral Monte Carlo calculations show good agreement with generalised GPE [H. Saito, J. Phys. Soc. of Jap. **85**, 053001 (2016)]

mean field phase diagram

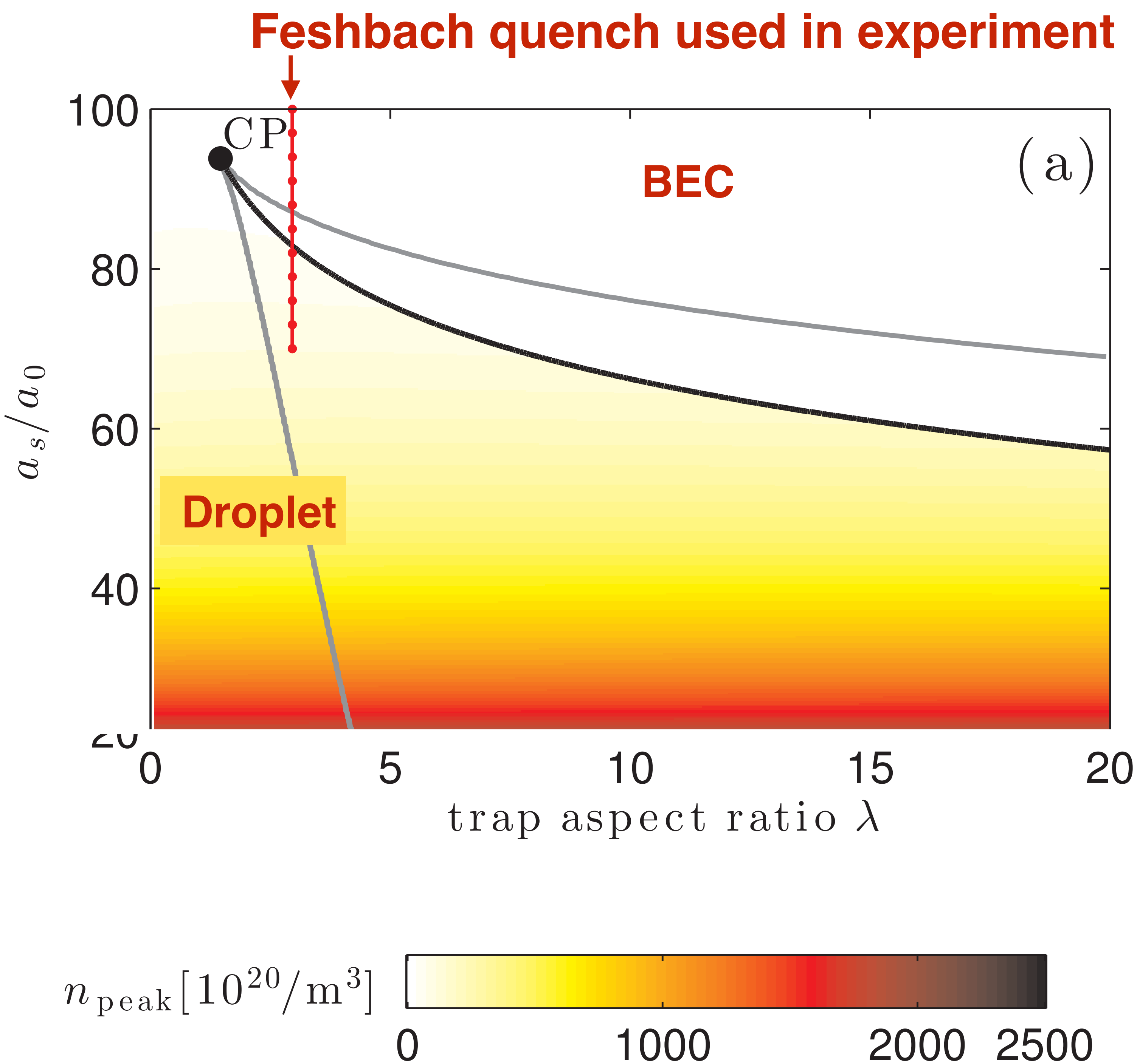


mean field phase diagram **with LHY**



Modeling the Stuttgart Experiment

Dy-164
parameters
$a_{\mathrm{dd}} = 130\, a_0$

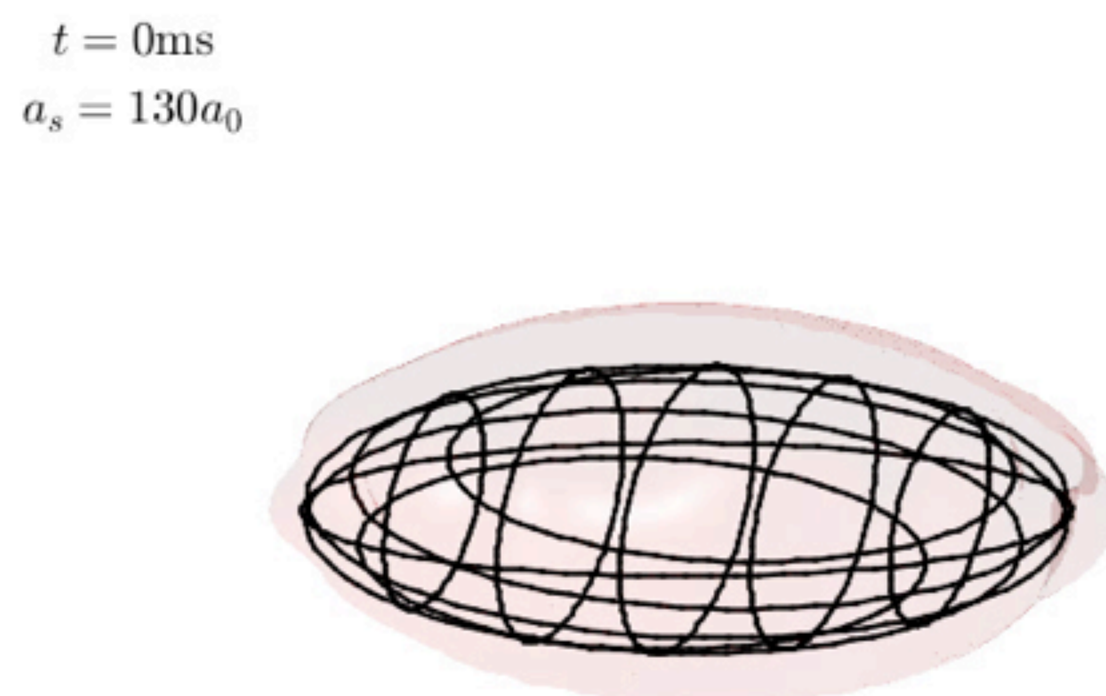


N=

Bisset *et al.*, Phys. Rev. A **94**, 033619 (2016)

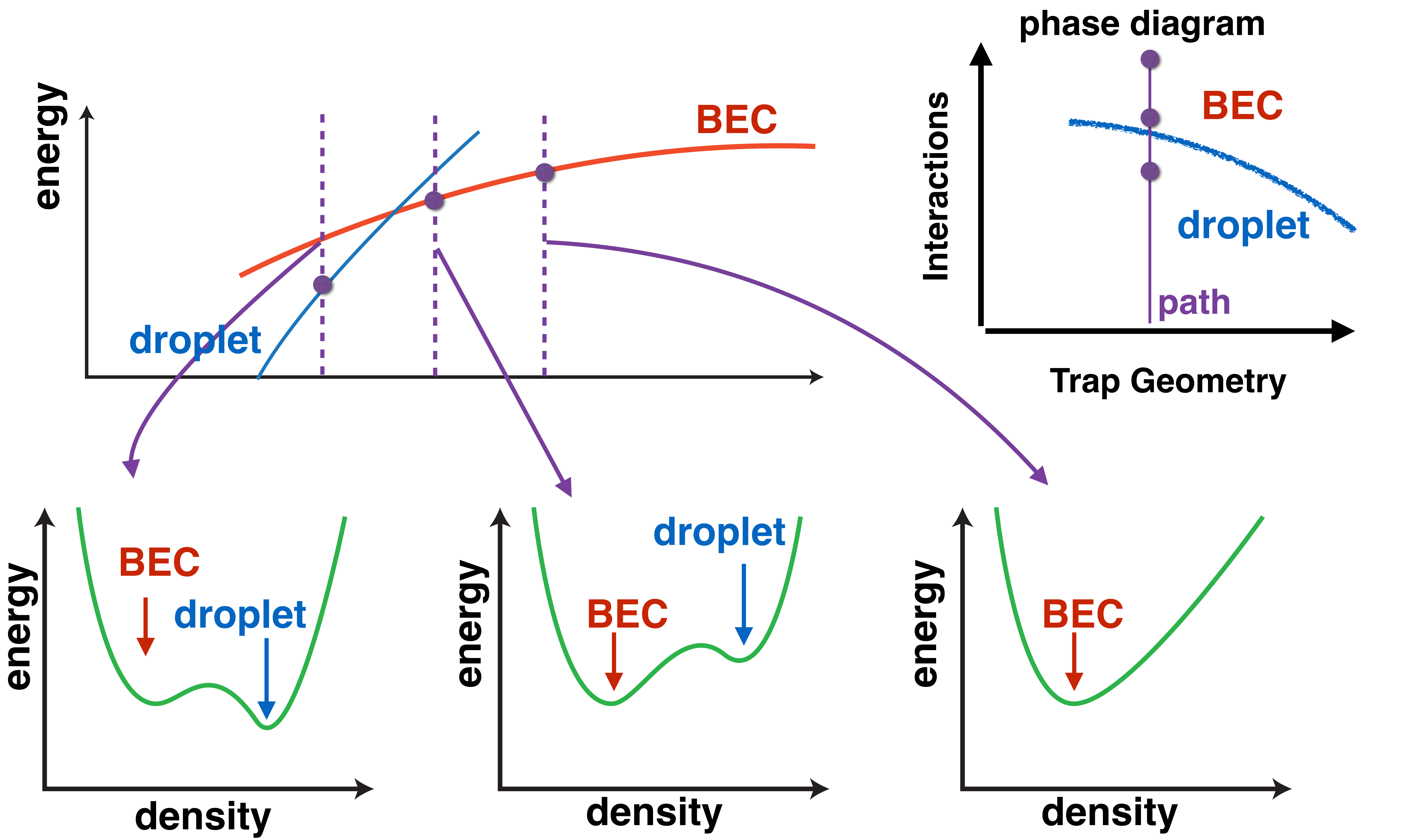
Phase diagram depends on N,
results here for $N = 15000, \bar{\omega}/2\pi = 64.6\,\mathrm{Hz}$.

Simulation of Stuttgart Experiment



Generalised GPE dynamics. Noise added to initial condensate
Scattering length quenched in 0.5 ms, trap left on

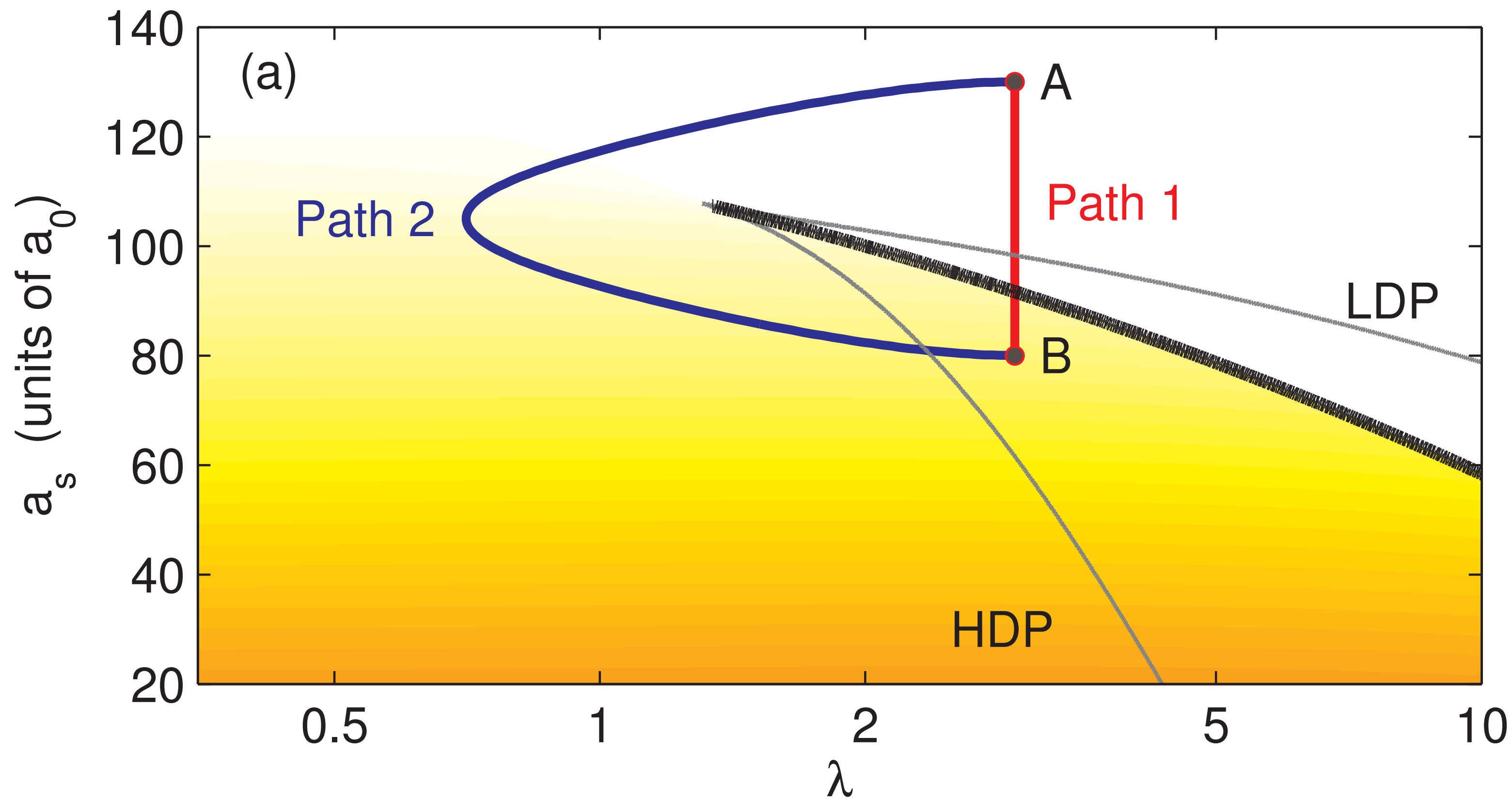
Discontinuous change in oblate trap



Droplet “crystal”

- Generalised GPE suggests droplets not phase coherent with each other (i.e.not a super-solid)
- Nucleation-like formation process.
- Size and properties of droplets in crystal not yet quantitatively understood (inter-droplet interactions important?)

path dependence?



geometric mean trap frequency held constant

- Path 1 is that considered in experiments
- Path 2 goes around the critical point
- Here use long quench time:

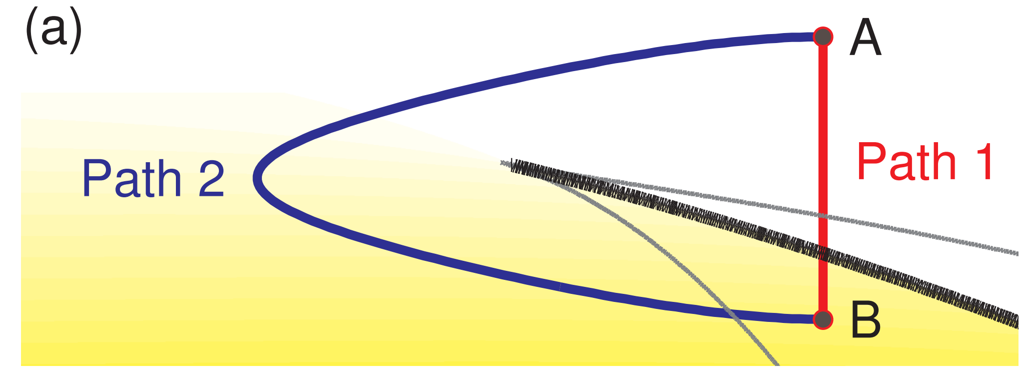
$$t_Q = 30\,\mathrm{ms}, \quad \mathrm{cf} \quad T_{\mathrm{trap}} = \frac{2\pi}{\bar{\omega}} = 16\,\mathrm{ms}$$

- Perform 3D solutions including higher order term, thermal & quantum initial noise $T = 20\,\mathrm{nK} \quad (N = 15 \times 10^3, \quad T_c \sim 70\,\mathrm{nK})$

P. B. Blakie, PRA **93**, 033644 (2016), March 2016

Path 1

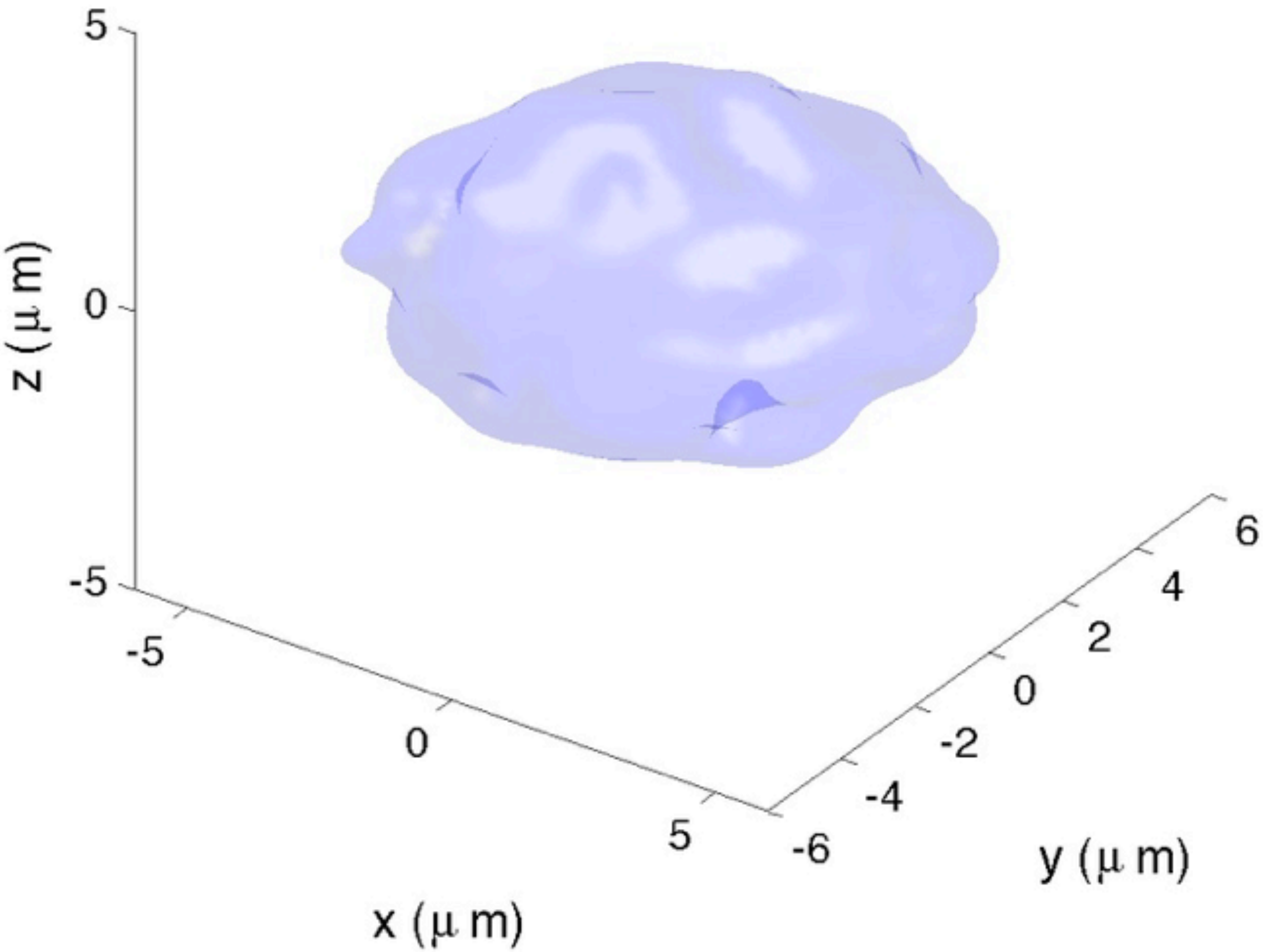
t=0ms



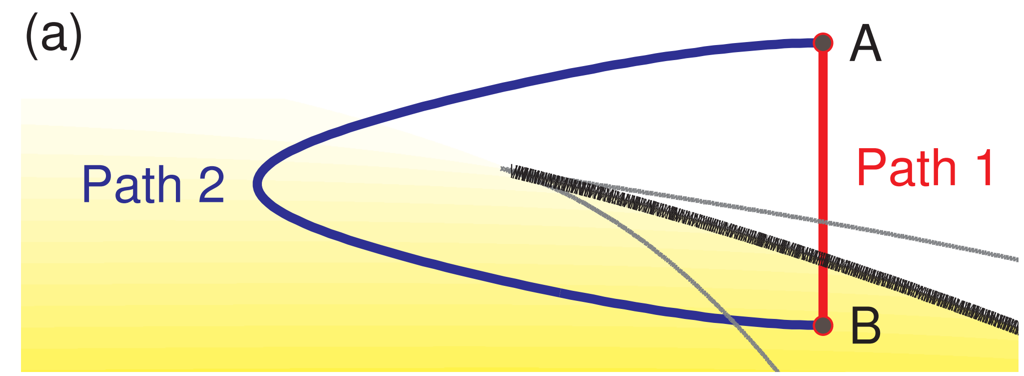
$$t_Q = 30\,\mathrm{ms}$$

blue density isosurface
$n = 2 \times 10^{19}\,\mathrm{m}^{-3}$

red density isosurface
$n = 2 \times 10^{20}\,\mathrm{m}^{-3}$



Path 2



$$t_Q = 30\,\mathrm{ms}$$

blue density isosurface
$n = 2 \times 10^{19}\,\mathrm{m}^{-3}$

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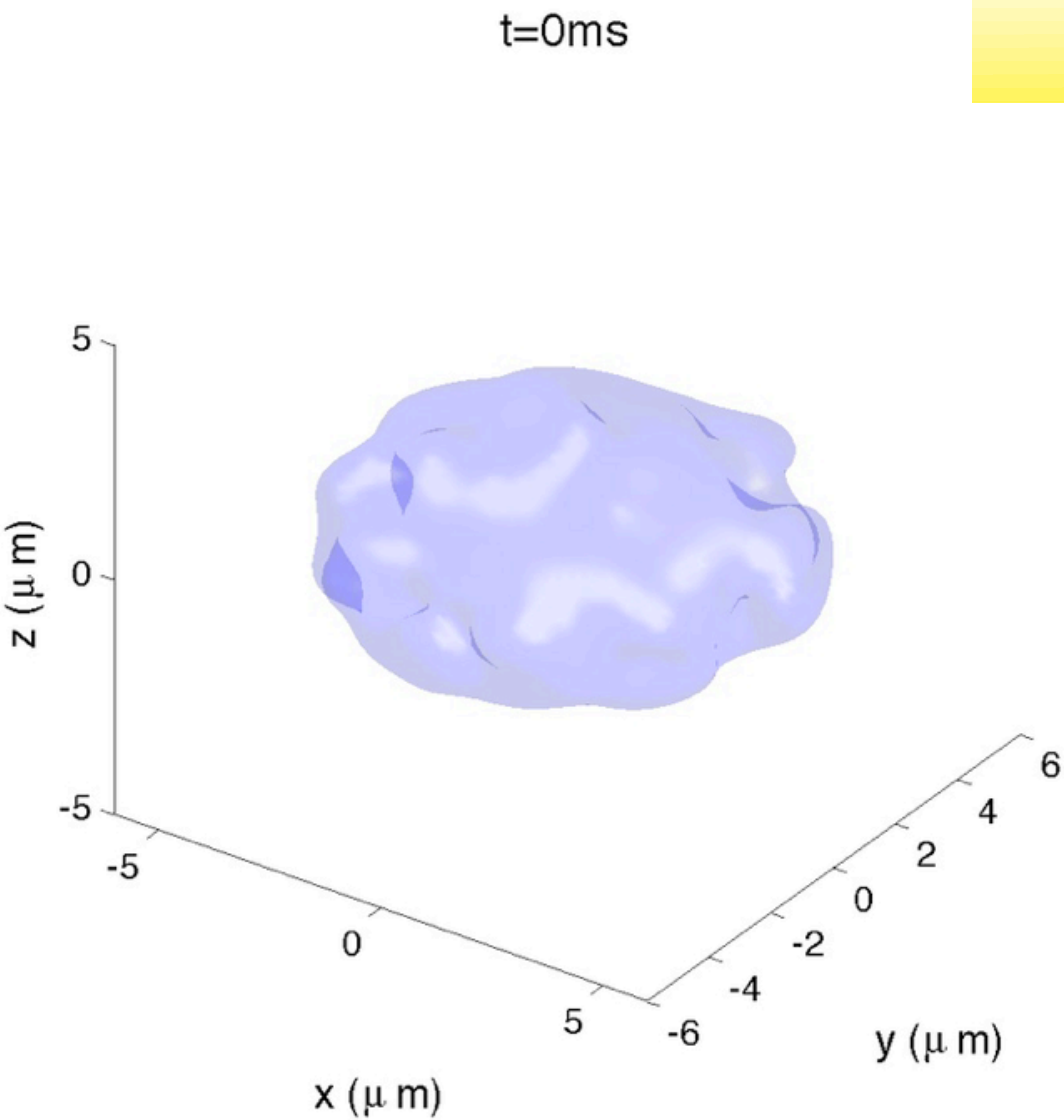
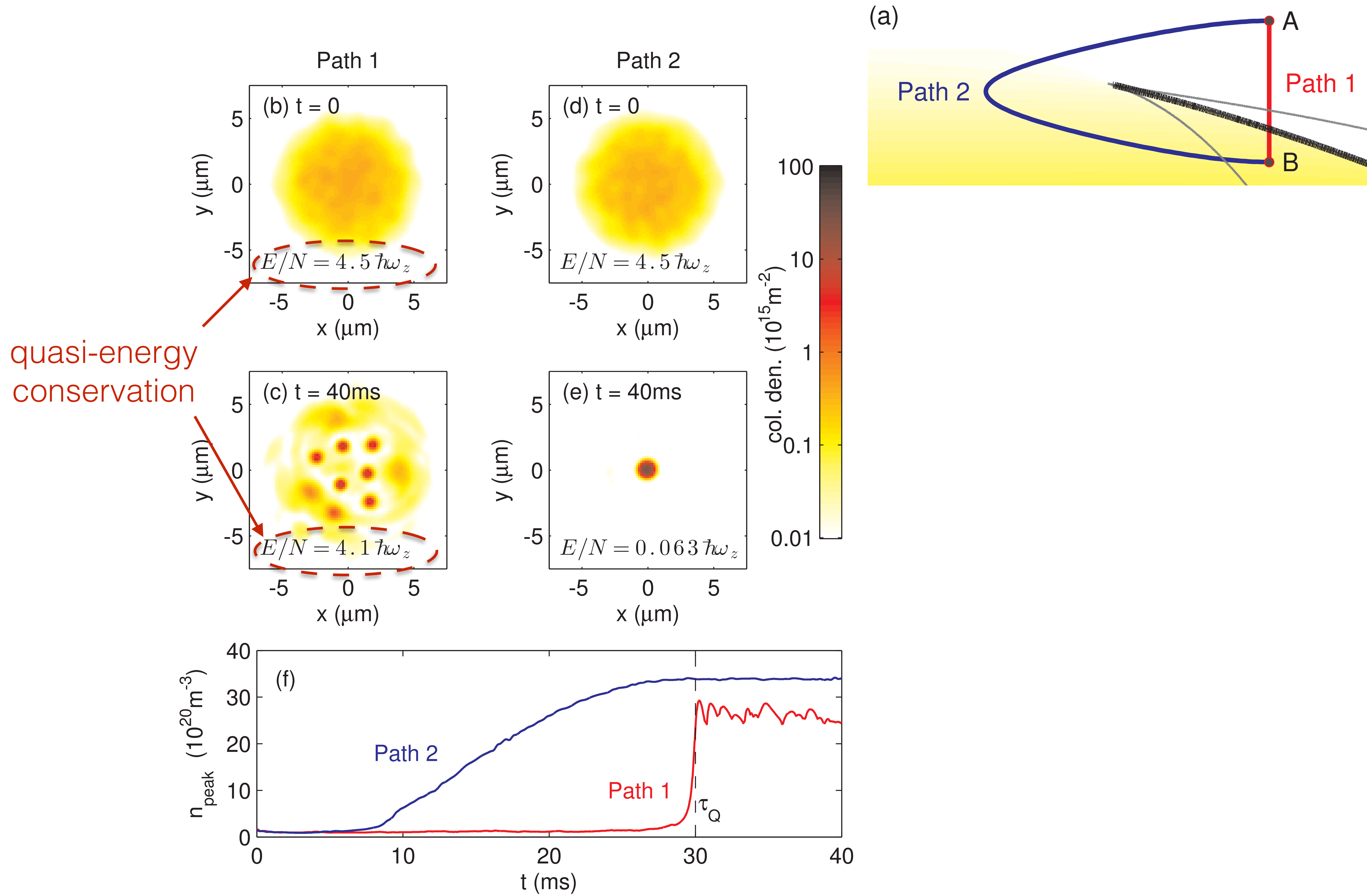
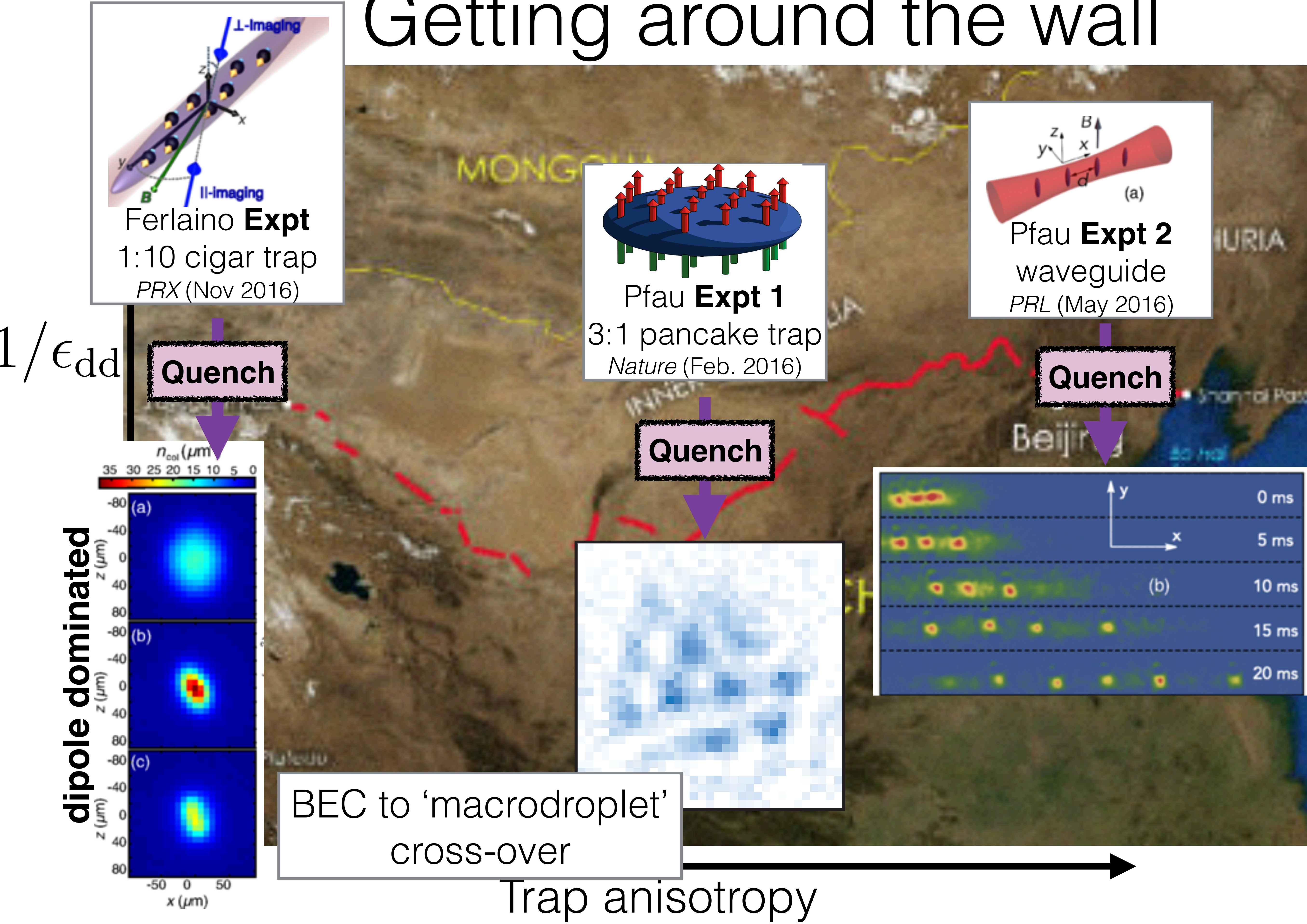


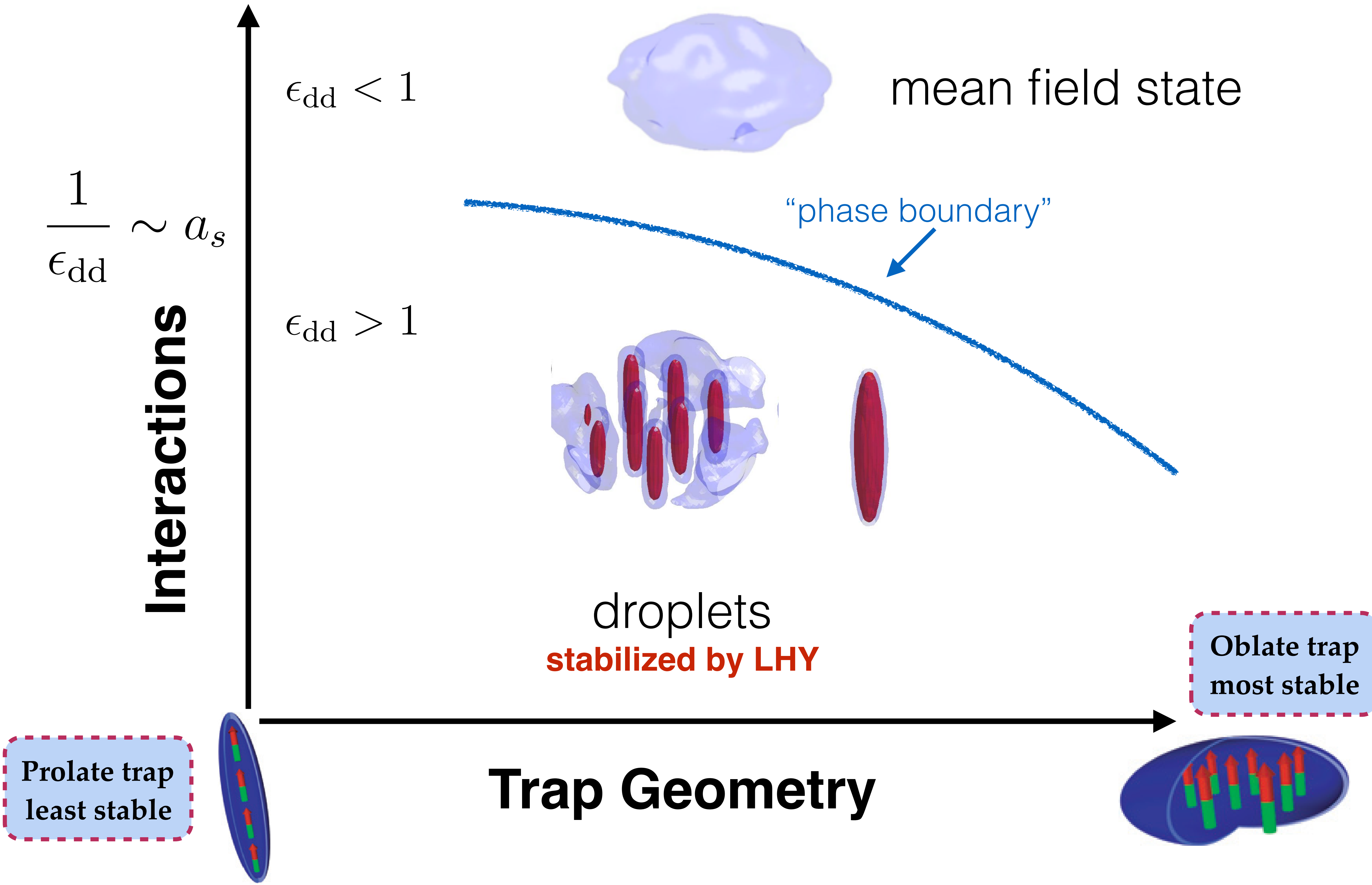
Path summary



Getting around the wall



mean field phase diagram **with LHY**



Do we need a trap?

In the absence of a trap and for fixed atomic number *N* there is a trivial uniform solution for the condensate wavefunction:

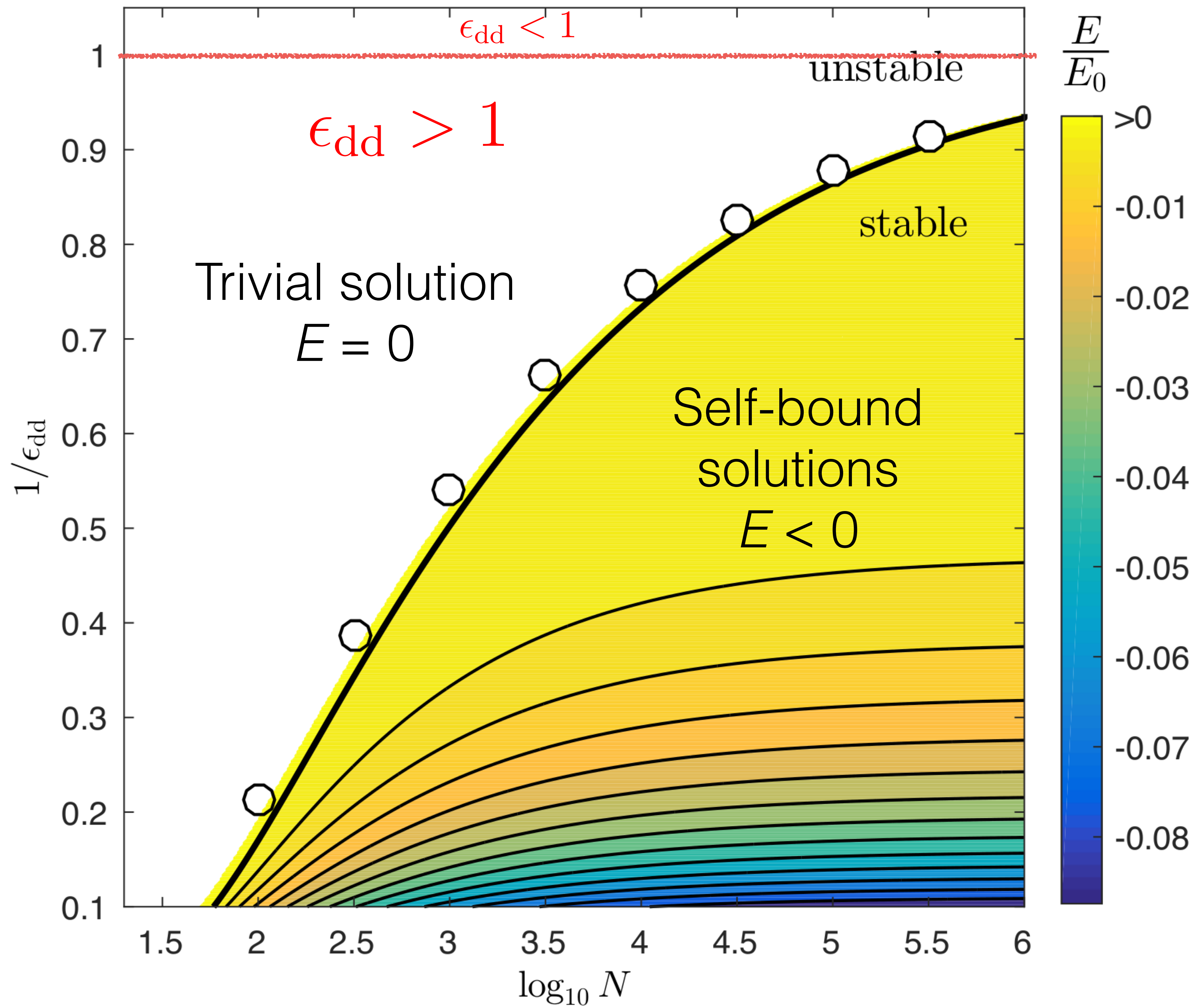
$$\psi \to 0 \qquad \text{with} \qquad E = 0$$

trivial 'dispersed' solution

Can we find *self-bound* localised solutions with

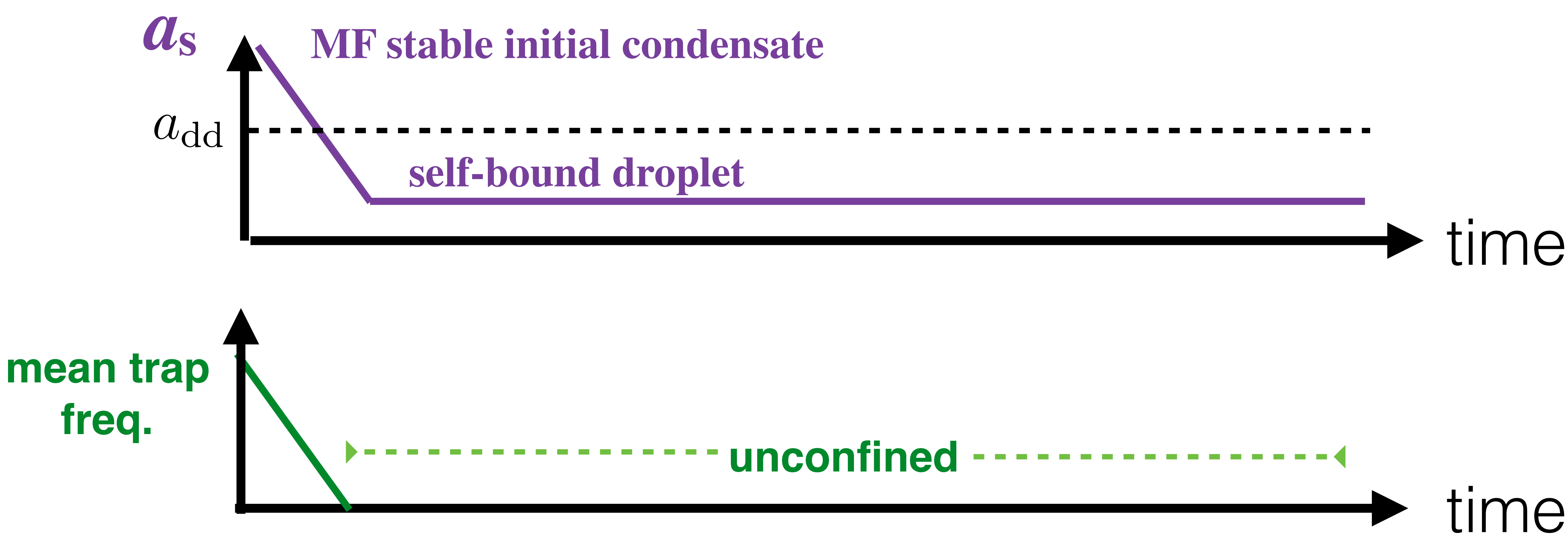
$$E < 0 \quad ?$$

Self-bound droplet phase diagram



self-bound droplet production

time sequence in **prolate** trap:



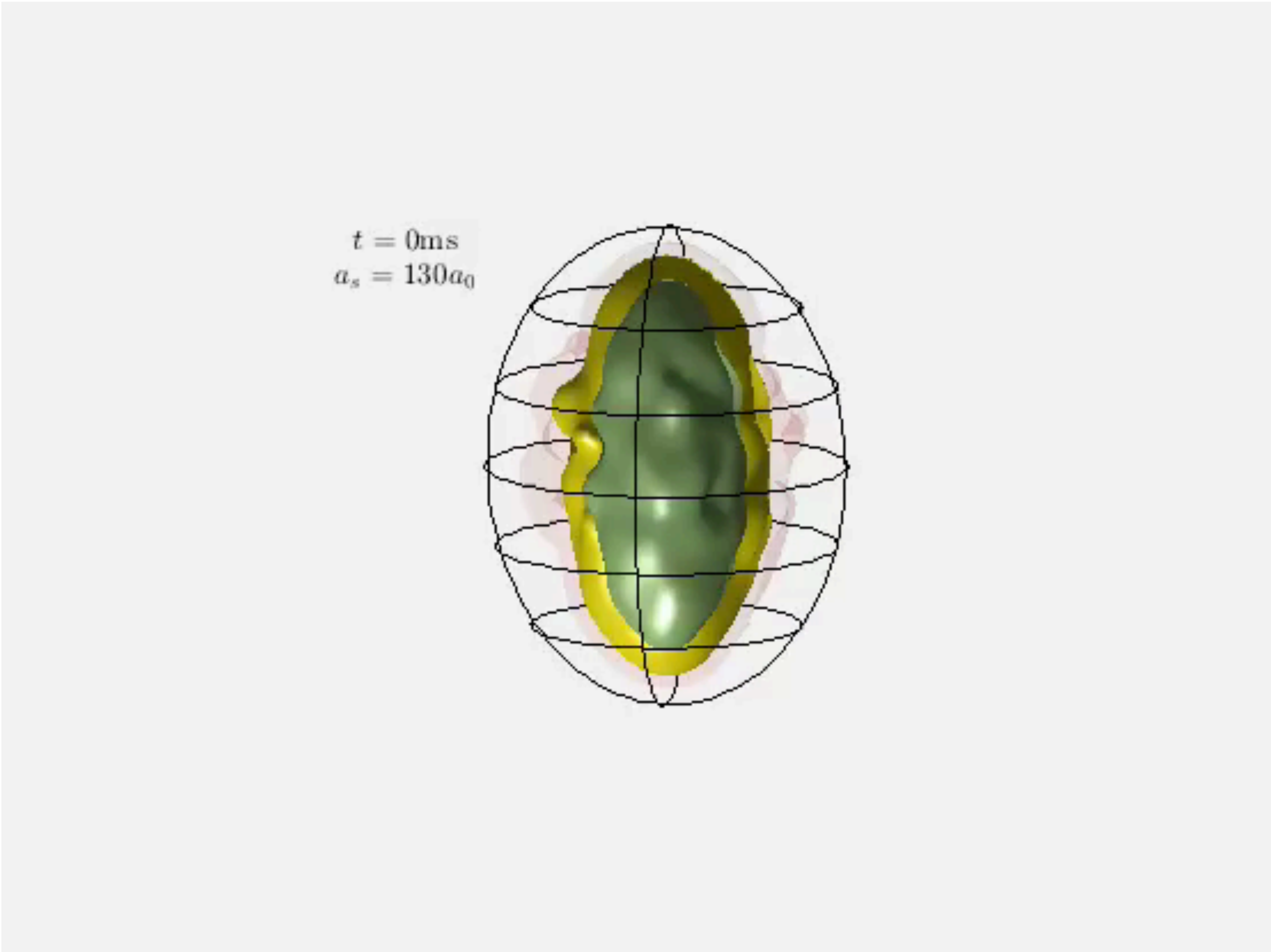
Dominant loss mechanism:
three-body recombination

$$i\hbar\frac{\partial\psi}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+\frac{m}{2}(\omega_\rho^2\rho^2+\lambda^2 z^2)+gn+\Phi_{dd}(\mathbf{r})+\gamma_{QF}n^{3/2}+\frac{i\hbar}{2}L_3 n^2\right]\psi$$

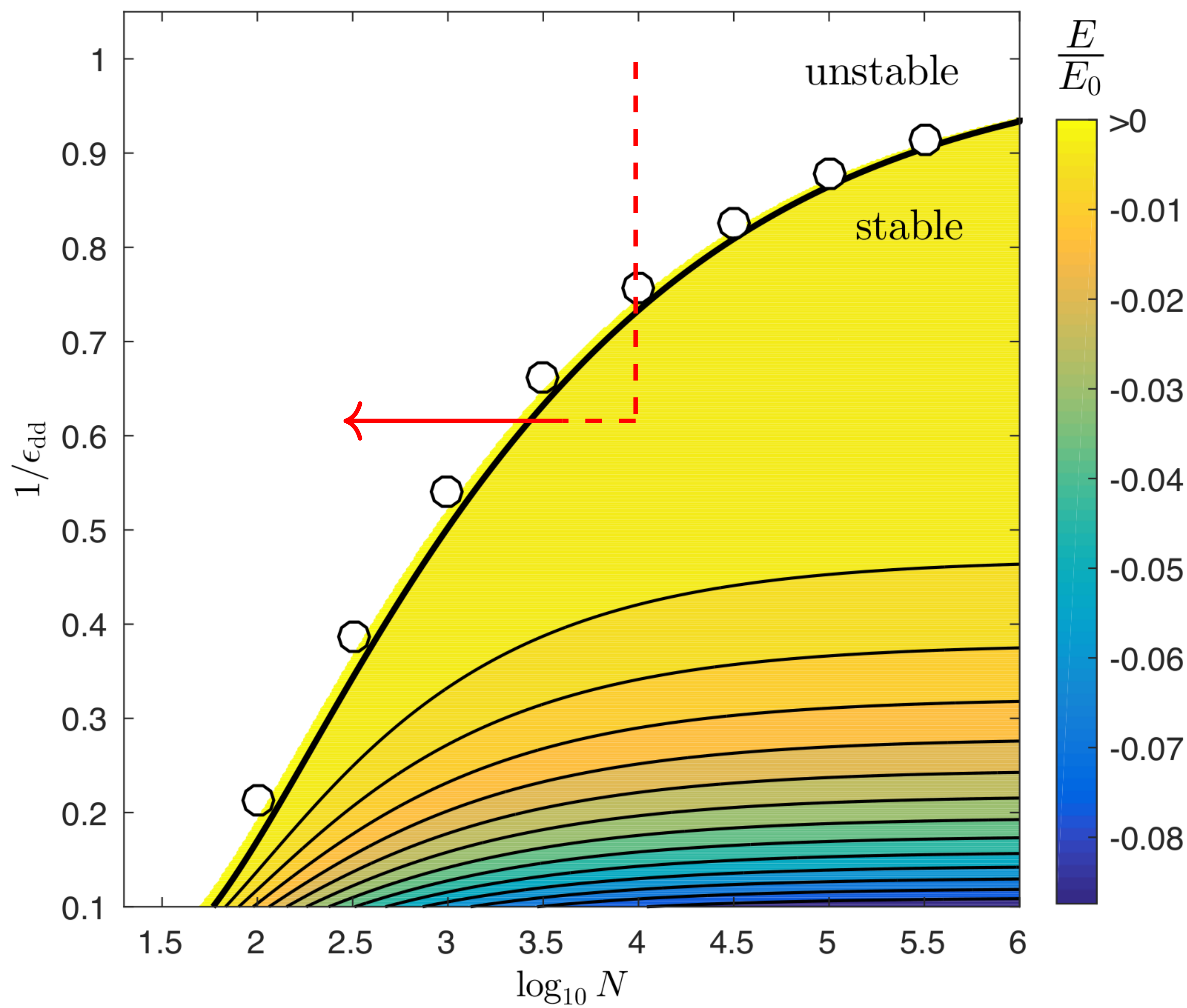
Generalised dipolar GPE including 3-body loss

3-body loss rate relatively low for Dy and Er

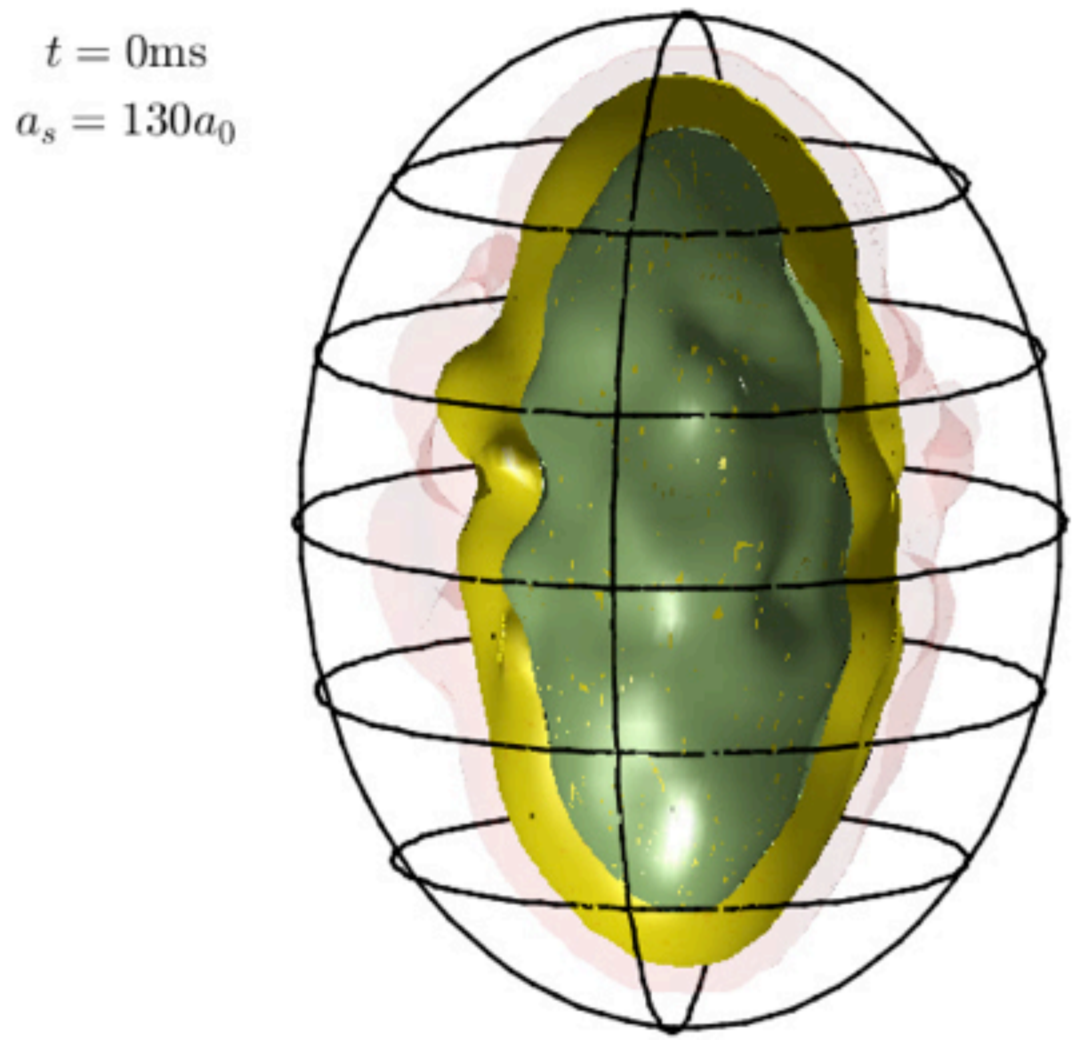
Quench to a_s $=80a_0$ and trap removal $(\epsilon_{\rm dd} \approx 1.6)$



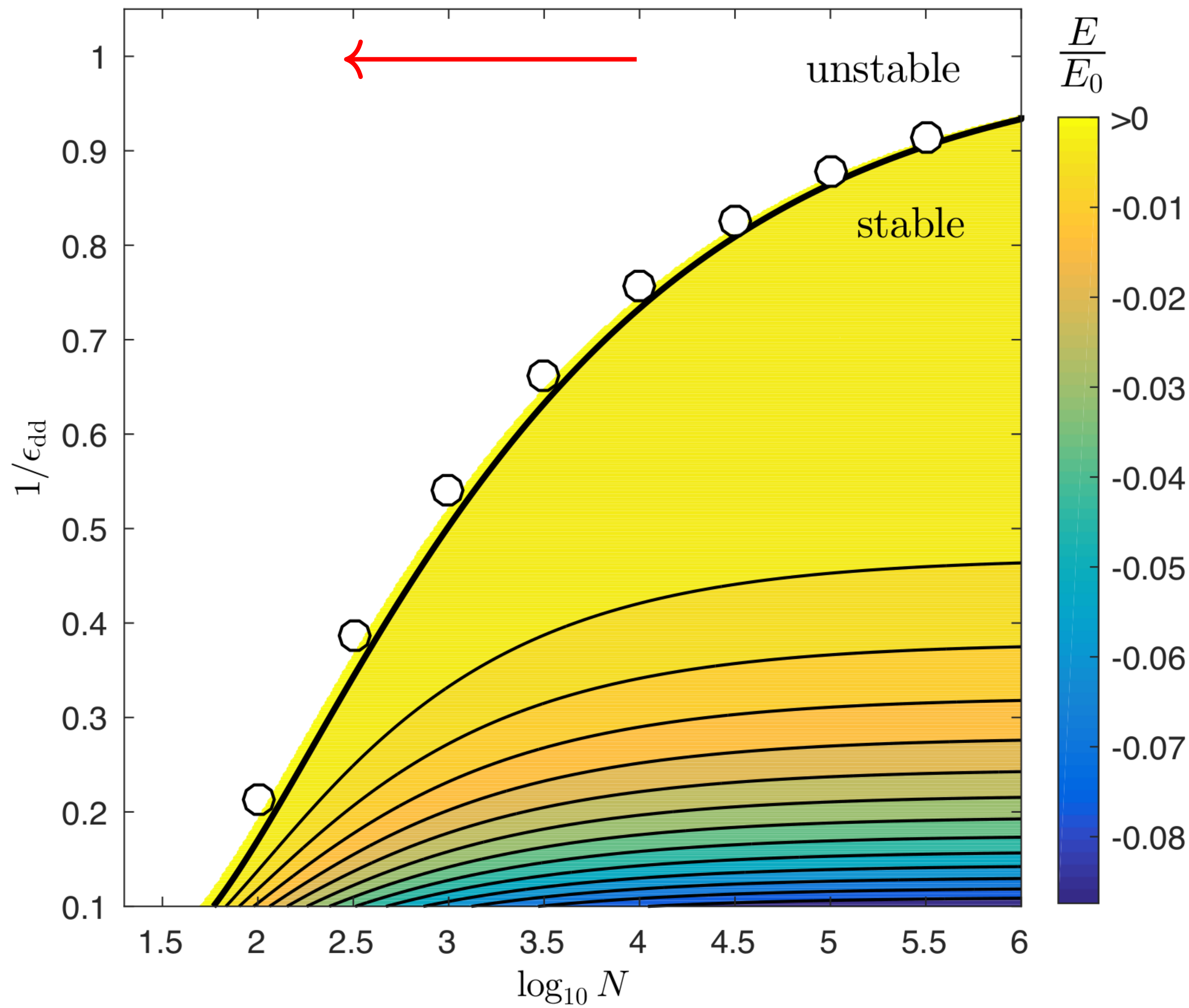
Phase diagram path: a_s quench and trap removal



Comparison: **no** a_s quench, **just** trap removal



Phase diagram path: **no** a_s quench, **just** trap removal

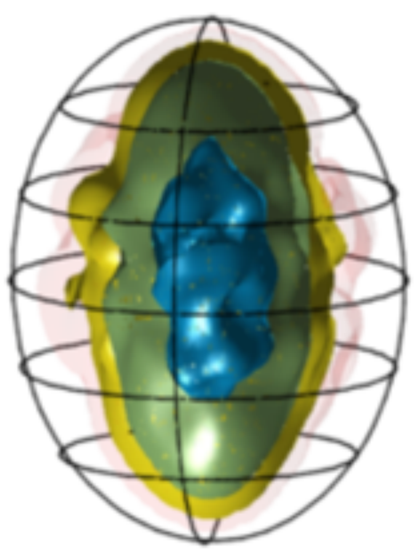


Editors' Suggestion **Rapid Communication**

Self-bound dipolar droplet: A localized matter wave in free space

D. Baillie, R. M. Wilson, R. N. Bisset, and P. B. Blakie

Phys. Rev. A **94**, 021602(R) (2016) – Published 11 August 2016

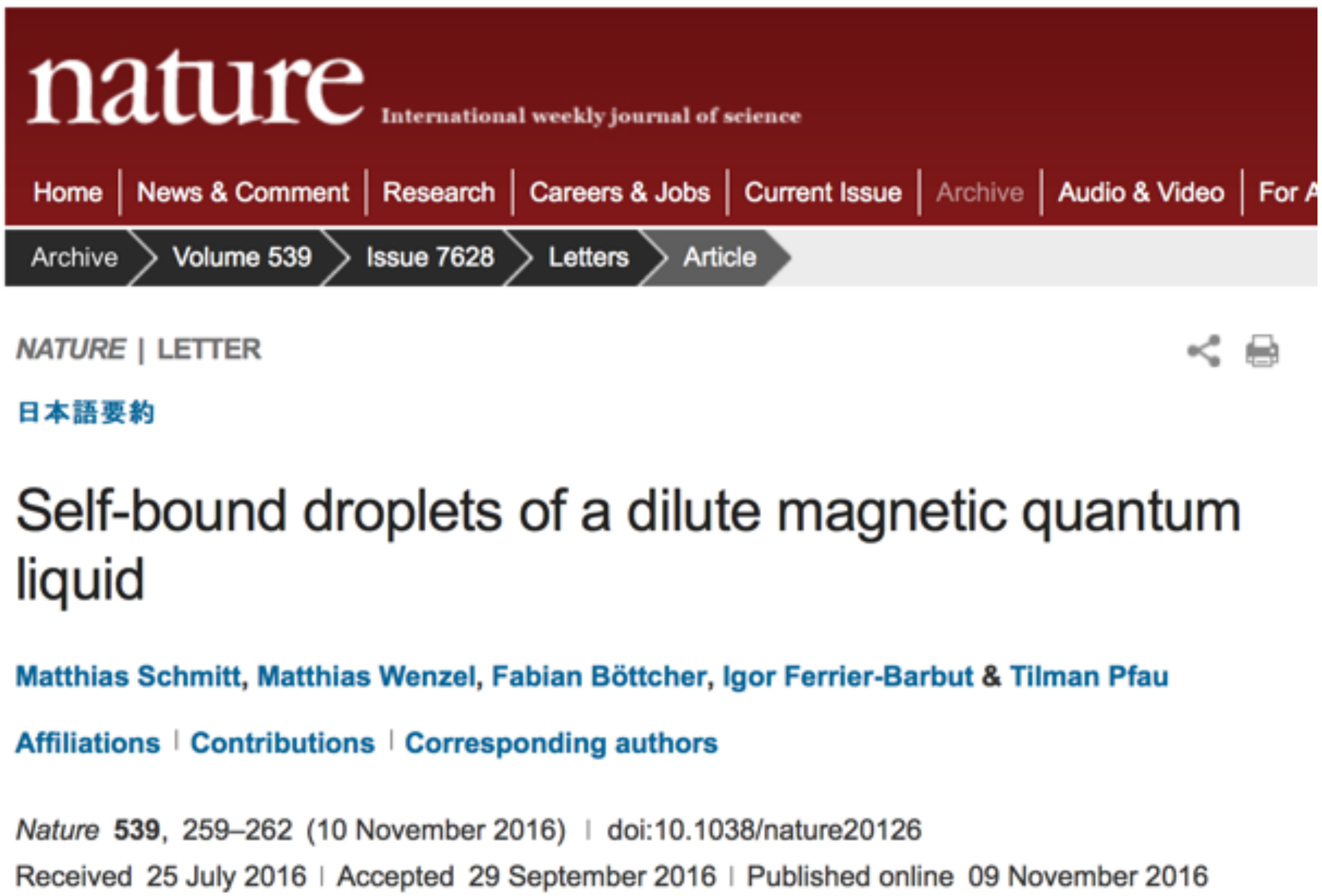


A liquid droplet is a self-bound phase of matter that holds itself together in the absence of a container. Without a container a gas will normally expand to fill space. A method is proposed to produce a self-bound dilute quantum gaseous dipolar Bose-Einstein condensate.

Show Abstract +

Experiments

- Pfau Group in Stuttgart



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Archive > Volume 539 > Issue 7628 > Letters > Article

NATURE | LETTER

日本語要約

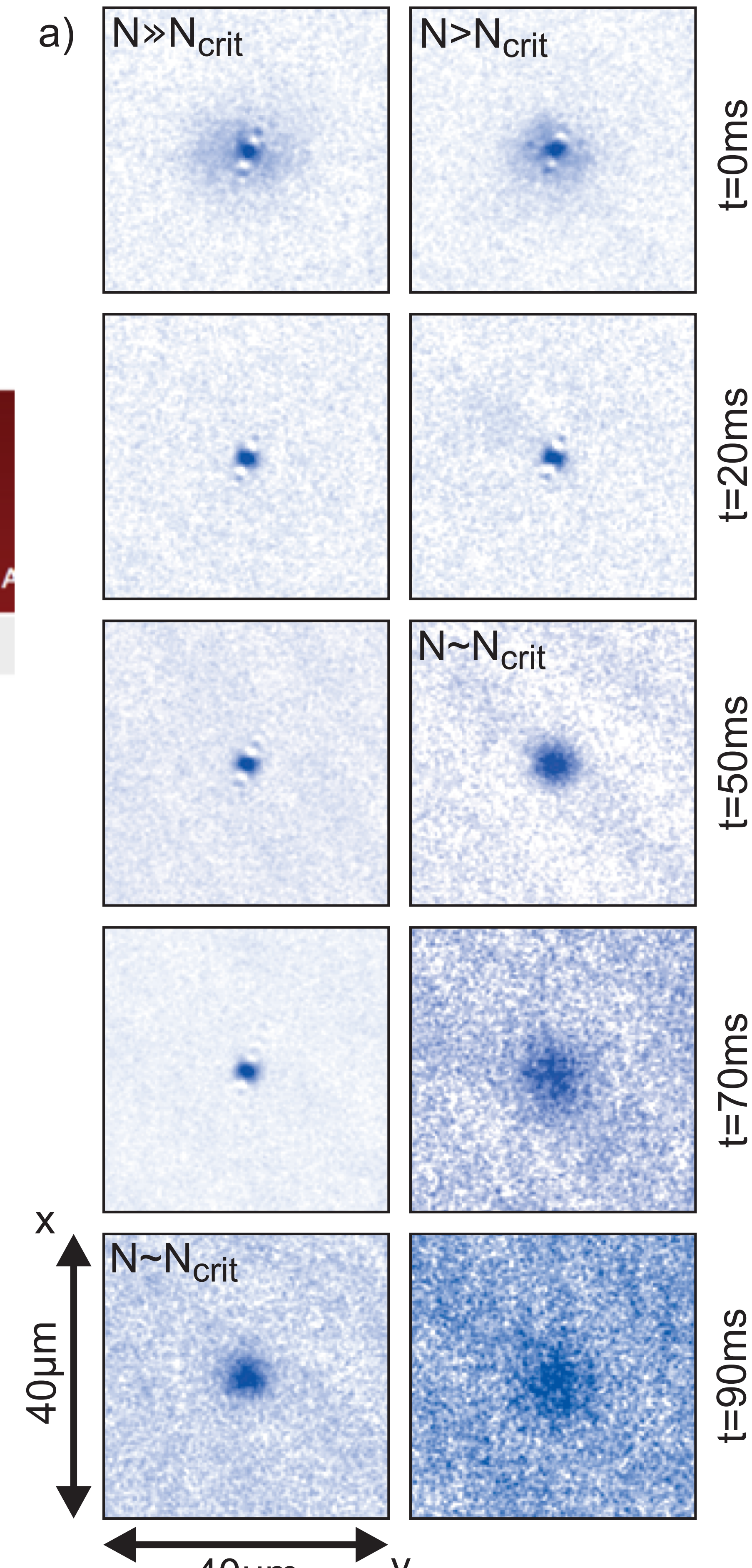
Self-bound droplets of a dilute magnetic quantum liquid

Matthias Schmitt, Matthias Wenzel, Fabian Böttcher, Igor Ferrier-Barbut & Tilman Pfau

Affiliations | Contributions | Corresponding authors

Nature **539**, 259–262 (10 November 2016) | doi:10.1038/nature20126

Received 25 July 2016 | Accepted 29 September 2016 | Published online 09 November 2016



Excitations

Mostly for self-bound droplets

Initial work: shape oscillations

PHYSICAL REVIEW X **6,** 041039 (2016)

Quantum-Fluctuation-Driven Crossover from a Dilute Bose-Einstein Condensate to a Macrodroplet in a Dipolar Quantum Fluid

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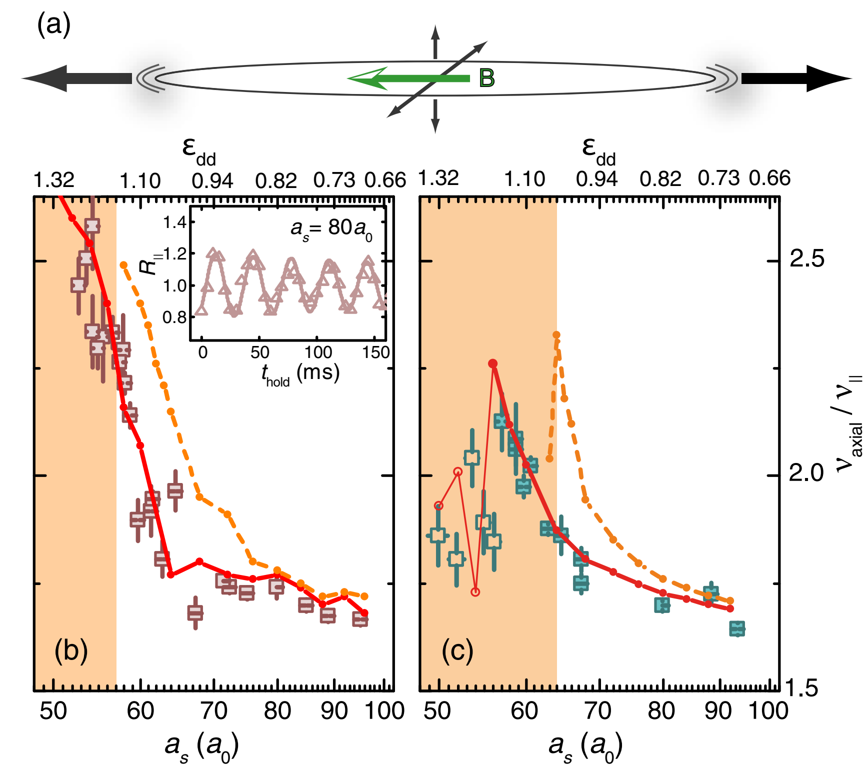
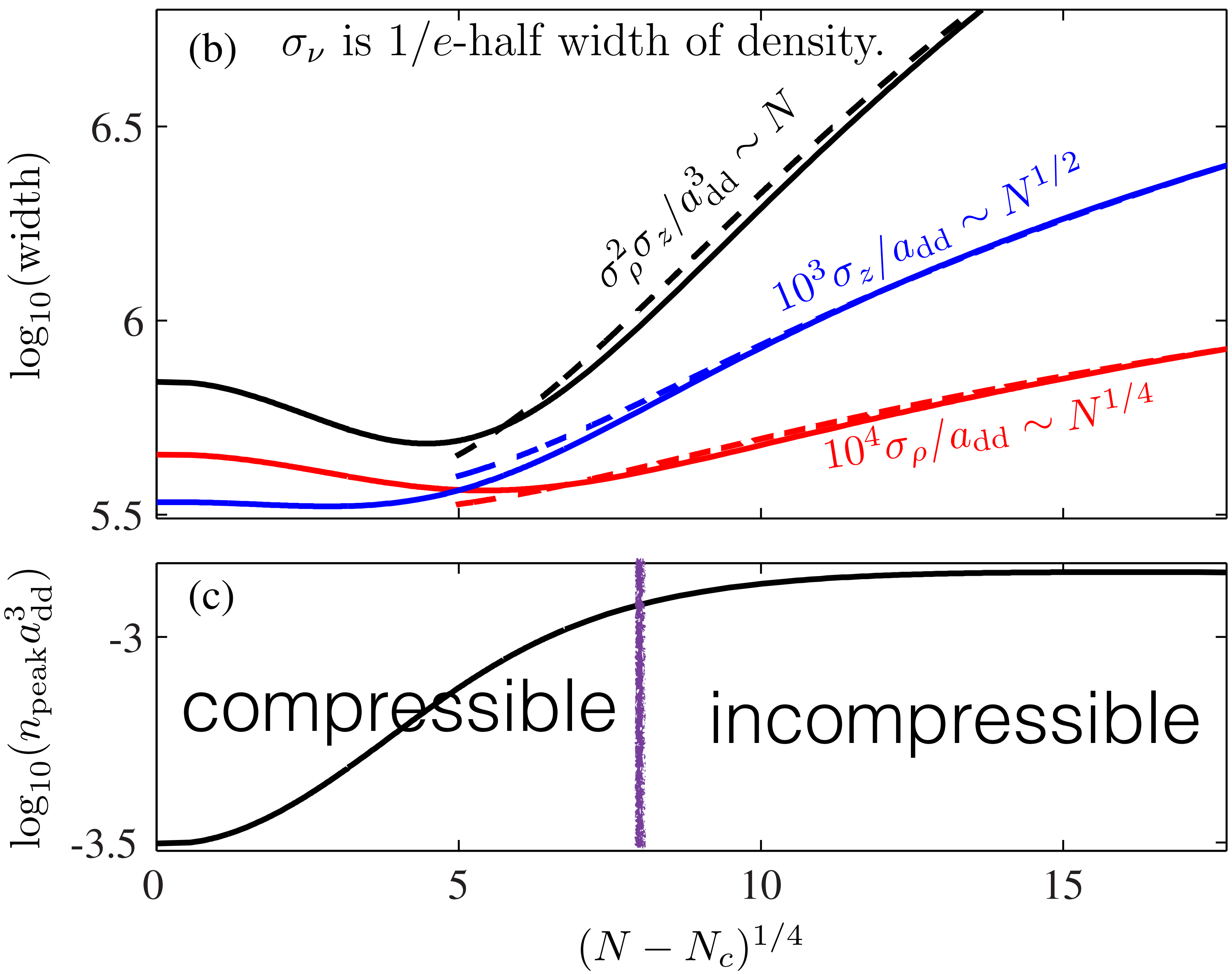
(Received 22 July 2016; revised manuscript received 15 September 2016; published 22 November 2016)

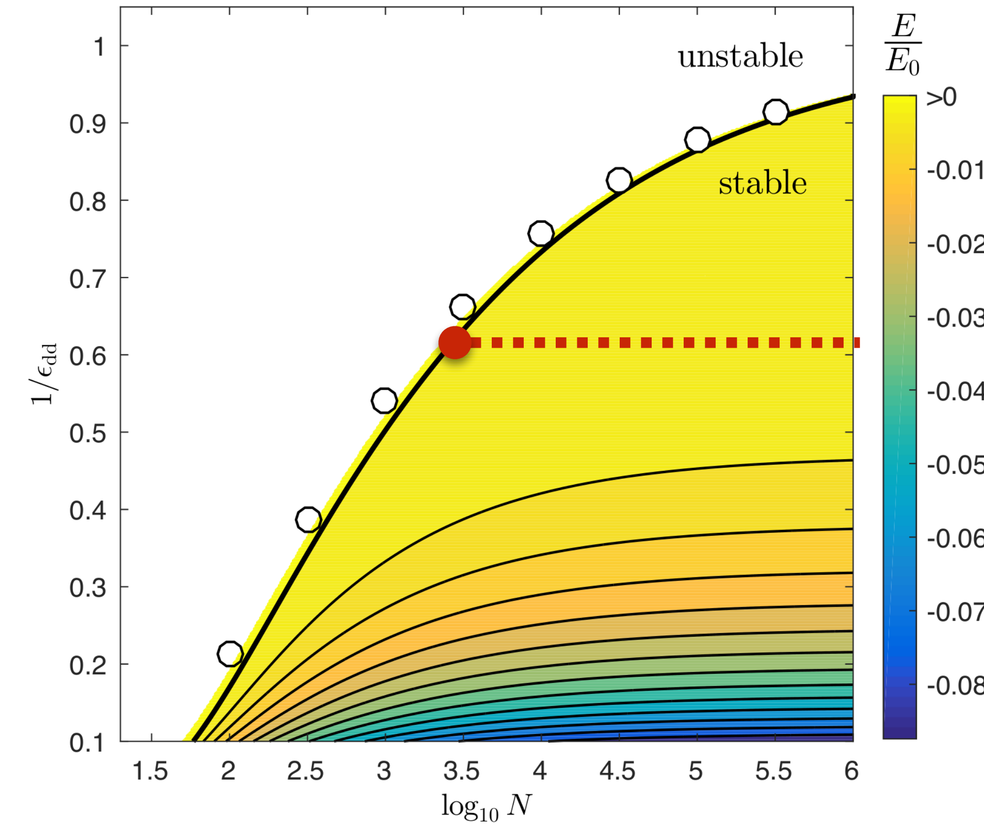


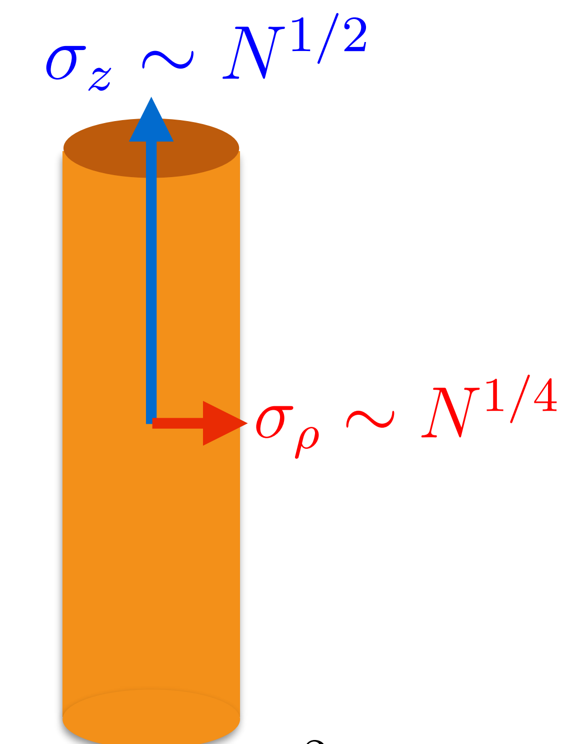
"axial" shape mode

Compressible to Incompressible

^{164}Dy atoms with $a_s = 80\,a_0$ ($\epsilon_{\rm dd} = 1.6$), $N_c = 1899$







effective volume $\sigma_\rho^2\sigma_z \sim N$

Excitations

- Linearize GPE $i\hbar\dot{\psi} = \mathcal{L}_{\rm GP}\psi$ about droplet:

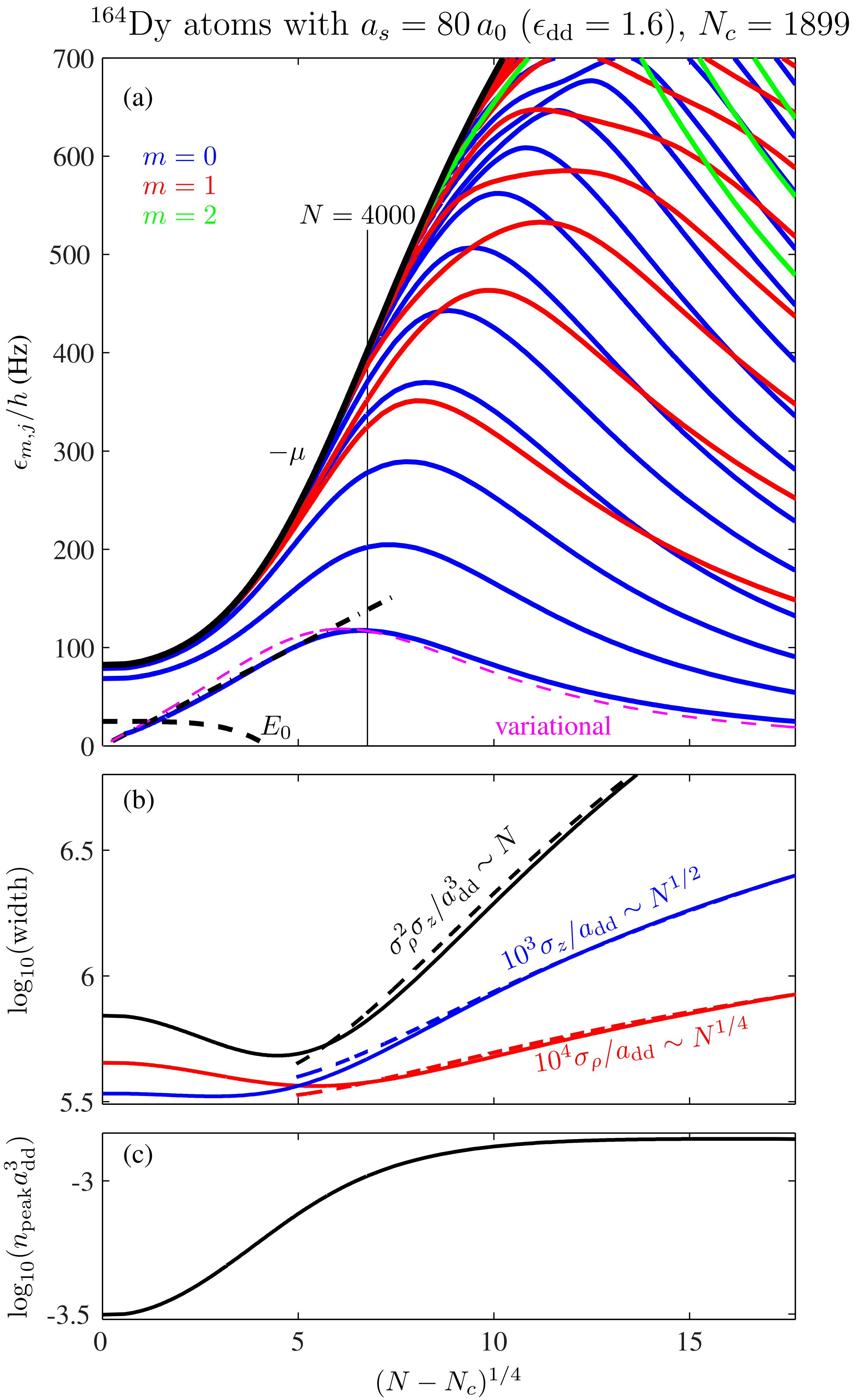
$$\psi = e^{-i\mu t/\hbar}\left[\psi_0 + \sum_\nu \left(\lambda_\nu u_\nu e^{-i\epsilon_\nu t/\hbar} - \lambda_\nu^* v_\nu^* e^{i\epsilon_\nu t/\hbar}\right)\right]$$

- Excitations satisfy Bogoliubov-de Gennes equations

$$\begin{pmatrix} \mathcal{L}_{\rm GP} - \mu + X & -X \\ X & -(\mathcal{L}_{\rm GP} - \mu + X) \end{pmatrix}\begin{pmatrix} u_\nu \\ v_\nu \end{pmatrix} = \epsilon_\nu \begin{pmatrix} u_\nu \\ v_\nu \end{pmatrix}$$

Exchange operator:

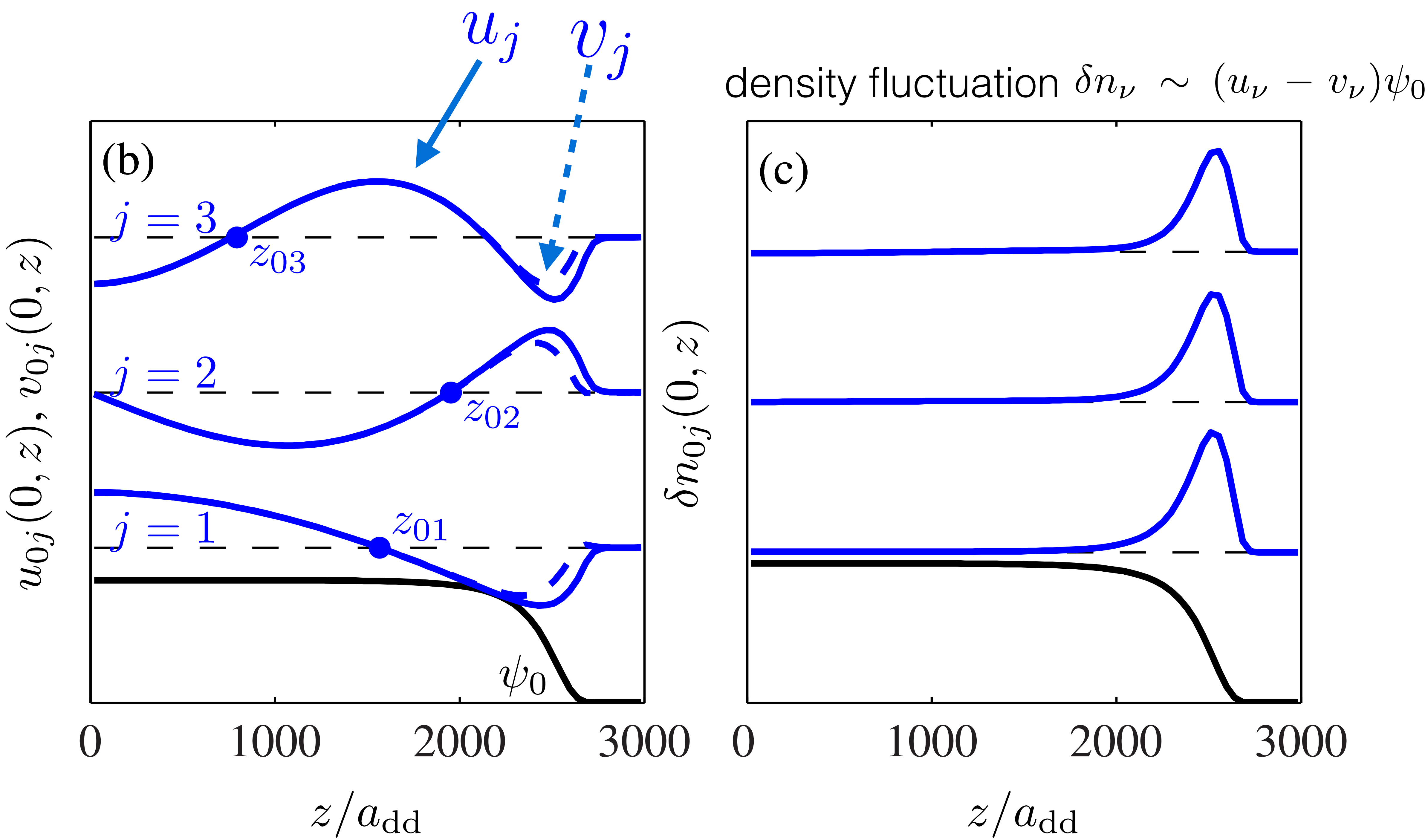
$$Xf \equiv \psi_0 \int d\mathbf{x}' U(\mathbf{x}-\mathbf{x}') f(\mathbf{x}')\psi_0^*(\mathbf{x}') + \tfrac{3}{2}\gamma_{\rm QF}|\psi_0|^3 f.$$



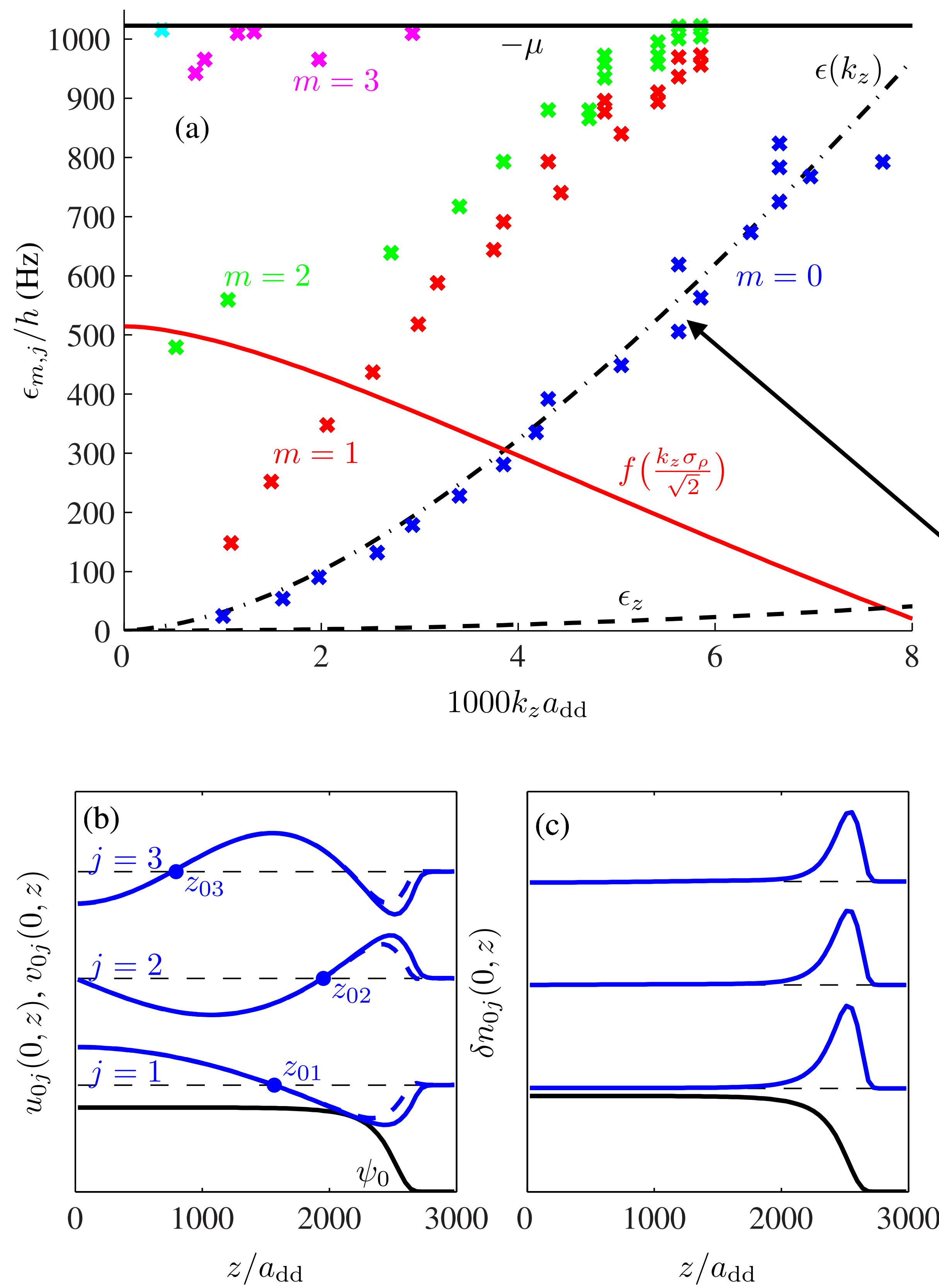
Excitations

- Excitations bound by condensate $-\mu$ i.e. $\epsilon_j - \mu < 0$
- Condensate energy can be positive (metastable)
- Lowest mode monopole-like near N_c
- Quadrupole-like for $N \gtrsim 4 \times 10^3$
- **Ladder** of excitations for each angular momentum

Density and excitation profiles



Droplet as a phonon waveguide



Dispersion mapping

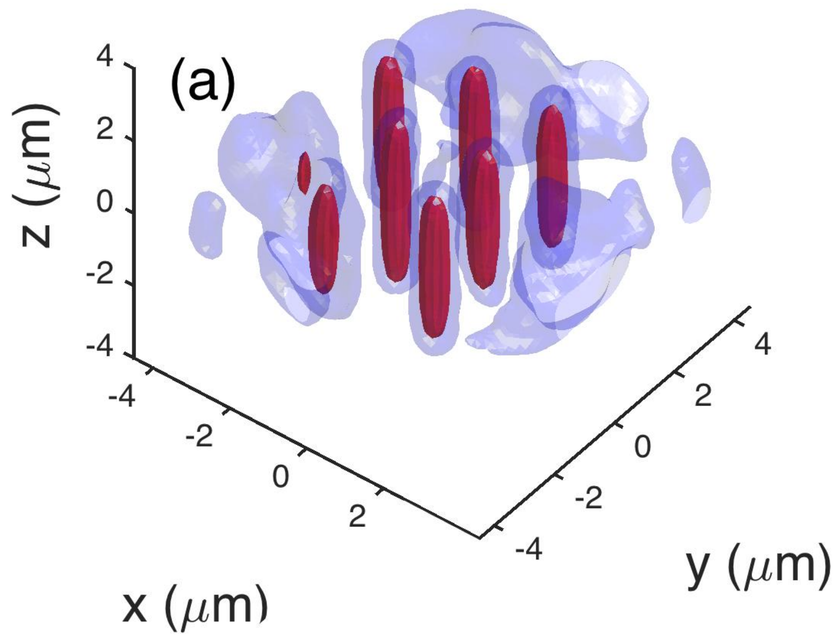
$$k_z = \pi/2z_\nu$$

Analytic result

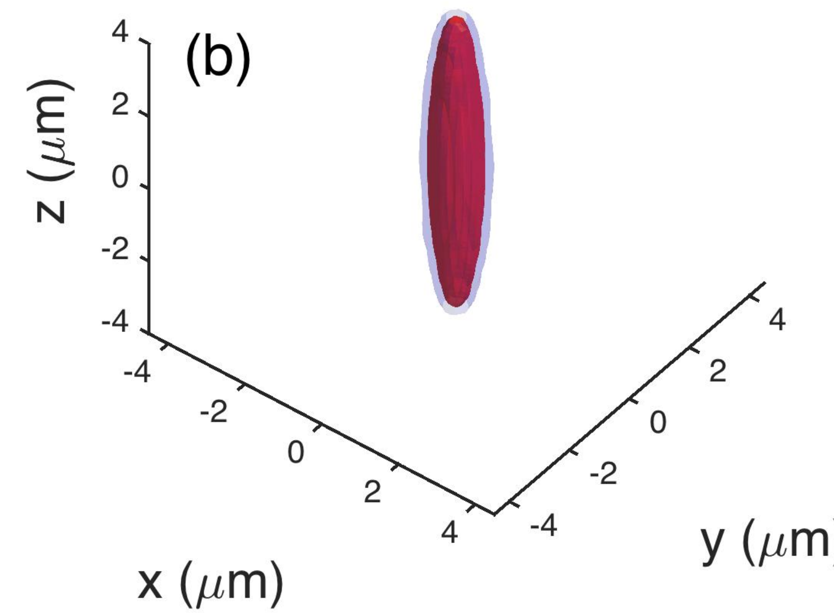
$$\epsilon(k_z) = \sqrt{\epsilon_z^2 + 2\epsilon_z n_{\text{peak}}\left[\frac{g_s}{2} - \frac{g_{\text{dd}}}{2} f\left(\frac{k_z\sigma_\rho}{\sqrt{2}}\right) + \frac{3}{5}\gamma_{\text{QF}} n_{\text{peak}}^{1/2}\right]},$$

$$\epsilon_z = \hbar^2 k_z^2/2M \qquad f(q) = 1 + 3q^2 e^{q^2}\,\text{Ei}(-q^2)$$

Assuming Gaussian radial profile of the condensate and excitations with width σ_ρ



Summary



- The LHY corrections stabilise a new *droplet* phase in dipolar condensates
- Stable self-bound droplets in absence of trapping potentials
- Liquid-like incompressible behaviour and droplets act as a waveguide for the phonon excitations