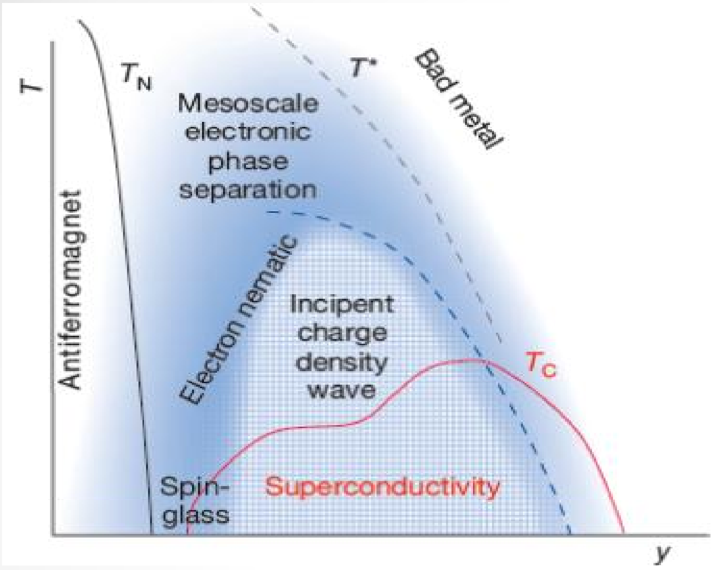
Superconductivity in the doped t-J and Hubbard model on the square lattice

Hong-Chen Jiang

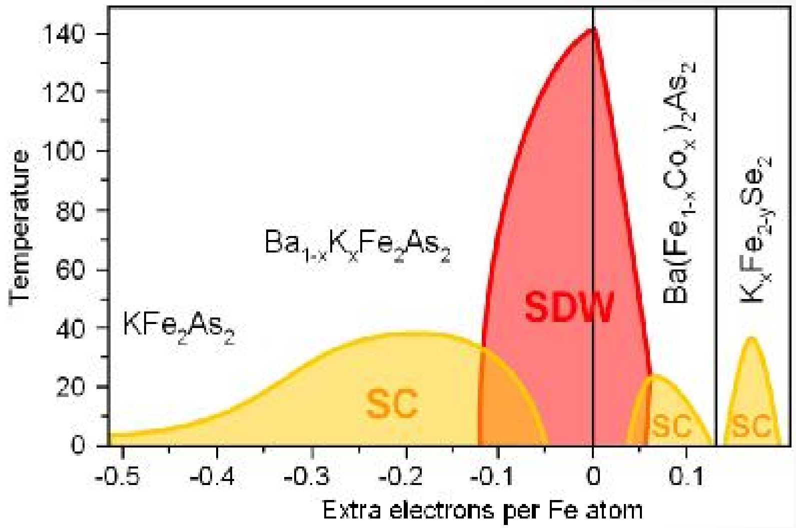
Hong-Chen Jiang, Thomas Devereaux, arXiv:1806.01465
Hong-Chen Jiang, Zheng-Yu Weng, and Steven A. Kivelson, arXiv: 1805.11163

Phase diagrams of Strongly correlated electron systems are complex

Cuprate Superconductor



Iron-based Superconductor



Fradkin and Kivelson, Nat. Phys. 2012; Lee, RMP 2006; etc

- ***Spin-density-wave (SDW) order***
- ***Superconductivity (SC)***
- ***Charge-density-wave (CDW) order***

- ***Cuprate (YBCO, LSCO, LBCO etc)***
- ***Iron-based Superconductor***
- ***Heavy fermion***
- ***Organic superconductors etc***

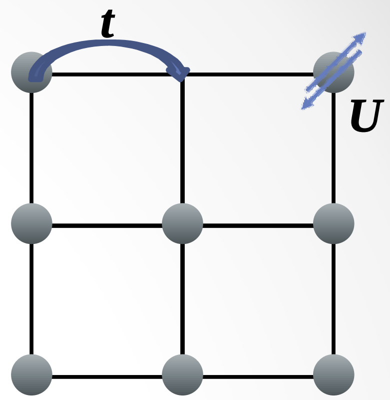
Question: There are hundreds of high-temperature superconducting compounds till now , the underlying microscopic mechanism of superconductivity is still much debated. General theoretically controlled methods seems still lacking for this class of problem.

Anderson, Science 1987; Kivelson et al., PRB, 1987; etc

Models for strongly correlated electronic system

Hubbard model

$$H = -\sum_{ij,\sigma} t_{ij}\left(c_{i\sigma}^{+} c_{j\sigma} + h.c.\right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



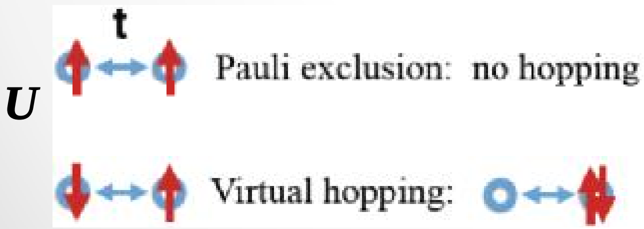
Possibly the most important model in condensed matter physics—widely regarded as the starting point for understanding the high-Tc superconductors

For the high-Tc cuprates:
U/t ~ 8-12, $0 < \delta < 0.3$
The t and U terms compete.
Bandwidth W = 8t, maximal competition W ~ U, or U=8t

Two limits:
(1) U<<t: Quasiparticles, Fermi surfaces …
(2) U>>t: Exchange, mapping to Heisenberg and t-J models (still hard to solve!)

t-J model

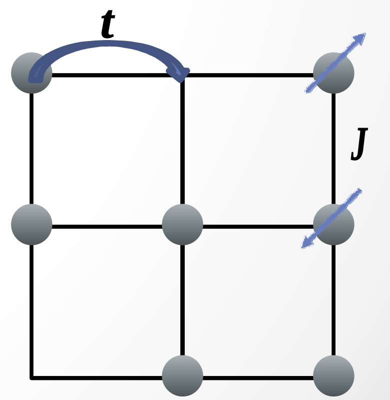
$$H = -\sum_{ij} t_{ij}\left(c_{i\sigma}^{+} c_{j\sigma} + h.c.\right) + \sum_{ij} J_{ij}\left(S_i \cdot S_j - \frac{n_i n_j}{4}\right)$$



spin exchange interaction

$$J_{ij} = 4t_{ij}^{2}/U$$

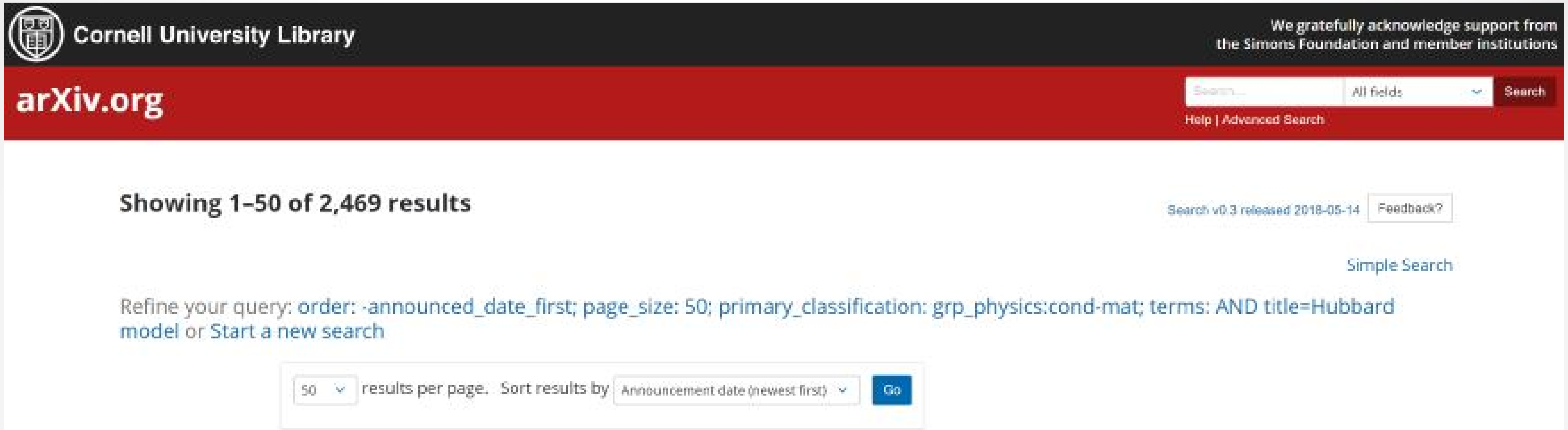
Favor antiferromagnetism



Anderson, Science 1987; Kivelson et al., PRB, 1987; etc

Previous model studies

Hubbard model $H = -\sum_{ij,\sigma} t_{ij}(c_{i\sigma}^{+} c_{j\sigma} + h.c.) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$



t-J model $H = -\sum_{ij} t_{ij}(c_{i\sigma}^{+} c_{j\sigma} + h.c.) + \sum_{ij} J_{ij}(S_i \cdot S_j - \frac{n_i n_j}{4})$

Previous model studies

- ED, QMC, VMC, DMRG, DMFT, DMET, PEPS etc.
- Slave-particle, Mean-field theory, Phase-String theory, Gauge theory, etc.

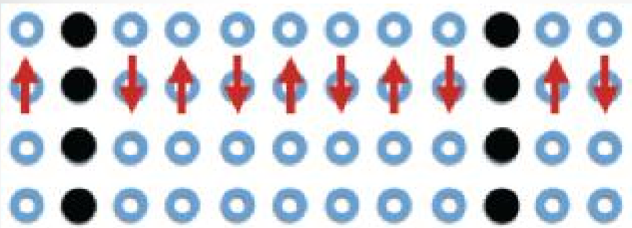
P. W. Anderson, Z. Y. Weng, S. White, D. Scalapino, S. Kivelson, T. Devereaux, T. Xiang, T. Li, M. Troyer, P. A. Lee, X. G. Wen, T. Senthil, A. Mills, E. Fradkin, J. Tranquata, R. M. Noack, P. Corboz and many others experts

Interesting results that have obtained

- ***Striped ground state with unidirectional charge-density-wave (CDW) order.***

  For instance, vertical charge stripe for cylindrical or latter geometry

"Filled" charge stripe

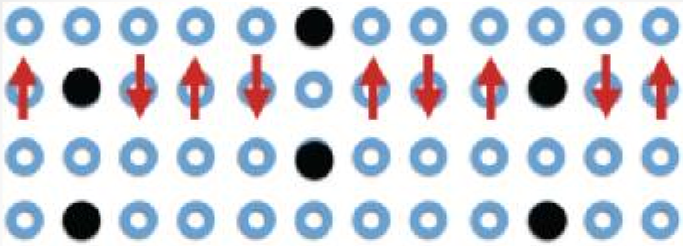


n=1 hole per CDW unit cell

$\lambda = 1/\delta$, e.g., $\lambda = 8$ at $\delta = 1/8$ doping

Hartree-Fock approximation in late 80's (*Jan Zaanen etc. PRB 40, 7391 (1989)*)

"Half-filled" charge stripe



n=1/2 hole per CDW unit cell

$\lambda = 1/2\delta$, e.g., $\lambda = 4$ at $\delta = 1/8$ doping

In some cuprates in the mid 90's (*John Tranquada etc.*), and confirmed by DMRG on the t-J model (*White & Scalapino*)

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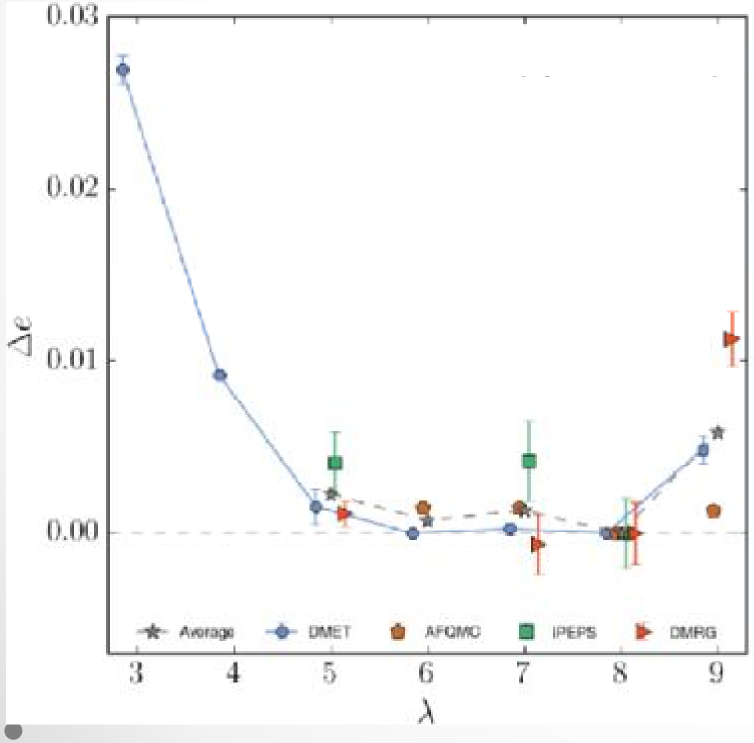
- ***Striped ground state with unidirectional charge-density-wave (CDW) order.***
- ***Many low-lying states with very close energy***



Hubbard model ($\delta = 1/8$, U/t=8)

- Remarkable near-degenerate states with different charge stripe wavelengths, with λ=8 slightly lower in energy, and λ=4 significantly higher.

B. X. Zheng, etc. Science 358, 1155 (2017)

Previous model studies

- ED, QMC, VMC, DMRG, DMFT, DMET, PEPS etc.
- Slave-particle, Mean-field theory, Phase-string, Gauge theory, etc.

P. W. Anderson, S. White, D. Scalapino, S. Kivelson, T. Devereaux, Z. Y. Weng, M. Troyer, P. A. Lee, X. G. Wen, T. Senthil, A. Mills, E. Fradkin, J. Tranquata, R. M. Noack, P. Corboz and many others experts

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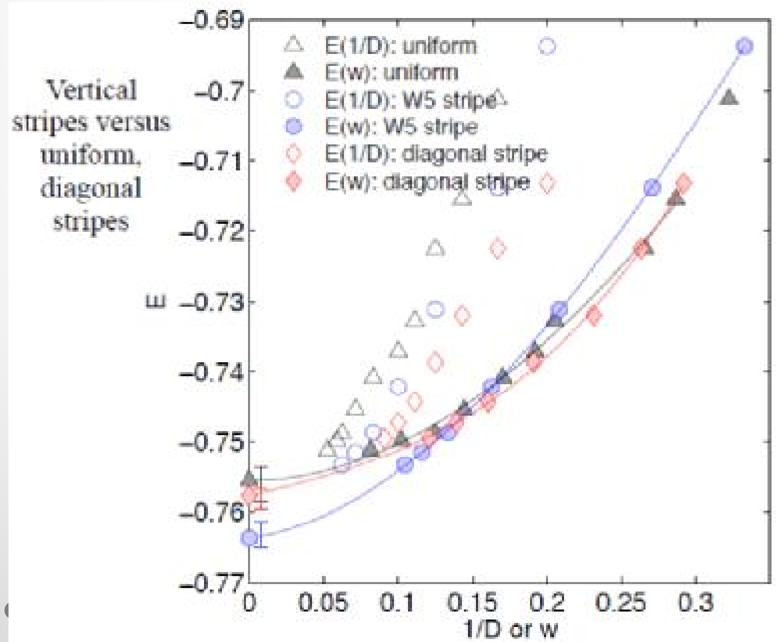
- ***Striped ground state with unidirectional charge-density-wave (CDW) order.***
- ***Many low-lying states with very close energy***
- ***Superconductivity (SC) is likely but no direct evidence for system wider than 2-leg***
- ***The nature of CDW, long-range, short-range or power-law?***



Hubbard model ($\delta = 1/8$, U/t=8)

- **iPEPS** (*P. Corboz etc.*) suggests the state of d-wave superconductivity with vertical stripe is lower in energy, however, conclusion depends on extrapolation procedure.

B. X. Zheng, etc. Science 358, 1155 (2017)

P. Corboz, etc, PRL 113, 046402 (2014)

Previous model studies

- Ø ED, QMC, VMC, DMRG, DMFT, DMET, PEPS etc.
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P. W. Anderson, S. White, D. Scalapino, S. Kivelson, T. Devereaux, Z. Y. Weng, M. Troyer, P. A. Lee, X. G. Wen, T. Senthil, A. Mills, E. Fradkin, J. Tranquata, R. M. Noack, P. Corboz and many others experts

Interesting results that have obtained

- Ø ***Striped ground state with unidirectional charge-density-wave (CDW) order.***
- Ø ***Many low-lying states with very close energy***
- Ø ***Superconductivity (SC) is likely but no direct evidence for system wider than 2-leg***
- Ø ***The nature of CDW, long-range, short-range or power-law?***

Hubbard model ($\delta = 1/8$, U/t=8)

- ➢ DMRG only sees short-range superconductivity
- ➢ Long-range CDW order

B. X. Zheng, etc. Science 358, 1155 (2017)
E. Ehlers, etc, PRB 95, 125125 (2017)

Similar situation for t-J model ***(DMRG)***

- Ø d-wave pairing is seen, and SC is likely, however, no true long-range SC was found (*Steve White, Doug Scalopino etc.*)

Questions that we want to focus on and possibly answer

- Do we have superconductivity in lightly doped t-J model in system wider than 2-leg?
- Do we have superconductivity in lightly doped Hubbard in system wider than 2-leg?
- How about charge-density-wave order?

Outline

- ***DMRG study of lightly doped t-J model on 4-leg cylinder***
- ***DMRG study of lightly doped Hubbard model on 4-leg cylinder***

DMRG study of doped t-J model on 4-leg cylinders

$$H = -t \sum_{\langle ij \rangle \sigma} \left(\hat{c}^{+}_{i\sigma} \hat{c}_{j\sigma} + h.c. \right) + J \sum_{\langle ij \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{\hat{n}_i \hat{n}_j}{4} \right)$$

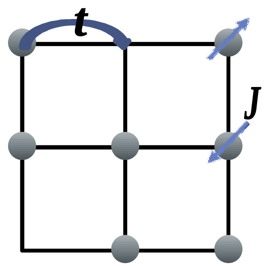
Numerical parameters

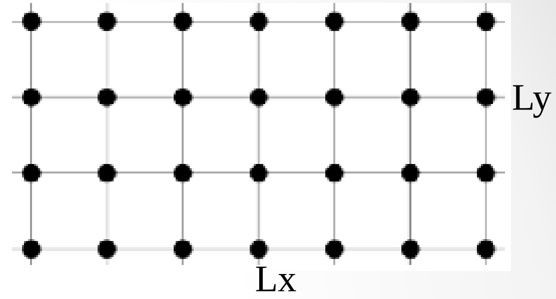
(1) *Ly*=4, *Lx*=16~160

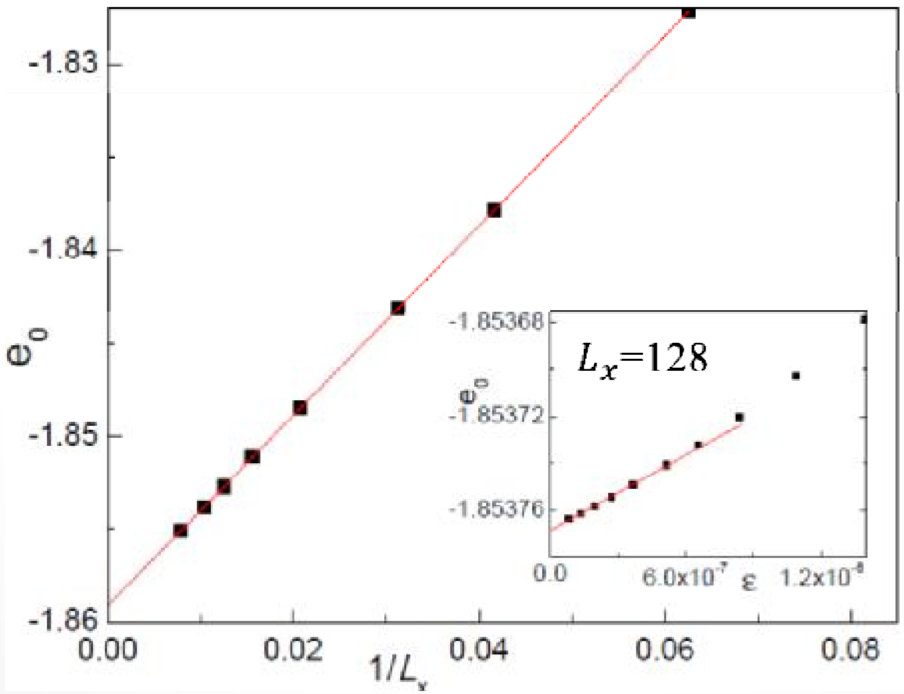
(2) t/J=3

(3) Doping level δ=5.0%~12.5%

(4) Keep m=2187~15000 states with~100 sweeps





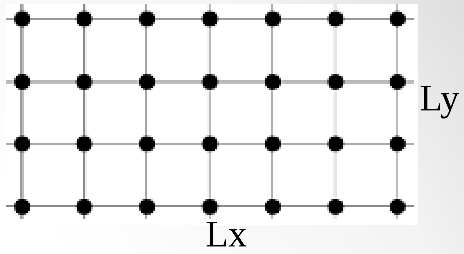


t/J=3, $\delta = 1/8$

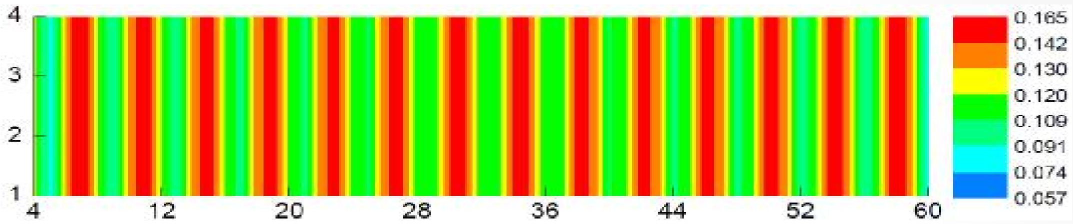
HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

DMRG study of doped t-J model on cylinders

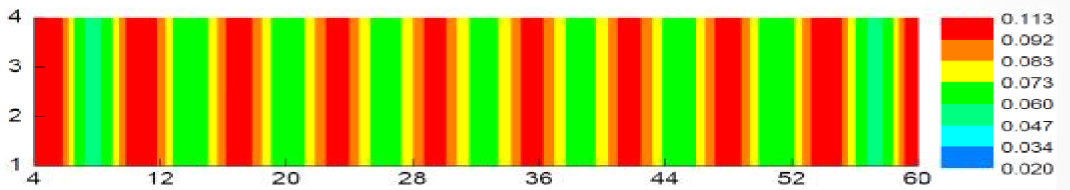
$$H = -t \sum_{\langle ij \rangle \sigma} (\hat{c}_{i\sigma}^{+} \hat{c}_{j\sigma} + h.c.) + J \sum_{\langle ij \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{\hat{n}_i \hat{n}_j}{4} \right)$$



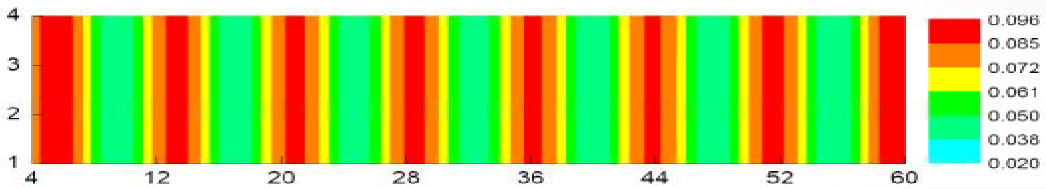
$\delta = 12.5\%$ $\quad n(x,y)$



$\delta = 7.81\%$



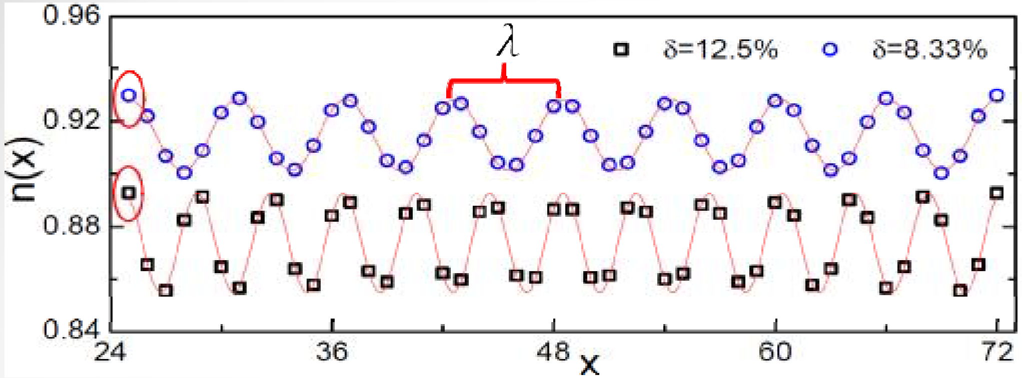
$\delta = 6.25\%$



- ✓ Vertical charge stripe of wavelength $\lambda = 1/2\delta$, consistent with previous studies. For instance, $\lambda = 4$ for $\delta = 1/8$ hole doping concentration
- ✓ Can be understood using Phase-String theory

HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

Charge-density wave



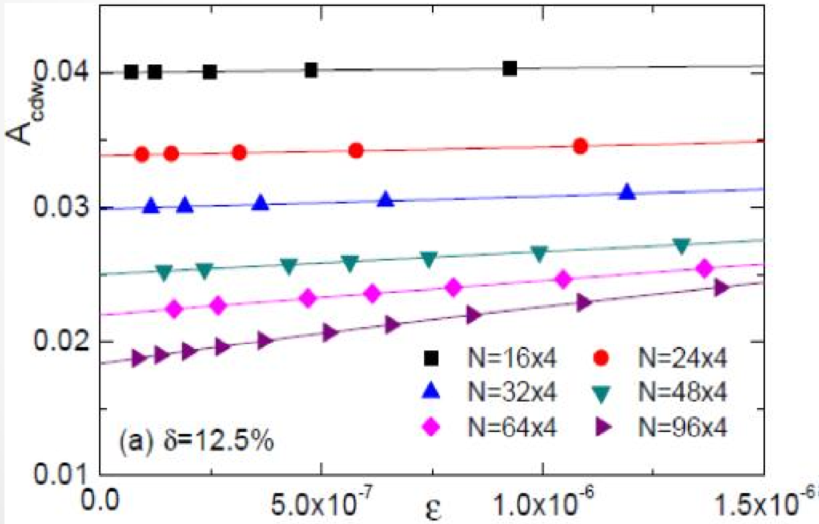
$$n(x) = \frac{1}{L_y}\sum_y n(x,y)$$

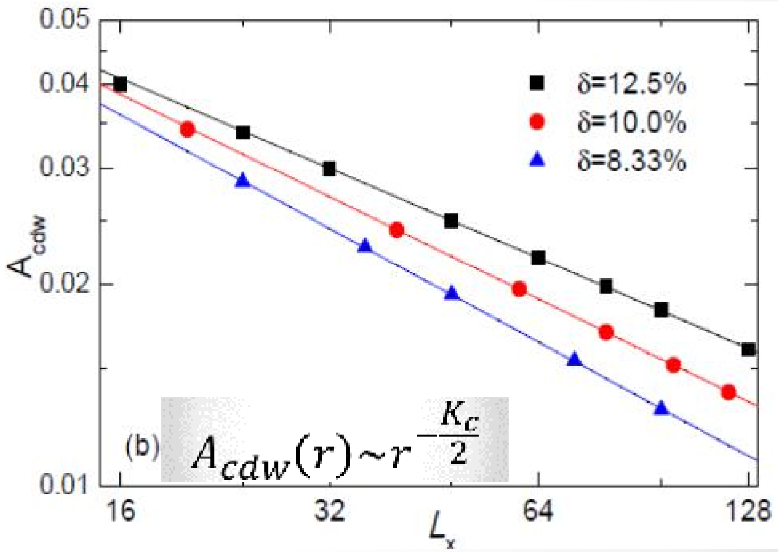
$$n(x) = \delta_0 + A_{cdw}\cos(\frac{2\pi}{\lambda}x + \phi)$$

A_{cdw}: CDW order parameter

Finite-entanglement scaling

Finite-size scaling

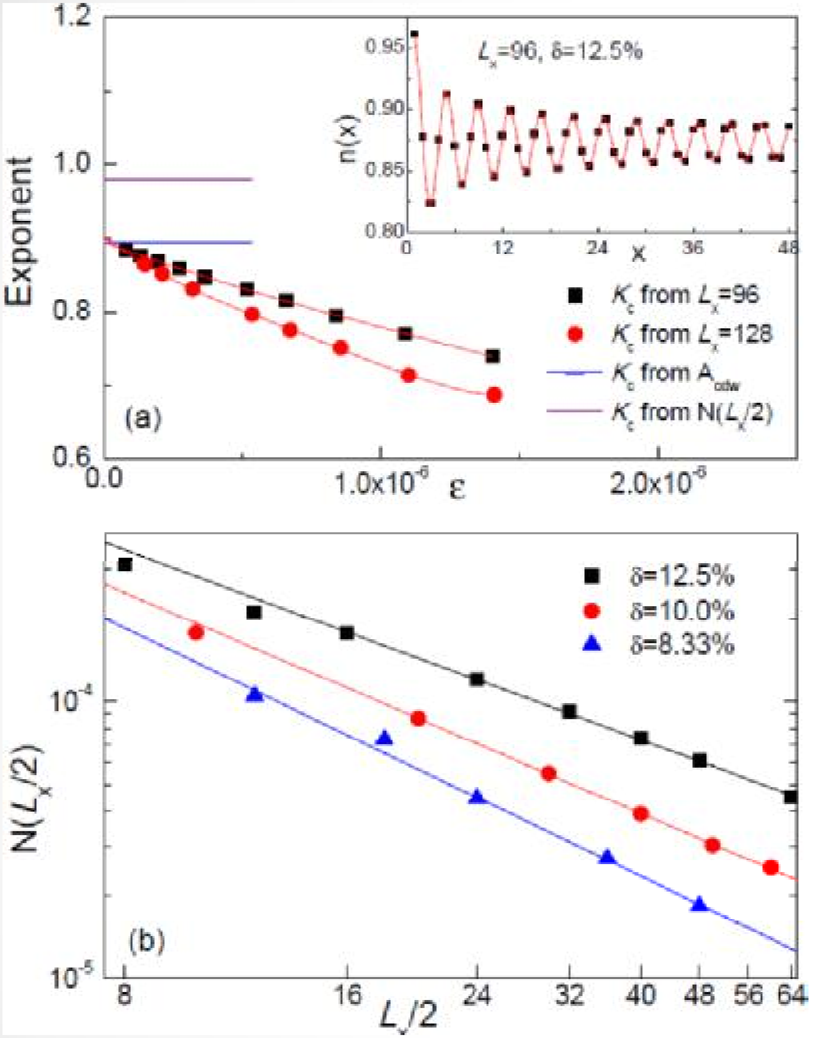




ü ***Quasi-long-range charge-density-wave order, no true-long-range CDW***

HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

Charge-density wave



- Friedel oscillation

$$n(x) = n_0 + \delta n * \cos(2k_F x + \phi) x^{-K_c/2}$$

- Charge density-density correlation

$$N(x) = \langle n(0) - \langle n(0) \rangle \langle n(x) - \langle n(x) \rangle \rangle$$

Here, we set $x = \frac{L_x}{2}$

ü *Quasi-long-range charge-density-wave order, no true-long-range CDW*

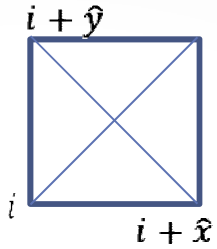
HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

Superconducting correlation

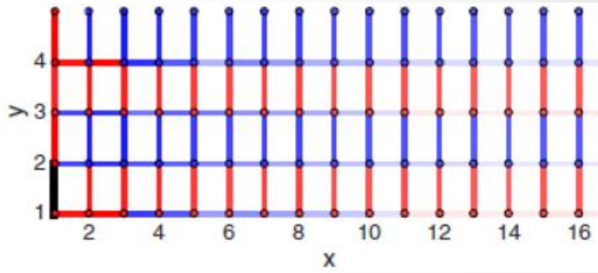
Superconducting correlation $(\alpha = \hat{x}, \hat{y})$

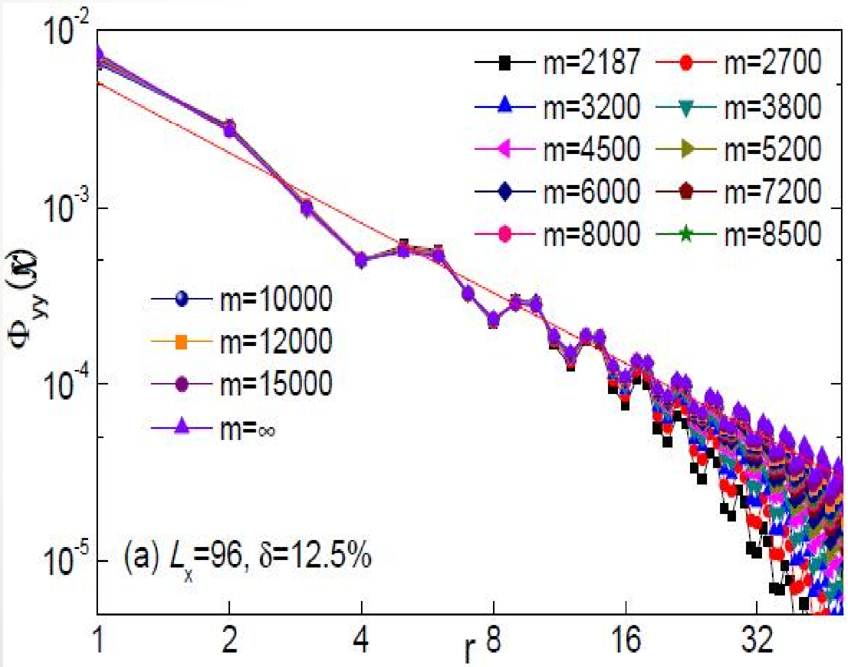
$$\Phi_{\alpha\beta}(x) = \frac{1}{L_y} \sum_{y=1}^{L_y} \langle \Delta_\alpha^\dagger(x_0, y) \Delta_\beta(x_0 + x, y) \rangle$$

$$\Delta_\alpha^\dagger(x,y) = \frac{1}{\sqrt{2}} [c^\dagger_{(x,y),\uparrow} c^\dagger_{(x,y)+\alpha,\downarrow} - c^\dagger_{(x,y),\downarrow} c^\dagger_{(x,y)+\alpha,\uparrow}]$$



D-wave symmetry

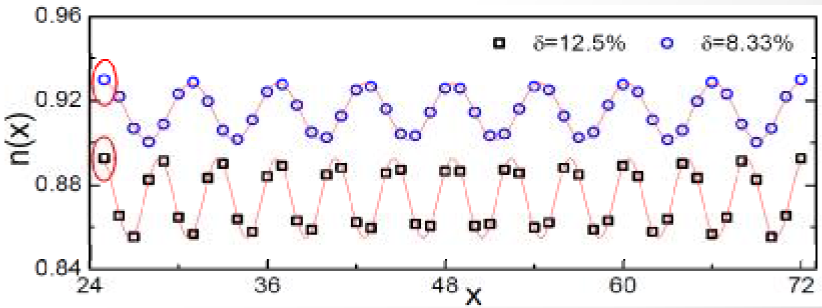




Quasi-long-range superconductivity

$$\Phi_{yy}(x) \sim |x|^{-K_{sc}}$$

Improved procedure to determine the decaying behavior of superconducting Correlation, by using $\Phi_{yy}(L_x/2)$



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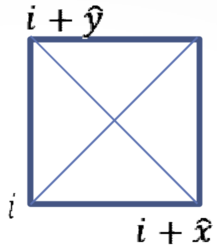
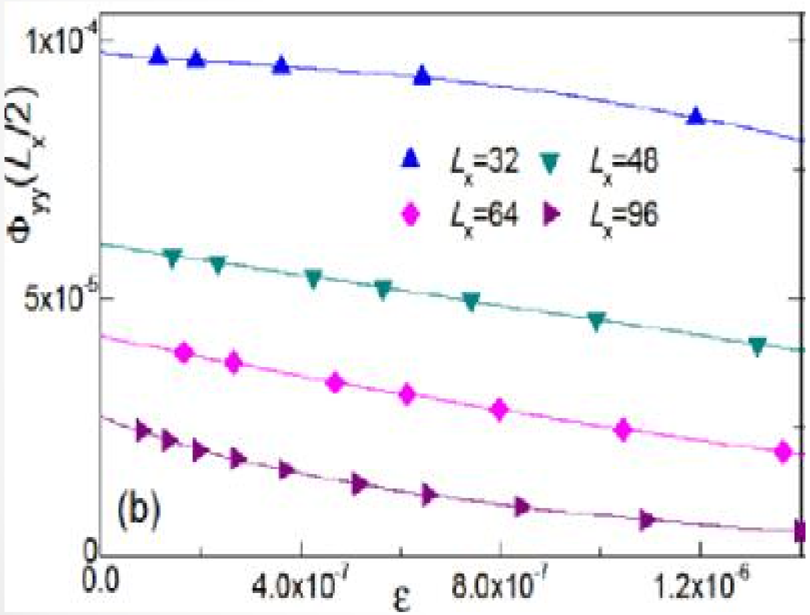
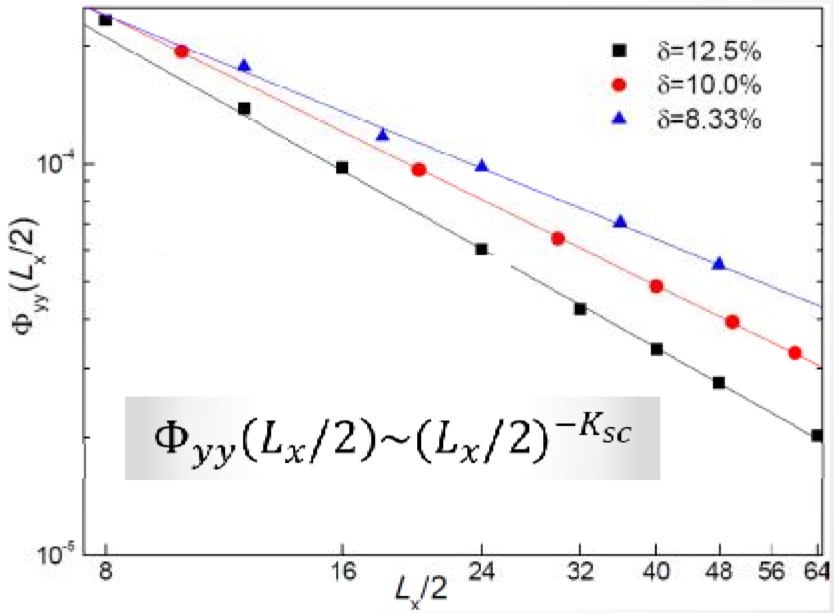


Finite-entanglement scaling



Finite-size scaling



ü ***Quasi-long-range superconducting correlation***

HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

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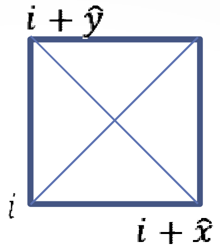
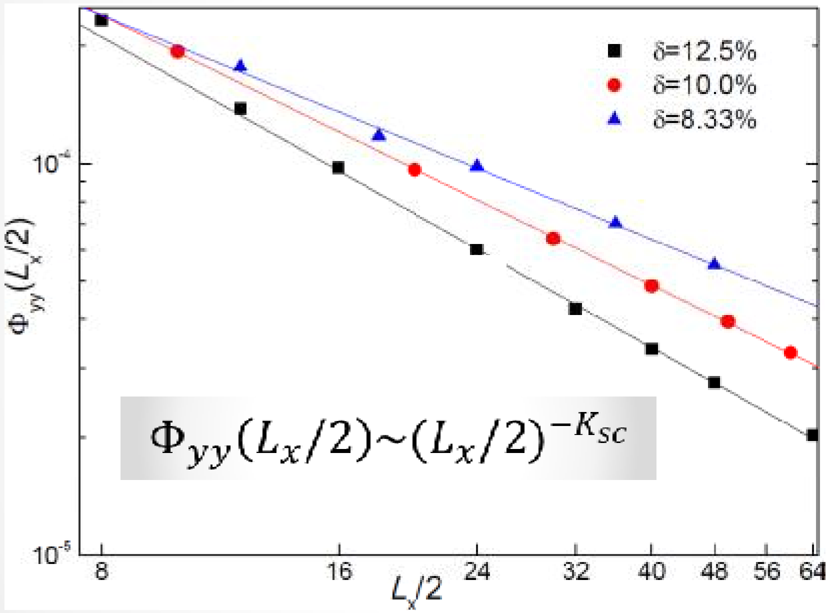
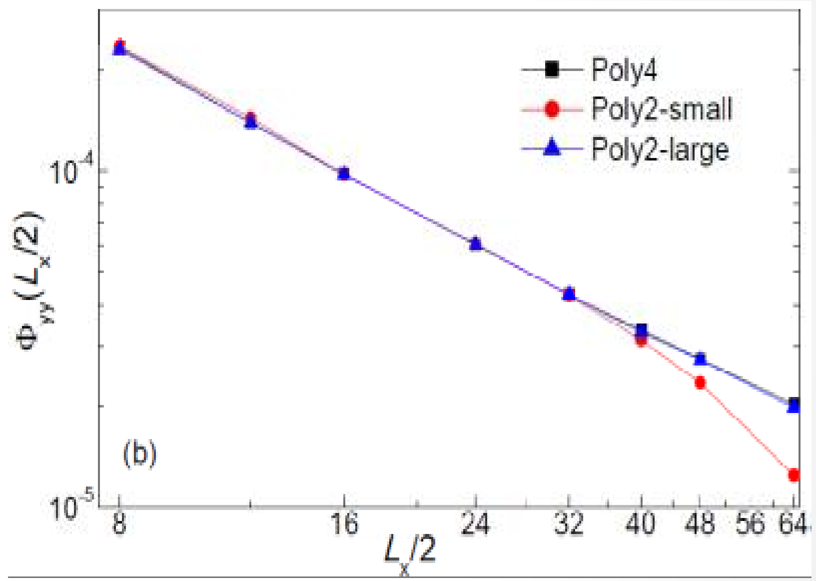


Finite-size scaling



4th order polynomial with m=3200~15000

Comparison between different fittings



Poly4: 4th order polynomial with m=3200~15000

Poly2-small: 2^{nd} order polynomial $m \leq 8000$

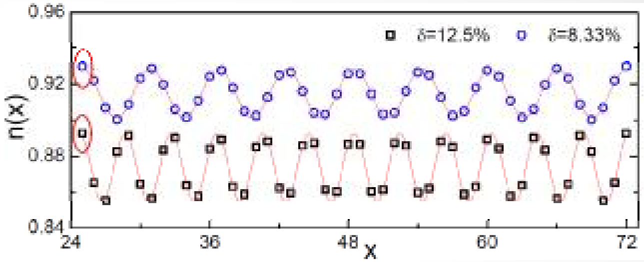
Poly2-large: 2^{nd} order polynomial with 5 largest m

- ***Quasi-long-range superconducting correlation***

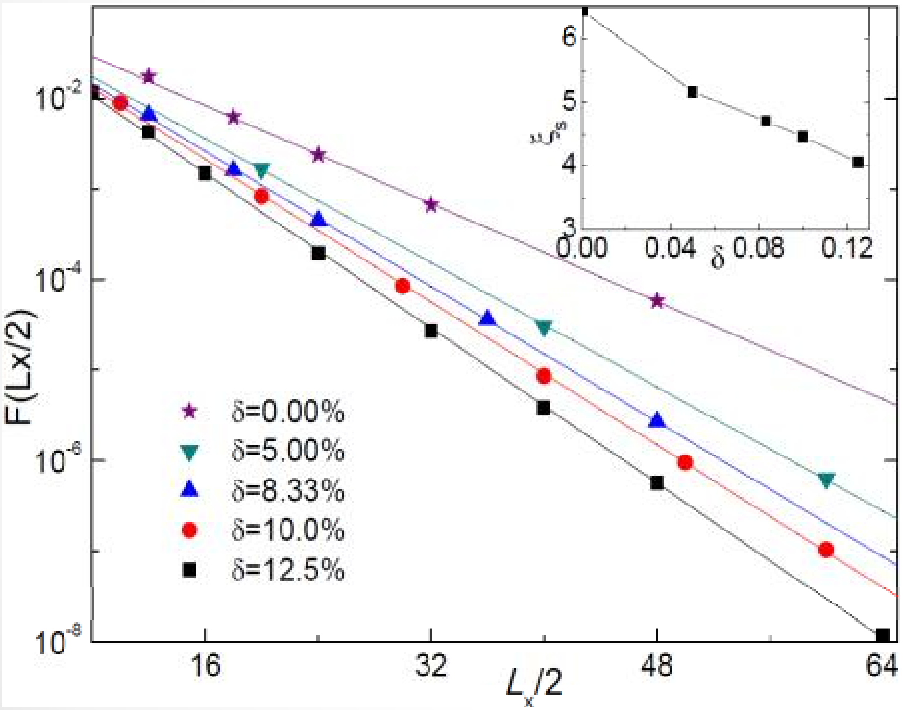
HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

Spin-density-wave correlation

$$F(x) = \frac{1}{L_y}\sum_{y=1}^{L_y} |\langle \vec{S}_{x_0,y} \cdot \vec{S}_{x_0+x,y}\rangle|$$



Finite-size scaling



$$F\left(\frac{L_x}{2}\right) \sim e^{-L_x/2\xi_s}$$

Spin-spin correlation decays exponentially with a correlation length $\xi_s = 4 \sim 5$ lattice spacings

ü ***Short-range spin-spin correlation with a finite spin gap***

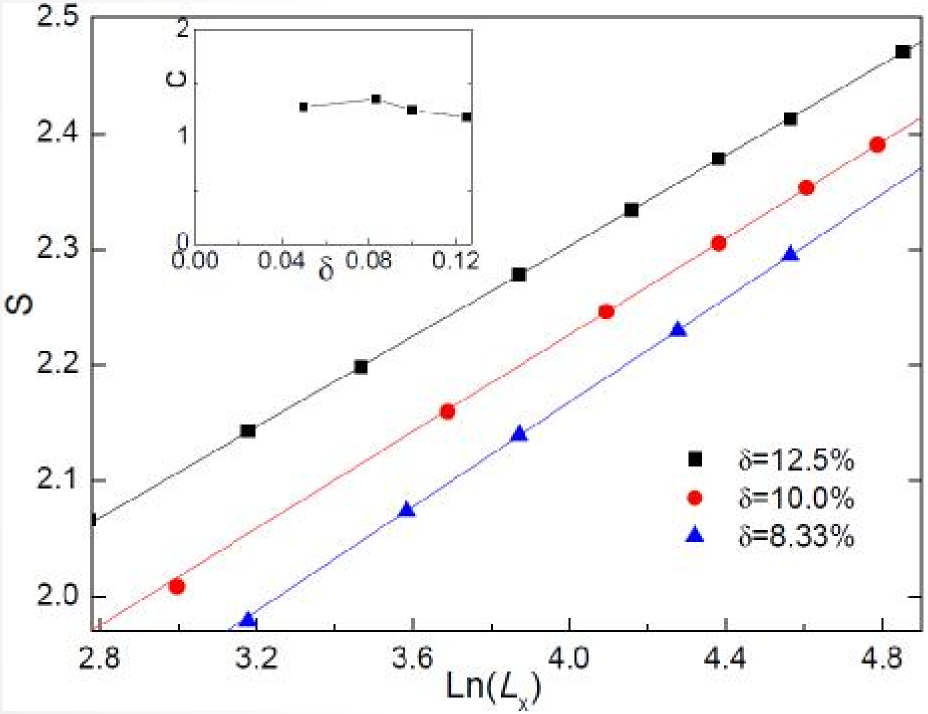
HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

Von Neumann entanglement entropy

$S = -\mathrm{Tr}\rho\ln\rho$

$$S\left(\frac{L_x}{2}\right) = \frac{c}{6}\ln(L_x) + \tilde{c}$$

c is the central charge



One gapless spinless charge mode with central charge c=1

HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

Luther-Emery liquid state

Luther-Emery liquid is a 1D analog of superconductors, which exhibits a spin gap and no charge gap, in which both electron density-density and superconducting pair-pair correlation decay algebraically.

A. Luther and V. J. Emery, PRL 33, 589 (1974)
*L. Balents and M. P. A. Fisher, PRB **53**, 12133 (1996) ……*

$$\langle (n(0).n(L_x/2)\rangle \sim (L_x/2)^{-K}$$

$$A_{cdw}(L_x) \sim L_x^{-K/2}$$

$$\Phi(L_x/2) \sim (L_x/2)^{-1/K}$$

$$K_c = K$$

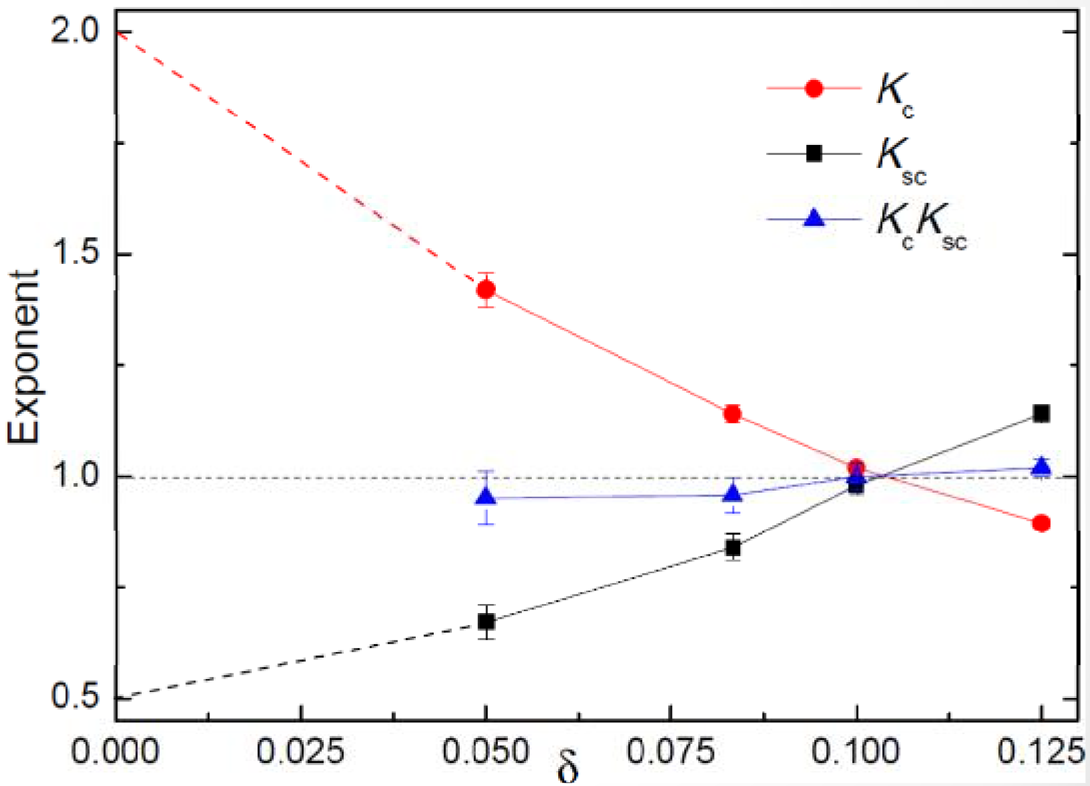
$$K_{sc} = \frac{1}{K}$$

$$K_c \times K_{sc} = 1$$

In the limit $\delta \to 0$

$K = 2, \frac{1}{K} = 0.5,$

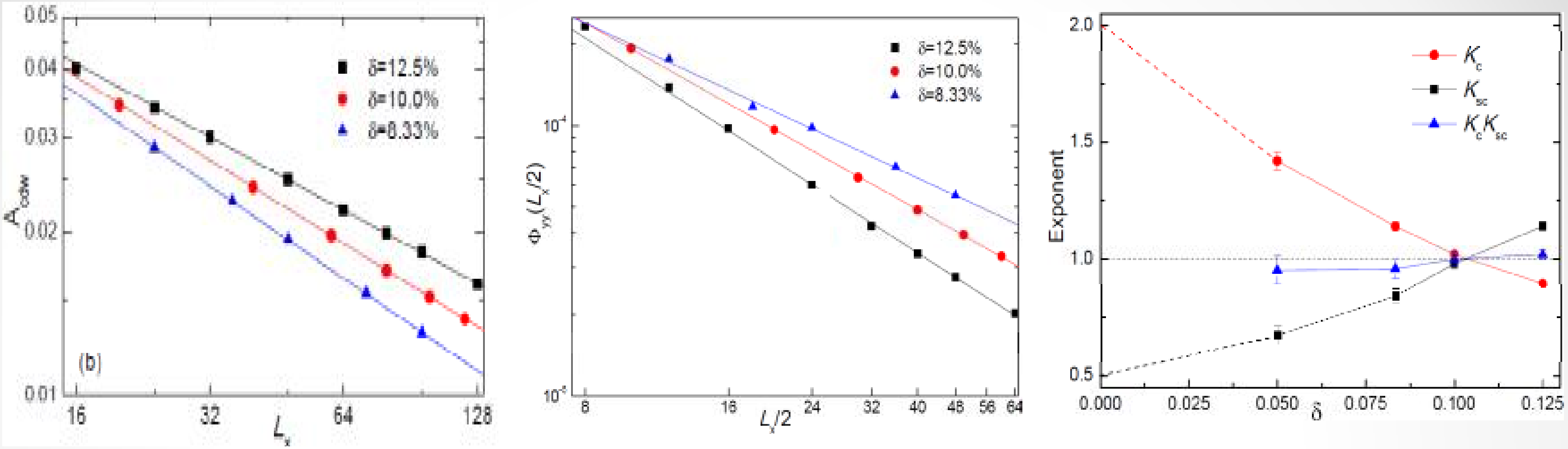
$K_c = 2, K_{sc} = 0.5$



ü ***Ground state of doped t-J model is a Luther-Emery liquid***

HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

Summary for t-J model



- Ø The ground state of lightly-doped t-J model on the 4-leg cylinder is a Luther-Emery liquid, with a finite spin gap but no charge gap.
- Ø Quasi-long-range charge-density-wave order and superconducting order
- Ø Our results suggest that the ground state of t-J model in 2D maybe superconducting

HCJ, Z.Y. Weng, S. Kivelson, arXiv: 1805.11163

Outline

- ***DMRG study of lightly doped t-J model on 4-leg cylinder***

- ***DMRG study of lightly doped Hubbard model on 4-leg cylinder***

DMRG study of doped Hubbard model on 4-leg cylinders

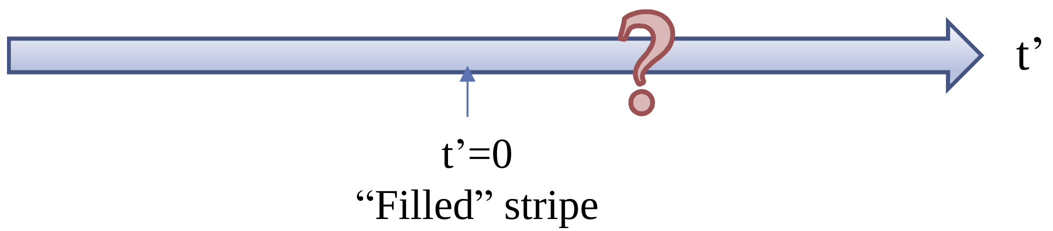
$$H = -\sum_{ij,\sigma} t_{ij}\left(c_{i\sigma}^{+} c_{j\sigma} + h.c.\right) + U\sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Hubbard model at doping $\delta = 1/8$, t=1, U=8-12

Previous study: $\delta = 1/8$, t=1, U=8

- "Filled" stripe
- No superconductivity

⇨ ***Is this generally true ?***



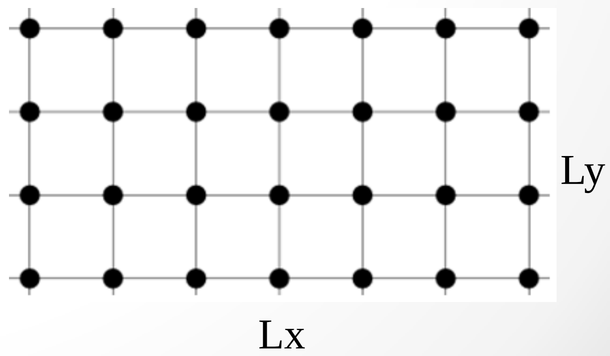
Numerical parameters

(1) *Ly*=4, *Lx*=16~64

(2) t=1, U=8-12, t'=-0.25

(3) Doping level δ=1/8

(4) Keep m=4096~20000 states with~100 sweeps



HCJ, T. P. Devereaux, arXiv:1806.01465

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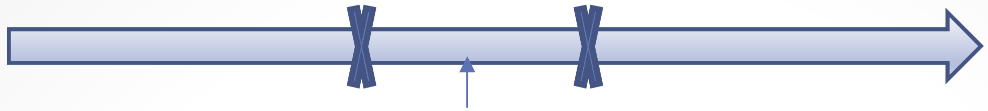


t’

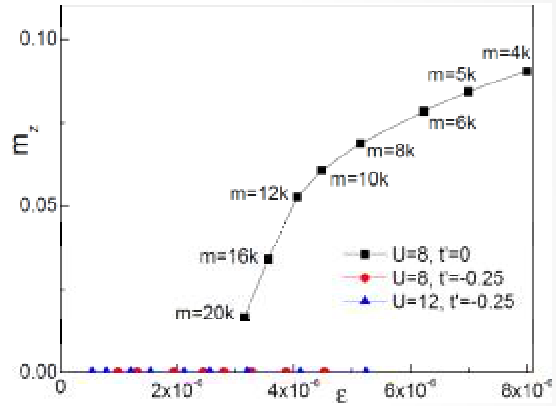
$t' < t'_c < 0$
“Half-filled” stripe

t’=0
“Filled” stripe

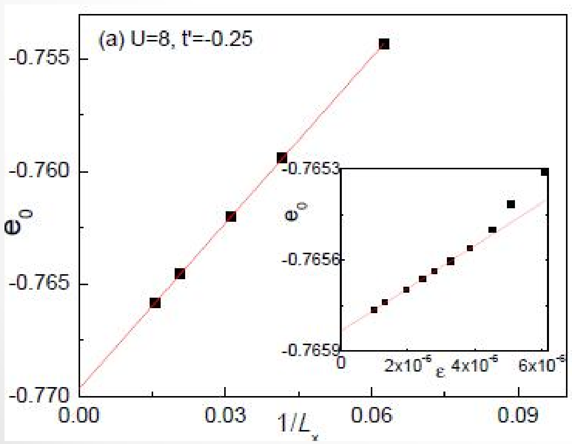
$t' > t'_c > 0$
“Half-filled” stripe

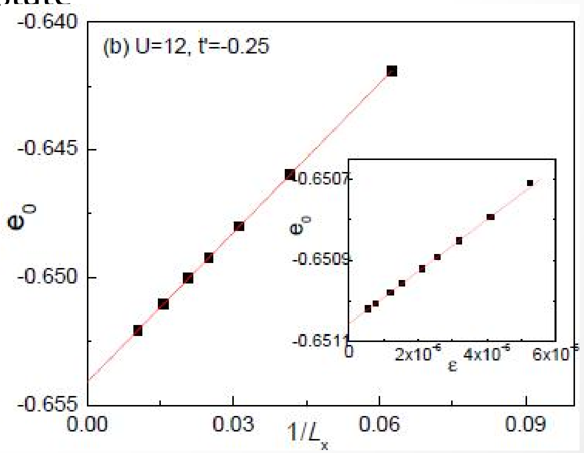
Criteria for convergence

- Ground state should preserve all the symmetries of the Hamiltonian on finite lattice
- Translational symmetry around the cylinder
- Reflection symmetry along the cylinder
- Spin SU(2) symmetry $m_z = \sum_{i=1}^{N} |\langle S_i^z \rangle| / N$



- t’=0, very hard to converge to the true ground state
- t’=-0.25, much easier to converge to the true ground state





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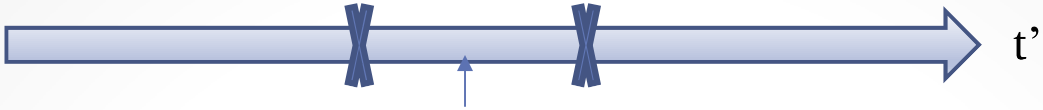
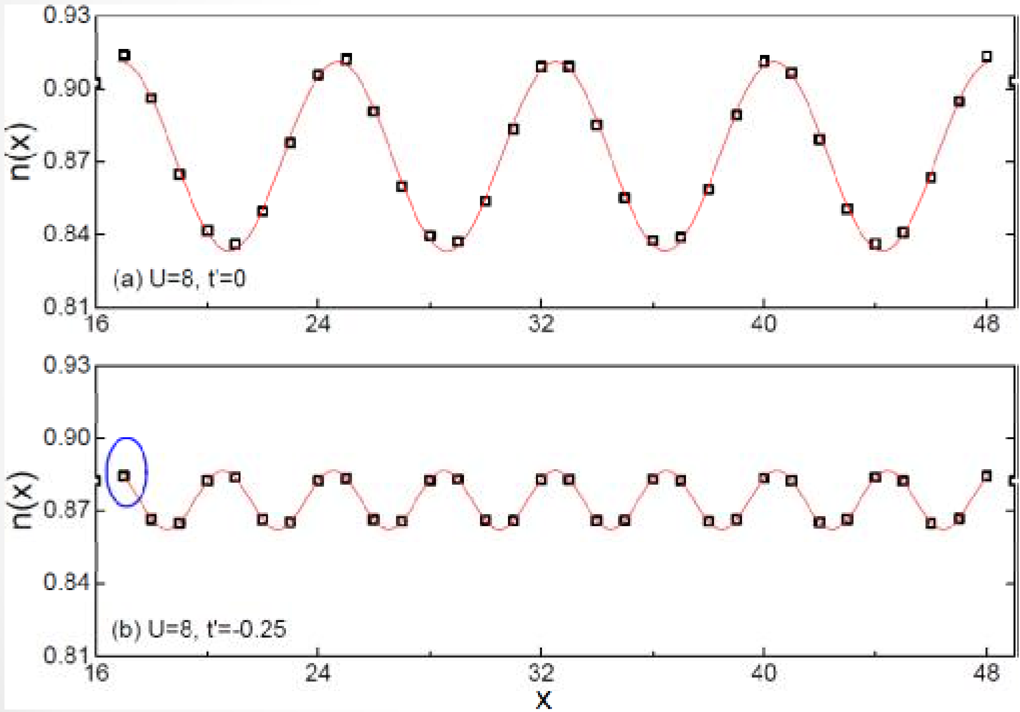
DMRG study of doped Hubbard model on 4-leg cylinders



$t' < t'_c < 0$
"Half-filled" stripe

t'=0
"Filled" stripe

$t' > t'_c > 0$
"Half-filled" stripe



$$n(x) = \frac{1}{L_y}\sum_y n(x,y)$$

$$n(x) = \delta_0 + A_{cdw}\cos\left(\frac{2\pi}{\lambda}x + \phi\right)$$

A_{cdw}: CDW order parameter

Doping concentration $\delta = 1/8$

- t'=0, "Filled" stripe of wavelength $\lambda = 8$, consistent with previous study
- t'=-0.25, "Half-filled" stripe of wavelength $\lambda = 4$

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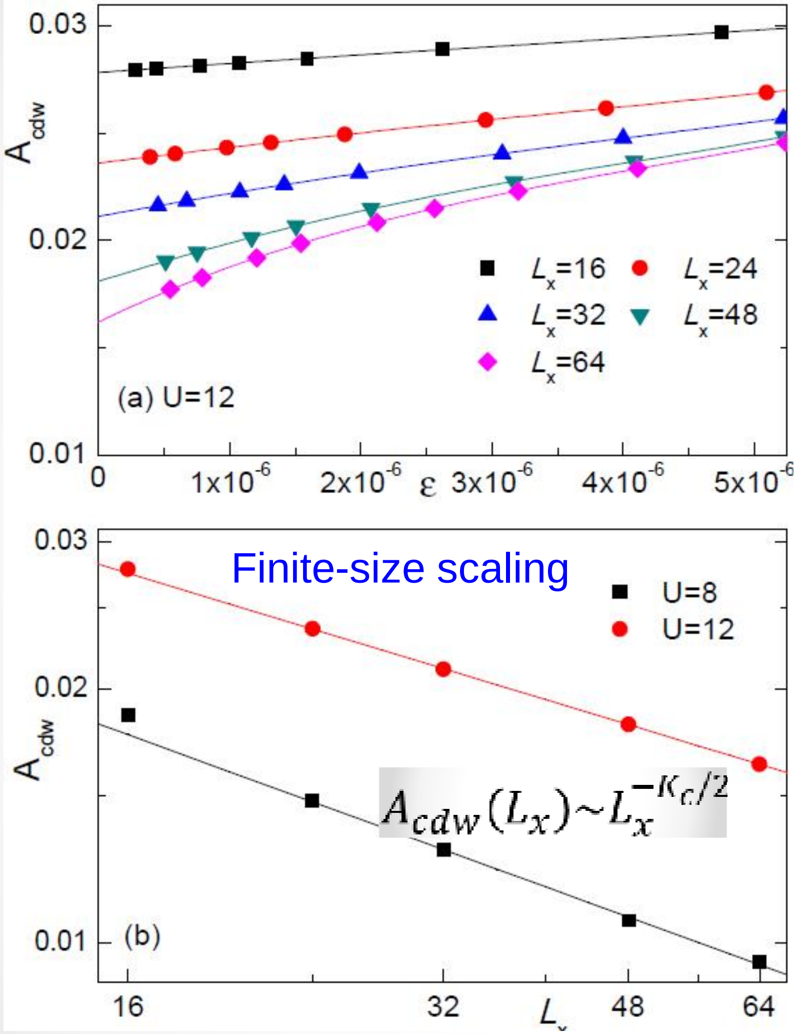
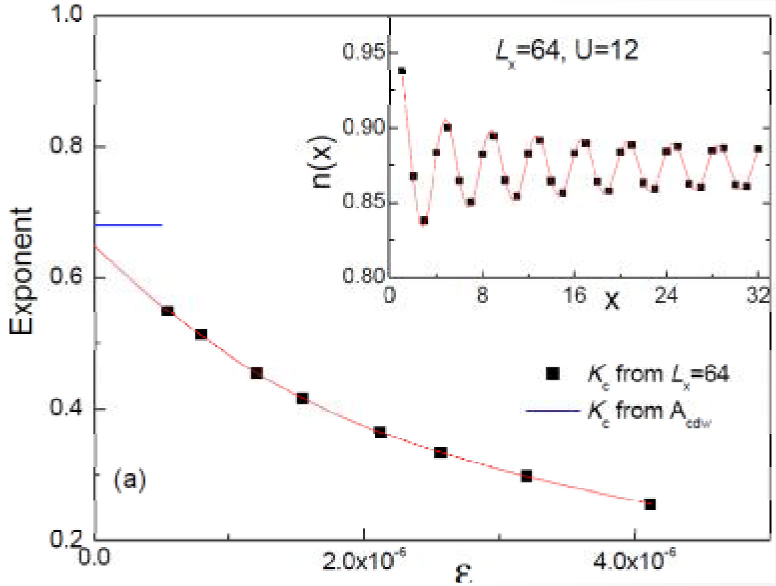
Charge-density wave

Finite-entanglement scaling



➢ Friedel oscillation

$$n(x) = n_0 + \delta n * \cos(2k_F x + \phi) x^{-K_c/2}$$



ü ***Quasi-long-range charge-density-wave order, no true-long-range CDW***

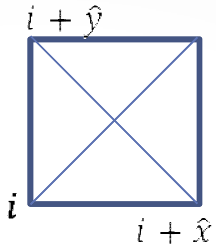
HCJ, T. P. Devereaux, arXiv:1806.01465

Superconducting correlation

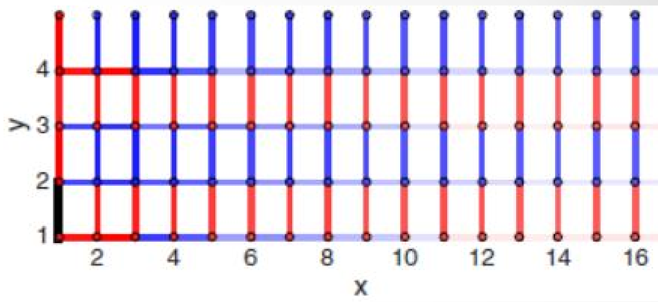
Superconducting correlation ($\alpha = \hat{x}, \hat{y}$)

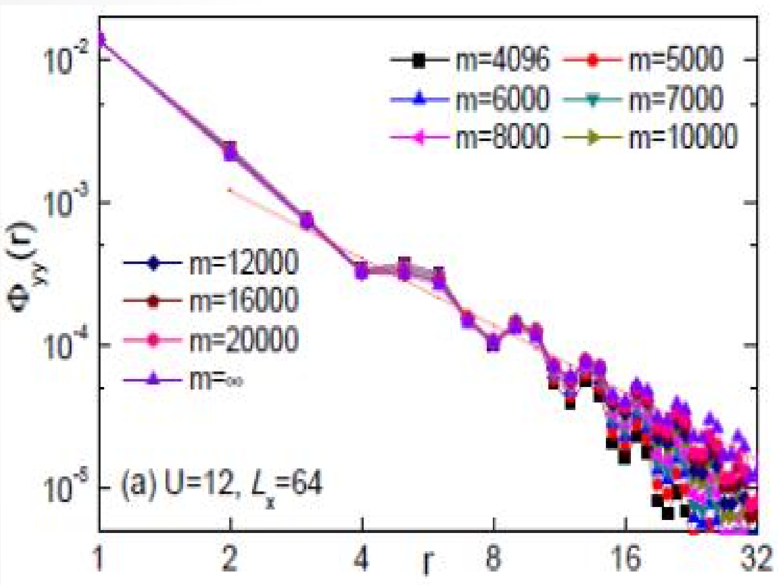
$$\Phi_{\alpha\beta}(x) = \frac{1}{L_y}\sum_{y=1}^{L_y} \langle \Delta_\alpha^\dagger(x_0, y)\Delta_\beta(x_0 + x, y)\rangle$$

$$\Delta_\alpha^\dagger(x,y) = \frac{1}{\sqrt{2}}[c^\dagger_{(x,y),\uparrow} c^\dagger_{(x,y)+\alpha,\downarrow} - c^\dagger_{(x,y),\downarrow} c^\dagger_{(x,y)+\alpha,\uparrow}]$$



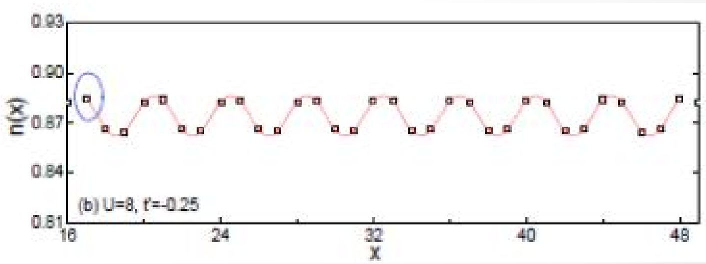
D-wave symmetry





Quasi-long-range superconductivity
$\Phi_{yy}(r) \sim |r|^{-K_{sc}}$

Improved procedure to determine the decaying behavior of superconducting Correlation, by using $\Phi_{yy}(L_x/2)$



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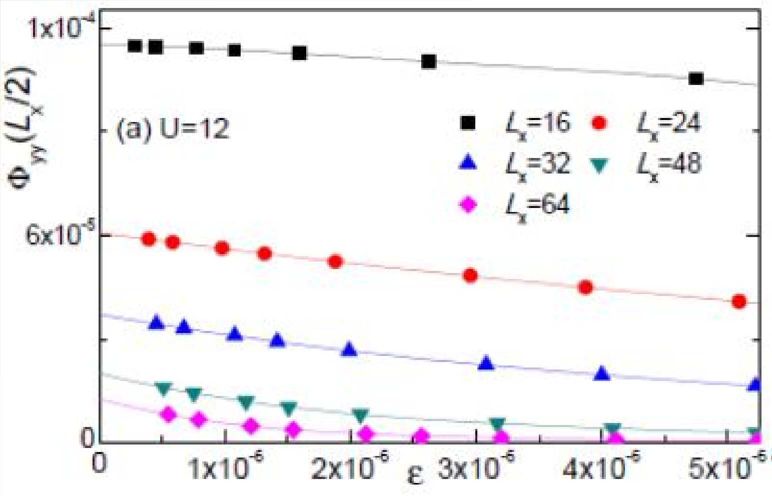
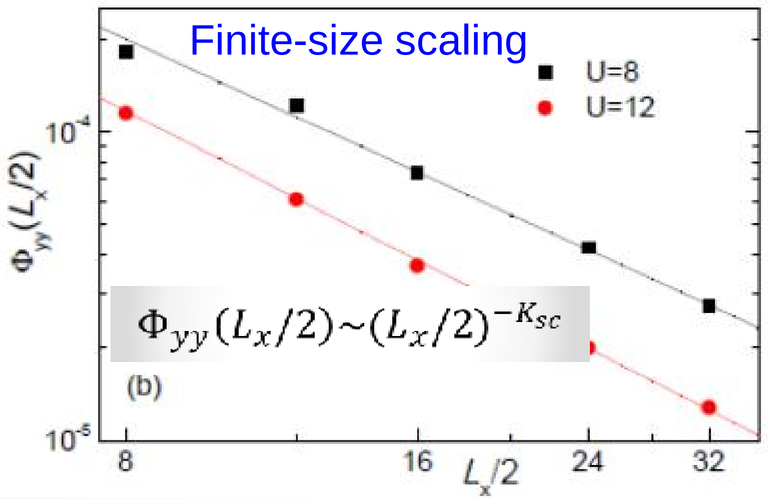
Superconducting correlation

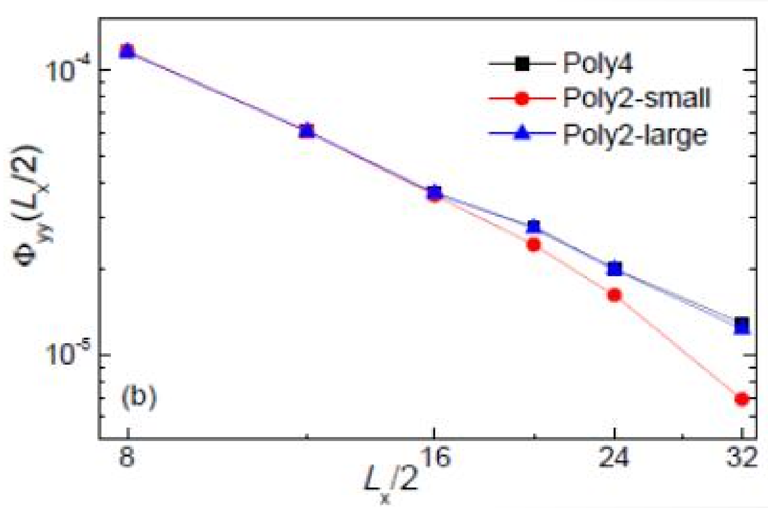
Finite-entanglement scaling





4-th order polynomial fitting

Comparison between different fittings



Poly4: 4th order polynomial with m=6000~20000

Poly2-small: 2nd order polynomial $m \leq 10000$

Poly2-large: 2nd order polynomial with 5 largest m

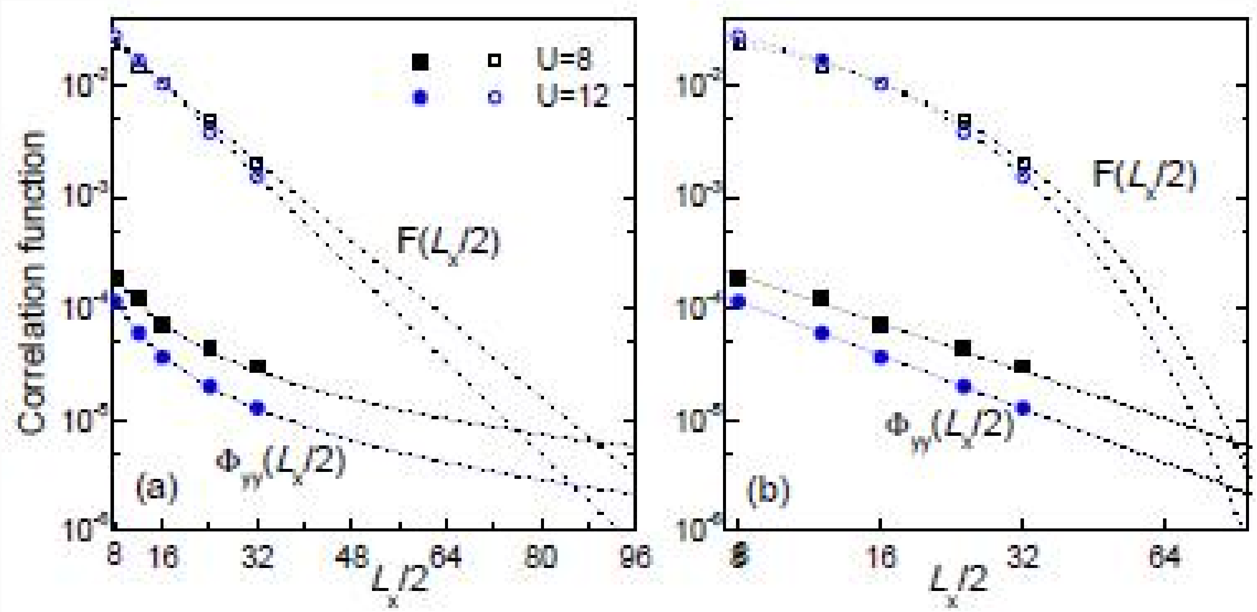
ü ***Quasi-long-range superconducting correlation***

HCJ, T. P. Devereaux, arXiv:1806.01465

Spin-density-wave correlation

$$F(x) = \frac{1}{L_y}\sum_{y=1}^{L_y} |\langle \vec{S}_{x_0,y} \cdot \vec{S}_{x_0+x,y} \rangle|$$

$$F(L_x/2) \sim e^{-L_x/2\xi_s}$$



- **Ø** Spin-spin correlation dominates relatively long short-range physics
- **Ø** Superconducting correlation dominates long-range physics

✓ Short-range spin-spin correlation with a finite spin gap and correlation length $\xi_s = 8 \sim 10$

HCJ, T. P. Devereaux, arXiv:1806.01465

Luther-Emery liquid state

Luther-Emery liquid is a 1D analog of superconductors, which exhibits a spin gap and no charge gap, in which both electron density-density and superconducting pair-pair correlation decay algebraically.

A. Luther and V. J. Emery, PRL 33, 589 (1974)
*L. Balents and M. P. A. Fisher, PRB **53**, 12133 (1996) ……*

$$A_{cdw}(L_x) \sim L_x^{-K/2}$$

$$\Phi(L_x/2) \sim (L_x/2)^{-1/K}$$

$$K_c = K$$

$$K_{sc} = \frac{1}{K}$$

$$K_c \times K_{sc} = 1$$

U	K_c	K_{sc}	K_cK_{sc}	ξ_s
8	0.90(6)	1.43(8)	1.3 (2)	9.8(6)
12	0.75(6)	1.60(7)	1.2(2)	8.3(4)

TABLE I: List of exponents K_c and K_{sc}, and spin-spin correlation length ξ_s of the Hubbard model at doping level $\delta = 12.5\%$ and $t' = -0.25$. Here $t = 1$.

✓ Ground state of doped Hubbard model at $\delta = 1/8$ doping is a Luther-Emery liquid

Summary and conclusion

Both Hubbard model and t-J model support superconductivity and CDW order, and can be considered as the starting point in understanding high-Tc superconductivity, such as cuprates. !

Thanks for your attention!